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Пgβ-connectedness in Intuitionistic Fuzzy Topological Spaces

T. Jenitha Premalatha[1, *], S. Jothimani[2]

[1]Department of Mathematics, Tips Global Institute, Coimbatore, India
[2]Department of Mathematics, Government Arts College, Coimbatore, India

Email address:
joel.jensi@gmail.com (T. J. Premalatha), jothimanis09@gmail.com (S. Jothimani)
[*]Corresponding author

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Abstract: The paper aspires to discuss the basic properties of connected spaces. Also the concept of types of intuitionistic fuzzy πgβ-connected and disconnected in intuitionistic fuzzy topological spaces are introduced and studied. The research paper of topological properties is introducedby making the idea of being connected. It turns out to be easier to think about the property that is the negation of connectedness, namely the property of disconnectedness and separable. Also the concepts of intuitionistic fuzzy $\pi g\beta C_5$-connectedness, intuitionistic fuzzy $\pi g\beta C_S$-connectedness, intuitionistic fuzzy $\pi g\beta C_M$-connectedness, intuitionistic fuzzy πgβ-strongly connectedness, intuitionistic fuzzyπ β-super connectedness and obtain several properties and some characterizations concerning connectedness in these spaces are explored.

Keywords: Intuitionistic Fuzzy Connected, Intuitionistic Fuzzy πgβ-connected, Intuitionistic Fuzzy $\pi g\beta C_5$-connectedness, Intuitionistic Fuzzy $\pi g\beta C_S$-connectedness, Intuitionistic Fuzzy $\pi g\beta C_M$-connectedness, Intuitionistic Fuzzy πgβ-Super Connectedness and Intuitionistic Fuzzy πgβ–strongly Connected

1. Introduction

A predominant characteristic of a topological space is the concept of connectedness and disconnectedness. The former is one of the topological properties that is used to distinguish topological spaces. Connectedness [3] is a powerful tool in topology. Many researchers have investigated the basic properties of connectedness. The first attempt to give a precise definition of these spaces was made by Weierstrass who in fact instigated the notion of arc wise connectedness. However, the notion of connectedness which is used today was introduced by Cantor (1883) in general topology, Later onZadeh [12] introduced thenotionof fuzzysets. Fuzzytopological space was further developed by Chang [5]. Coker [6] introduced the intuitionistic fuzzy topological spaces. Connectedness in intuitionistic fuzzy special topological spaces was introduced byOscag and Coker [6]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Turnali and Coker [11] and studies these spaces very extensively and also delved into various generalization too of these spaces. Recently JenithaPremalatha and Jothimani [7] proposed herald into a new class of sets called πgβ-closed sets in intuitionistic fuzzy topological space, and these concepts have been used to define and analyse many topological properties. The aim of this paper is to study πgβ-connectedness and the notions of Intuitionistic fuzzy πgβ-separated sets, Intuitionistic fuzzy πgβ-connectedness and Intuitionistic fuzzy πgβ-disconnectedness is dealt with in detail. Some of their types and their characterizations in Intuitionistic fuzzy topological spaces is studied. The problem focuses on the results when connectedness is replaced with πgβ–connectedness in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1: [2] An intuitionistic fuzzy (IF) set A in X is an objecthaving the form $A=\{<x, \mu_A(x), \nu_A(x)>/x\in X\}$ where $\mu_A(x)$ and $\nu_A(x)$ denote the degree ofmembership and non-membership respectively, and $0\leq \mu_A(x)+\nu_A(x) \leq 1$

Definition 2.2: [2] Let A and B be IFSs of the form A

$=\{<x, \mu_A(x), \nu_A(x)>/ x\in X\}$ and $B=\{<x, \mu_B(x), \nu_B(x)>/x\in X\}$. Then (i) $A\subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^c = \{< x, \nu_A(x), \mu_A(x)>/ x \in X\}$

(iv) $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)> / x \in X\}$

(v) $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)> / x\in X\}$.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim\in \tau$

(ii) $G_1\cap G_2\in \tau$ for any $G_1, G_2\in \tau$

(iii) $\cup G_i\in \tau$ for any family $\{G_i / i\in I\}\subseteq\tau$.

Definition 2.4: [4] Let (X,τ) be an IFTS and $A =<x, \mu_A, \nu_A>$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$Int(A) = \cup\{G / G$ is an IFOS in X and $G \subseteq A\}$

$Cl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$

Definition 2.5: An IF subset A is said to be IF regular open [8] if A = Int(Cl(A))

The finite union of IF regular open sets is said to be IFπ-open [8].

The complement of IFπ- open set is said to be IFπ- closed [8].

Definition 2.6: A is said to be IFβ-open [1] if $A\subseteq Cl(Int(Cl(A)))$. The family of all IFβ-open sets of X is denoted by IFβO(X).

The complement of a IFβ–open set is said to be IFβ–closed [1]. The intersection ofall IFβ–closed sets containing A is called IFβ–closure [2] of A, and is denoted by IFβ-Cl(A).

The IFβ-Interior [2] ofA, denoted by IFβ-Int(A), is defined as union of all IFβ-open sets contained in A.

It is well known IFβ-Cl(A) $=A\cup Int(Cl(Int(A)))$ andIFβ-Int(A)= $A\cap Cl(Int(Cl(A)))$.

Definition 2.7 ([6]): A mapping f: (X, τ)→(Y, σ) is called an intuitionistic fuzzy πgβ-continuousmappingif $f^{1}(V)$ is an IFπgβ-closed Set in (X,τ) for every IFCS V of (Y, σ).

Definition 2.8([6]): A mapping f: (X,τ)→(Y,σ) is called an intuitionistic fuzzy πgβ-irresolute if $f^{1}(V)$ is an IF-πgβ closed Setin (X,τ) for every IF-πgβ closed set V of (Y, σ).

Definition 2.9([10]): Two IFSs A andB in X are said to be q-coincident (AqB) if andonly if there exists an element $x\in X$ such that $\mu_A(x)>\nu_A(x)$ or)$\nu_B(x)<\mu_B(x)$

Definition2.10 ([10]): Two IFSs A and B in X are said to be not q-coincident (Aq^CB) if and only if $A\subseteq B^C$.

3. Intuitionisttic Fuzzy πGβ Connected Spaces and Its Types

Definition 3.1. [9]: Two subsets A and B in a IF space (X,τ) are said to be IF-πgβ -separated if and only if

$A\cap$πgβ-C l(B) =0 ~and πgβ-C l(A)$\cap$B= 0 ~. If A andB are not IF πgβ–separated then it is said to be intuitionistic fuzzy πgβ–connected.

Remark: 3.1: Each two IFπgβ-seperated sets are always disjoint, since $A\cap B\subseteq A\cap$πgβ-Cl(B)= 0 ~.

Theorem 3.1 Let A and B be nonempty sets in an IF space (X,τ). The following statements hold:

i) If A and B are IFπgβ-seperated and $A_1\subseteq A$ and $B_1\subseteq B$ then A and B1 are also IFπgβ-seperated.

ii) If $A\cap B$=0~ such that each of A and B are both πgβ-closed (πgβ-open) then A and B are IFπgβ separated.

iii) If each of Aand Bare bothIFπgβ -closed (πgβ -open) and if H =A∩(X−B) and G= B∩(X−A), then Hand G are IF πgβ -separated.

Proof:

(i) Since $A_1\subseteq A$, then πgβ-C l(A_1)$\subseteq$πgβ-Cl(A).

ThenB∩πgβ-C l(A) = 0 ~ implies $B_1\cap$ πgβ C l(A) = 0 ~ and $B_1\cap$ πgβ-C l(A_1) = 0 ~. Similarly $A_1\cap$ πgβ-Cl(B_1)= 0 ~. Hence $A_1$and B_1 are IFπgβ-separated.

(ii) Since A = πgβ-C l(A) and B = πgβ-Cl(B) and $A\cap B$=0 ~, then πgβ-C l(A) ∩B = 0 ~and πgβ-C l(B) ∩ A = 0 ~.

Hence A and Bare IFπgβ -separated. If A andBare IFπgβ -open, then their complements are IFπgβ -closed.

(iii) If A and B are IFπgβ -open, then X − A and X − B are IFπgβ-closed. Since $H\subseteq X\backslash B$, πgβ-Cl(H) $\subseteq$πgβ-Cl(X\B)=X\B and so πgβ-Cl(H)$\cap$B=0~. Thus G∩πgβ Cl(H) = 0 ~. Similarly, H∩πgβ- Cl(G) = 0 ~. Hence H and G are IFπgβ-separated.

Theorem 3.2: The sets A and B of a IF space X are IFπgβ -separated if and only if there exist U and V in IF πgβ-O(X) such that $A\subset U$, $B\subset V$ and $A\cap V$=0~, $B\cap U$=0~.

Proof: Let A and B be IFπgβ –separated sets. Let V = X\πgβ-Cl(A) and U = X\πgβ-Cl(B). Then U,V∈πGβO(X) Such that $A\subset U$, $B\subset V$ and $A\cap V$=0~, $B\cap U$=0~. On the other hand, let U,V∈πgβO(X) such that $A\subset U$, $B\subset V$, V and $A\cap V$=0~, $B\cap U$=0. Since X −V and X −U are IFπgβ-closed, then πgβ-Cl(A)$\subset$X\V$\subset$X\B and πgβ-Cl(B)$\subset$X\U$\subset$X\A.

Thus πgβ-Cl(A)$\cap$B=0~andπgβ-Cl(B)$\cap$A=0~.

Definition 3.2: A point $x\in X$ is called aIF πgβ-limit point of a set $A\subset X$, if every IFπgβ -open set $U\subset X$ containing x contains a point of A other than x.

Theorem 3.3: Let A and B be nonempty disjoint subsets of a spaceX and $E=A\cup B$. Then A and B are IFπgβ -separated if and only if each of A and B is IF πgβ -closed (IFπgβ -open) in E.

Proof: Let A and B are IFπgβ -separated sets. By Definition 3.1, A contains no IF πgβ -limit points of B. Then B contains all IFπgβ-limit points of B which are in $A\cup B$ and B is IFπgβ-closed in $A\cup B$.

Therefore B is IFπgβ -closed in E. Similarly A is IF πgβ -closed in E.

*Definition 3.3:*A subset S of a space X is said to be IFπgβ -connected relative to X if theredoes not exist two IF πgβ -separated subsets A and B relative to X and $S=A\cup B$, Otherwise S is said to be IFπgβ disconnected.

Definition 3.4: An intuitionistic fuzzy topological space (X,τ) issaid to be intuitionistic fuzzy πgβ-disconnected if there exists an intuitionistic fuzzy πgβ -open sets A, B in X, A≠0~, B≠0~, such that $A\cup B$=1~ and $A\cap B$=0~. If X is not IF πgβ-disconnected then it is said to be intuitionistic fuzzy πgβ–connected.

Theorem3.4: Let $A\subseteq B\cup C$ such that A be a nonempty

IFπgβ -connected set in a space X and B,C are IFπgβ-separated. Then only one of the following conditions holds:

(i) A⊆B and A∩C= 0 ~.

(ii) A ⊆ C and A∩B = 0 ~.

Proof: Since A∩C=0~, then A⊆B. Also, if A∩B=0 ~, then A⊆C. Since A⊆B∩C, then both A∩B=0~ and A∩C=0 ~cannot hold simultaneously. Similarly suppose thatA∩B= 0 ~and A∩C=0 ~then by Thorem 3.1 (i), A∩B and A∩C are IFπgβ- separated such that A=(A∩B)∪(A∩C) which contradicts with theI F π g β-connectedness of A. Hence one of the conditions (i) and (ii) must be hold.

Theorem 3.5: Let A and Bbe subsets in IF space (X,τ) such thatA ⊂ B ⊂IF πgβ-Cl(A). If A is IFπGβ-connected then B isIFπGβ -connected.

Proof: If B is IFπgβ -disconnected, then there exists two IFπgβ -separated subsets Uand V relative to X such that B =U∪V. Then either A⊆U or A⊆V. Let A⊆U. As A⊆U⊆B thenπgβ-C l_B(A)⊆πgβ-C l_B(U)⊆πgβ C l(U).

Also πgβ-C l_B(A) = B ∩πgβ C l(A) = B⊇πgβ C l(U).

This implies B= πgβ-C l(U).

So U and V are not IFπgβ -separated and B is IFπgβ-connected.

Theorem 3.6: If E is IF πgβ -connected, then IFπgβ-C l(E) isIFπgβ -connected.

Proof: By contradiction, suppose that IFπgβ-C l(E) is IF πgβ -disconnected. Then there are two nonempty IFπgβ -separated sets G and Hin X such that IFπgβ-Cl(E)= G∪H. Since E=(G∩E)∪(H∩E) and IFπgβ-C l(G∩E)⊂ IFπgβ-Cl(G) and IF πGβ-Cl(H∩E)⊂ IF πgβ-C l(H) and G∩H=0~, then (IF πgβ-Cl(G∩E))∩H=0~.

Hence (IF πgβ-Cl(G∩E))∩H ∩E)= 0~. Similarly Hence (IF πgβ-Cl(H∩E))∩G ∩E) = 0~.

Therefore Eis IF πgβ–disconnected.

Definition 3.5: An IFTS (X, τ) is said to be an intuitionistic fuzzy πgβ-connected space if the only IF sets which are bothintuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ closed are 0~ and 1~.

Definition 3.6: An IFTS (X, τ) in X is saidtobeIFπgβ C_5-disconnected, if there exists an IF set A

in X, which is both intuitionistic fuzzy πgβ open and intuitionistic fuzzy πgβ- closed, such that 0~≠A≠1~.

X is called C5 -connected, if X is not C5 –disconnected.

*Definition3.7([10]):*An IFTS (X, τ) is said to be an intuitionistic fuzzy C_5 – connected space if the only IFSs which are bothintuitionisticfuzzy open and intuitionistic fuzzy closed are 0~ and 1~.

Theorem 3.7: Every intuitionistic fuzzy πgβ- connected space is an intuitionistic fuzzy πgβ C_5 -connected space.

Proof: Let (X, τ) be an intuitionistic fuzzy πgβ-connected space. Suppose (X,τ) is notan intuitionistic fuzzy πgβ C_5 -connectedspace, thenthereexists a properIF set A which bothintuitionistic fuzzy πgβ open andintuitionistic fuzzy πgβ closed in (X,τ). This implies that (X, τ) is not an intuitionistic fuzzy πgβ-connected space. This is a contradiction. Therefore (X, τ) is an intuitionistic fuzzy πgβ C_5 -connected space.

Definition 3.8 ([10]): An IFTS (X, τ) is said to be an intuitionistic fuzzy GO-(IFGO)-connected space if the only IFSs which are bothintuitionisticfuzzy generalized open and intuitionistic fuzzy generalizedclosed are 0~ and 1~.

Theorem 3.8: Every intuitionistic fuzzy πgβ-connected space is an intuitionistic fuzzy GO-connected space.

Proof: Let (X,τ) be anintuitionistic fuzzy πgβ- connected space. Suppose (X,τ) is not an intuitionistic fuzzy GO-connected space, thenthereexists a properIFSA which both intuitionistic fuzzy open andintuitionistic fuzzy closed in (X, τ). That is A is both intuitionistic fuzzy πgβ-open andintuitionistic fuzzy πgβ-closedin (X,τ). This implies that (X,τ) is not an intuitionistic fuzzy πgβ-connected space. This is a contradiction. Therefore (X, τ) is an intuitionistic fuzzy GO-connected space.

Theorem 3.9: An IFTS (X,τ) is an intuitionistic fuzzy πgβ-connected space if and only if there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that $A=B^C$.

Proof: Necessity: Let A and B be two intuitionistic fuzzy πgβ open sets in (X, τ) such thatA≠0~≠B and $A=B^C$. Therefore $B^C$is an intuitionistic fuzzy πgβ-open set. Since A≠0 implies B≠1~. This impliesB is a proper IFS which is both intuitionistic fuzzy πgβ-open and Intuitionistic fuzzy πgβ-closedin (X,τ). Hence (X,τ) is not an intuitionistic fuzzy πgβ- connected space. But this is a contradiction to our hypothesis thatthereexistno non-zerointuitionisticfuzzy πgβ-open sets A and B.

Sufficiency: Let A be both intuitionistic fuzzy πgβ open and intuitionistic fuzzy πgβ-closed in (X, τ) such that 1~≠A≠0~. Now let$B=A^C$. Then B is an intuitionistic fuzzy πgβ-open set and A≠1~.

This implies $B=A^C$≠0~, which is a contradiction to our hypothesis. Therefore, (X,τ) is an intuitionistic fuzzy πgβ-connected space.

Theorem 3.10: An IFTS (X, τ) is an intuitionistic fuzzy πgβ-connected space if and only if there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that $A=B^C$, $B=(β\text{-Cl}(A))^C$ and $A=(β\text{-Cl}(B))^C$.

Proof. Necessity: Assume that there exist IF sets A and B such that A≠B≠0~, $A=B^C$, $B=(β\text{-Cl}(A))^C$ and $A=(β\text{-Cl}(B))^C$. Since $B=β\text{-Cl}(A)^C$ and $A=β\text{-Cl}(B)^C$ are intuitionistic fuzzy πgβ-open sets in (X,τ), A and B are intuitionisticfuzzy πgβ-open sets in (X,τ) This implies (X, τ) is not an intuitionistic fuzzy πgβ-connectedspace, which is a contradiction.

Therefore there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X,τ) such that $A=B^C$,$B=(β\text{-Cl}(A))^C$ and $A=(β\text{-Cl}(B))^C$.

Sufficiency: Let A be both intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed in (X,τ) such that 1~≠A≠0~. Now by taking $B=A^C$, will lead to the contradiction to our hypothesis. Hence (X,τ) is an intuitionistic fuzzy πgβ- connected space.

Definition 3.9: An IFTS (X,τ) is said to be an intuitionistic fuzzy πgβ-$T_{1/2}$ space if every intuitionistic fuzzy πgβ-closed set is an intuitionistic fuzzy closed set in (X,τ).

Theorem 3.11: Let (X,τ) be an intuitionistic fuzzy πgβ-$T_{1/2}$ space, then the following are equivalent.

(i) (X, τ) is an intuitionistic fuzzy πgβ- connectedspace.

(ii) (X, τ) is an intuitionistic fuzzy GO-connected space.

(iii) (X, τ) is an intuitionistic fuzzy $\pi g\beta C_5$ -connectedspace.

Proof: (i)→(ii) is obvious from Theorem3.8

(ii)→(iii) is obvious.

(iii)→(i) Let (X, τ) be an intuitionistic fuzzy $\pi g\beta C_5$ -connectedspace. Suppose (X, τ) is not an intuitionistic fuzzy πgβ-connectedspace, thenthereexists a proper IFS A in (X, τ) which is both intuitionistic fuzzy πgβ-open andintuitionistic fuzzy πgβ-closed in (X,τ). Since (X, τ) is an intuitionistic fuzzy πgβ -T ½space, is both intuitionistic fuzzy open an intuitionistic fuzzy closed in (X, τ). This implies tha (X,τ) is not an intuitionistic fuzzy $\pi g\beta C_5$ -connected space, which is a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy πgβ-connected space.

Theorem 3.12: If f: (X, τ)→(Y, σ) is an intuitionistic fuzzy πgβ-continuous surjection and (X,τ) is an intuitionistic fuzzy πgβ-connected space, then (Y, σ) is an intuitionistic fuzzy πgβ C_5 –connected space.

Proof: Let (X,τ) be anintuitionistic fuzzy πgβ-connectedspace. Suppose (Y,σ) is not an intuitionistic fuzzy πgβ C_5 -connected space, then there exists a proper IF Set A which is both intuitionistic fuzzy πgβ open and intuitionistic fuzzy πgβ closed in (Y, σ). Since f is an intuitionistic fuzzy πgβ-continuous mapping, $f^{-1}(A)$ is both intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed in (X,τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an intuitionistic fuzzy πgβ C_5 -connected space.

Theorem 3.13: If f: (X, τ)→(Y, σ) is an intuitionistic fuzzy πgβ-irresolute surjection and (X,τ) is an intuitionistic fuzzy πgβ- connected space, then (Y, σ) is also an intuitionistic fuzzy πgβ -connectedspace.

Proof: Suppose (Y, σ) is not an intuitionistic fuzzy πgβ-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy πgβ open and intuitionistic fuzzy πgβ-closed in (Y, σ). Since f is an intuitionistic fuzzy πgβ-irresolutemapping, $f^{-1}(A)$ is bothintuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed in (X,τ). But this is a contradiction to hypothesis. Hence (Y, σ) is an intuitionistic fuzzy πgβ- connected space.

Definition 3.10: An IFTS (X, τ) is called intuitionistic fuzzy $\pi g\beta C_5$ -connected between two IFSs A andB if thereis no intuitionistic fuzzy open set Ein (X,τ) such that $A\subseteq E$ and$Eq^C B$.

Definition 3.11: An IFTS(X, τ) is called intuitionistic fuzzy πgβ connectedbetweentwo IFSs A andB if thereis no intuitionistic fuzzy πgβ open set E in (X, τ) such that $A\subseteq E$ and $Eq^C B$.

Theorem 3.14: If an IFTS (X, τ) is intuitionistic fuzzy πgβ connected between two IFSs A and B, then it is intuitionistic fuzzy $\pi g\beta C_5$ –connected between two IF Sets A and B.

Proof: Suppose (X, τ) is not intuitionistic fuzzy πgβ C_5 –connected between A and B, then there exists an intuitionistic fuzzy open set E in (X,τ) such that $A\subseteq E$ and $Eq^C B$. Since every intuitionistic fuzzy open set is intuitionistic fuzzy πgβ-open set, there exists an intuitionistic fuzzy πgβ-open set E in (X, τ) such that $A\subseteq E$ and $Eq^C B$. This implies (X, τ) is not intuitionistic fuzzy πgβ-connectedbetween A and B, a contradiction to our hypothesis. Therefore (X, τ) is intuitionistic fuzzy πgβ C_5 -connected between A and B.

Theorem 3.15: An IFTS (X, τ) is intuitionistic fuzzy πgβ-connected between two IFSs A and B if and only if there is no intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed set E in (X,τ) such that $A\subseteq E \subseteq B^C$.

Proof: Necessity: Let (X,τ) be intuitionistic fuzzy πgβ-connected between two IF Sets A and B. Suppose that there exists an intuitionistic fuzzy πgβ- open andintuitionistic fuzzy πgβ- closed set Ein (X,τ) such that $A\subseteq E \subseteq B^C$, then $A\subseteq E$ and $Eq^C B$. This implies (X, τ) is not intuitionistic fuzzy πgβ-connected between A and B, by Definition 3.11, A contradiction to our hypothesis. Therefore there is no intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed setE in (X,τ) such that $A\subseteq E \subseteq B^C$.

Sufficiency: Suppose that (X, τ) is not intuitionistic fuzzy πgβ-connected between A and B. Then there exists an intuitionistic fuzzy- open set E in (X, τ) such that $A\subseteq E$ and$Eq^C B$. Thisimplies that there is no intuitionistic fuzzyπgβ-open set Ein (X, τ) such that $A\subseteq E \subseteq B^C$.

But this is a contradiction to our hypothesis. Hence (X,τ) is intuitionistic fuzzy πgβ connected between A and B.

Theorem 3.16: If an IFTS(X,τ) is intuitionistic fuzzy πgβ-connected between two IFSs A and B, $A\subseteq A_1$ and $B\subseteq B_1$, then (X, τ) is intuitionistic fuzzy πgβ- connectedbetween A_1 and B1.

*Proof:*Suppose that (X, τ) is not intuitionistic fuzzy πgβ connected between A1 and B1, then by Definition 3.11, there exists an intuitionistic fuzzy πgβ openset E in (X, τ) such that $A_1\subseteq E$ and $Eq^C B_1$. This implies $E\subseteq B^C$ and $A1\subseteq E$ implies $A\subseteq A1\subseteq E$. That is $A\subseteq E$. Now let us prove that $E \subseteq B^C$, that is letus prove $Eq^C B$. Suppose that EqB, then by Definition 2.9, there exists an element x in X such that μE (x) > νB (x) and νE (x) < μB (x). ThereforeμE (x)> νB (x) >νB_1 (x) and νE (x)< μB (x)< μB1 (x), since $B\subseteq B_1$

Thus EqB_1. But $E\subseteq B_1$. That is $Eq^C B_1$, which leads to a contradiction. Therefore $Eq^C B$.

That is $E\subseteq B^C$. Hence (X, τ) is not intuitionistic fuzzy πgβ connectedbetween A and B, which is a contradiction Thus (X, τ) is intuitionistic fuzzy πgβ connected between A1and B1.

Theorem 3.17: Let (X,τ) be anIFTSandA andBbe IFSs in (X,τ). If AqB, then (X,τ) is intuitionistic fuzzy πgβ connected between A and B.

Proof: Suppose (X,τ) is not intuitionistic fuzzy πgβ connectedbetween A and B. Then there exists an intuitionistic fuzzy πgβ open set Ein (X,τ) such that $A\subseteq E$ and $E\subseteq B^C$. This implies that $A\subseteq B^C$, That is $Aq^C B$. But this is a contradiction to our hypothesis.

Therefore (X,τ) is intuitionistic fuzzy πgβ connected between A and B.

Definition 3.12: An intuitionistic fuzzy πgβ-open set A is called an intuitionistic fuzzy regular πgβ-open set if A=πgβ-Int (πgβ-Cl(A)). The complement of an intuitionistic fuzzy regular πgβ-open set is called an intuitionistic fuzzy regular πgβ-closed set.

Definition 3.13: An IFTS(X, τ) is called an intuitionistic fuzzy πgβ–super connected space if there exists no intuitionistic fuzzy regular πgβ open set in (X, τ).

Theorem 3.18: Let (X, τ) be an IFTS, then the following are equivalent.

(i)(X,τ) is an intuitionistic fuzzy πgβ-superconnectedspace.

(ii)For every non-zero intuitionistic fuzzy regular πgβopen set A, πgβ-Cl(A)=1~.

(iii)For every intuitionistic fuzzy regular πgβ closed set A withA= 1~, πgβ-Int(A)= 0~.

(iv) There exists no intuitionistic fuzzy regular πgβ open sets A and B in (X,τ) such that A=0~=B, $A\subseteq B^C$.

(v)There exists no intuitionistic fuzzy regular πgβ open setsA and B in(X,τ), Such that A=0~ = B, B = πgβ-Cl(A), A= πgβ-Cl(B).

(vi) There exists no intuitionistic fuzzy regular πgβ closed sets A and B in (X,τ) such that A=1~=B, B= πgβ-Int(A), A= πgβ-Int (B).

*Proof: (i)⇒(ii)*Assume that there existes an intuitionistic fuzzy regular πgβ-open set A in (X,τ) such that A=0~ and πgβ-Cl(A)= 1~. Now let B= πgβ-Int(πgβ-Cl))^C. Then B is a proper intuitionistic fuzzy regular πgβ-open set in (X,τ). But this is a contradiction to the fact that (X,τ) is an intuitionistic fuzzy πgβ-super connected space. Therefore πgβ-Cl(A)= 1~.

(ii)⇒(iii) Let A=1~, be an intuitionistic fuzzy regular πgβ-closed set in (X,τ). If B= A^C, then B is anintuitionistic fuzzyregularπgβ-opensetin(X, τ) with B=0~. That is πgβ-Int(B^C)= 0~Hence πgβ-Int(A)= 0~.

(iii)⇒(iv) Let A and B be two intuionitic fuzzy regular πgβ open sets in (X,τ) such that A=0~=B, $A\subseteq B^C$.

Since B is an intuitionisticfuzzy regularπgβ-closed set in (X, τ) and B=0~, implies B=1~, B^C=πgβ-Cl(πgβ-Int(B^C)) and we have,πgβ-Int(B^C)= 0~. But $A\subseteq B^C$.

Therefore0~=A=πgβ-Int(πgβ-Cl(A))⊆πgβ-Int(πgβ-Cl(B^C)) =πgβ-Int(πgβ-Cl(πgβ-Cl(πgβ-Int(B^C))))=πgβ-Int(πgβ-Cl(πgβ-Int(B^C))) =πgβ -Int(B^C) =0~, Which is a contradiction. Therefore (iv) is true.

(iv)⇒(i) Let 0~=A=1~ be an intuitionistic fuzzy regular πgβ-open set in (X,τ).

If we take B=πgβ-cl(A))^C, then B is an intuitionistic fuzzy regular πgβ open set.

(iv)⇒(i) Let 0~=A=1~, be an intuitionistic fuzzy regular πgβ open set in (X,τ), If we take B=(πgβ-Cl(A))^C,

then B is an intuitionistic fuzzy regular πgβopen set, since πgβ-Int(πgβ-Cl(B))=πgβ-Int(πgβ-Cl(πgβ-Cl(A))^C) =πgβ-Int(πgβ-Int(πgβ-Cl(A)))^C=πgβ-Int(A^C)=πgβ-Cl(A))^C=B.

Also we getB=0~, thisimplies =(πgβ-Cl(A))^C,
Hence =πgβ-Cl(A)= 1~.
Hence A= πgβ-Int(πgβ-Cl(A))= πgβ-Int(1~)=1~.
This is A=1~, which is a contradiction.
Therefore B=0~ and $A\subseteq B^C$. But this is a contradiction to (iv). Therefore (X,τ) is anintuitionistic fuzzy πgβ-super connectedspace.

(i)⇒(v) Let A and B be two intuitionistic fuzzy regular πgβ open sets in (X,τ) such that A=0~, B=πgβ-Cl(A))^C and A = (πgβ -Cl(B))^C. We have πgβ -Int(πgβ -Cl(A)) =πgβ-Int(B^C) =(πgβ -Cl(B))^C =A, A=0~ andA=1~, since if A=1~, then 1~ =(πgβ -Cl(B))^C⇒πgβ-Cl(B)= 0 ~. ⇒B=0~. But B= 0~⇒ A=1~, which implies A is properIntuitionistic fuzzy regular πgβ-open set in (X,τ), which is a contradiction to (i). Hence (v) is true.

(v)⇒(i) Let A be an intuitionistic fuzzy regular πgβ- open set in (X,τ) such that A= πgβ-Int(πgβ -Cl(A) and 0~=A=1~. Now take B=πgβ -Cl(A)^C. In this case we get B= 0 ~ and B is intuitionistic fuzzy regular πgβ-open set in (X,τ) B= (πgβ-Cl(A))Cand(πgβ-Cl(B))^C =(πgβ -Cl(πgβ-Cl(A))^C)^C =πgβ-Int(πgβ-Cl(A)^C)^C =πgβ-Int(πgβ-Cl(A))=A. But this is a contradiction to(v). Therefore (X, τ) is an intuitionistic fuzzy πgβ -super connected space.

(v) ⇒(vi) Let A and B be two intuitionistic fuzzy regular πgβ closed sets in (X,τ) such that A=1~=B, B=(πgβ-Int(A))^C and A = (πgβ-Int(B))^C. Taking C=A^C and D=B^C, C and D become intuitionistic fuzzy regular πgβ open set in (X,τ) with C=0~=D, D=(πgβ-Cl(C)), C=πgβ-Cl(D))^C which is a contradiction to (v). Hence (vi) is true.

(vi)⇒(v) can be easily proved by the similar way as in (v)⇒(vi).

Definition 3.14: An IFTS (X,τ) is IF-πgβ-strongly connected if there exists no nonempty IF πgβ-closed sets A and B in X such that $\mu_A + \mu_B \subseteq 1$, $\nu_A + \nu_B \supseteq 1$. In other words, an IFTS (X,τ) is IF πgβ -strongly connected if there exists no nonempty IF πgβclosed sets A and B in X such that A∩B=0~.

Definition 3.15: An IFTS (X,τ) is IF πgβ-strongly connected if there exists no IF πgβ-open sets A and B in X, A≠ 1~≠ B such that $\mu_A + \mu_B \supseteq 1$, $\nu_A + \nu_B \subseteq 1$.

Theorem 3.19: Let f:(X,τ) →(Y,σ) be a IF πgβ -irresolute surjection. If X is an IF πgβ -strongly connected, then is also πgβ -strongly connected.

Proof: Suppose that Y is not IF πgβ -strongly connected then there exists IF πgβ –Closed Sets C and D in Y such that C ≠0 ~, D ≠0~, C∩D=0~. Since f is IF πgβ - irresolute, f^{-1}(C), f^{-1}(D) are IF πgβ-Closed Sets in X and f^{-1}(C)∩f^{-1}(D)=0~, f^{-1}(C) ≠0~, f^{-1} (D) ≠0~. (If f^{-1}(C) = 0~ then f(f^{-1}(C)) = C which implies f(0~) = C. So C = 0~ is a contradiction, Hence X is IF πgβ –strongly disconnected is a contradiction. Thus (Y,σ) is IF πgβ -strongly connected.

Definition 3.16: A and B are non-zero intuitionistic fuzzy sets in (X,τ). Then A and B are said to be IF πgβ-weakly separated if πgβ –Cl(A) ⊆ B^C and πgβ –Cl(B)⊆ A^C

(ii) IF πgβ-q-separated if (πgβ-Cl(A))∩B = 0~ = A∩(πgβ-Cl(B)).

Definition 3.17: An IFTS (X,τ) is said to be IF πgβ C_S-disconnected if there exists IFπgβ -weakly separated non-zero intuitionistic fuzzy sets A and B in (X,τ) such that A∪B=1~.

Definition 3.18: An IFTS (X,τ) is said to be IF C_M -disconnected if there exists IFπgβ-q-separated non-zero IFS's A and B in (X,τ) such that A∪B=1~.

Remark 3.2: An IFTS (X,τ) is be IF πgβ C_S-connected if and only if (X,τ) is IFπgβ C_M –connected.

Definition 3.19: An IFTS(X, τ) is said to be an intuitionistic fuzzy πgβ extremally disconnected if the πgβ closure of every intuitionistic fuzzy πgβ open set in (X, τ) is an intuitionistic fuzzy πgβ open set.

Theorem 3.20: Let (X, τ) be an intuitionistic fuzzy πgβ $T_{1/2}$ space, then the following are equivalent.

(i) (X,τ) is anintuitionistic fuzzy πgβ extremally disconnected space.

(ii) For each intuitionistic fuzzy πgβ closed set A, πgβ-Int(A) is an intuitionistic fuzzy πgβ closed set.

(iii) For each intuitionistic fuzzy πgβ open set A, $\pi g\beta cl(A)=(\pi g\beta cl(\pi g\beta cl(A))^C)^C$.

(iv) For each intuitionistic fuzzy πgβ open sets A and B with $\pi g\beta\text{-}Cl(A)=B^C$, $\pi g\beta\text{-}Cl(A)=\pi g\beta\text{-}Cl(B))^C$

Proof: (i) → (ii) Let A be any intuitionistic fuzzy πgβ closed set. Then A^C is an intuitionistic fuzzy πgβ open set. So (i) implies that $\pi g\beta\ \text{-}Cl(A^C)=(\pi g\beta\text{-}Int(A))^C$ is anintuitionistic fuzzy πgβ open set. Thus πgβ-Cl(A) is an intuitionistic fuzzy πgβ closed set in (X, τ).

(ii) →(iii) Let A be anintuitionistic fuzzy πgβ open set. Then wehave $\pi g\beta\text{-}Cl(\pi g\beta\text{-}Cl(A))^C=\pi g\beta\text{-}Cl(\pi g\beta\text{-}Int(A^{C)})$. Therefore $(\pi g\beta\text{-}Cl(\pi g\beta\text{-}Cl(A))^{C)C} =(\pi g\beta\text{-}Cl(\pi g\beta\ int(A^C)))^C$. Since A is an intuitionistic fuzzy πgβ open set, A^C is an intuitionistic fuzzy πgβ closed set. So by(ii) $\pi g\beta\text{-}Int(A^C)$ is an intuitionistic fuzzy πgβclosed set.
That is $\pi g\beta\text{-}Cl(\pi g\beta\text{-}Int(A^C)) = \pi g\beta\text{-}Int(A^C)$.
Hence $(\pi g\beta\text{-}Cl(\pi g\beta\text{-}Int(A^C)))^C=(\pi g\beta\text{-}Int(A^C))^C=\pi g\beta\text{-}Cl(A)$.

(iii)→(iv) Let A and B be any two intuitionistic fuzzy πgβ-open sets in (X,τ) such that $\pi g\beta\text{-}Cl(A)=B^C$.(iii) implies $\pi g\beta\text{-}Cl(A)=(\pi g\beta\text{-}Cl(\pi g\beta\text{-}Cl(A))^C)^C=(\pi g\beta\text{-}Cl(B^C)^C)^C=(\pi g\beta\text{-}Cl(B))^C$.

(iv) →(i) Let A be any intuitionistic fuzzy πgβ open set in (X, τ). Put $B=(\pi g\beta\text{-}Cl(A))^C$. Then $\pi g\beta\text{-}Cl(A)=B^C$. Hence by (iv), $\pi g\beta\text{-}Cl(A) = (\pi g\beta\text{-}Cl(B))^C$. Since πgβ-Cl(B) is an intuitionistic fuzzy πgβ-closed set as the space is an intuitionistic fuzzy πgβ-$T_{1/2}$space, it follows that πgβ-Cl(A) is an intuitionisticfuzzy πgβ open set. This implies that (X,τ) is an intuitionistic fuzzy πgβ extremally disconnected space.

4. Applications

While focusing on some applications of connectedness, fixed point theorems in connection with application of connectedness. Fixed point theorems are useful in obtaining the (unique) solutions of differential and integral equations. In robotic motion planning, the connectedness of the configuration space conveys that one can reach the desired arrangement of solid objects from any initial arrangement.

5. Conclusion

The πgβ closed sets are used to introduce the concepts πgβ -connected space. Also, the characterization and thetypes of πgβ -connected spaces have been framed and analyzed. In general, the entire content will be a successful tool for the researchers for finding the path to obtain the results in the context of connected spaces in bi topologyand can be extended to Group theory. Also it is believed that this approach will prove useful for studying structures in the phase space of dynamical systems.

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