Πgβ-connectedness in Intuitionistic Fuzzy Topological Spaces

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Abstract: The paper aspires to discuss the basic properties of connected spaces. Also the concept of types of intuitionistic fuzzy Πgβ-connected and disconnected in intuitionistic fuzzy topological spaces are introduced and studied. The research paper of topological properties is introduced by making the idea of being connected. It turns out to be easier to think about the property that is the negation of connectedness, namely the property of disconnectedness and separable. Also the concepts of intuitionistic fuzzy ΠgβC5-connectedness, intuitionistic fuzzy ΠgβC5-connectedness, intuitionistic fuzzy Πgβ-connectedness, intuitionistic fuzzy Πgβ-strongly connected, intuitionistic fuzzy Πgβ-M-connectedness, intuitionistic fuzzy Πgβ-Super Connectedness, and intuitionistic fuzzy Πgβ-strongly Connected into various generalization too of these spaces. Recently Jenitha Premalatha and Jothimani [7] proposed herald into a new class of sets called Πgβ-closed sets in intuitionistic fuzzy topological space, and these concepts have been used to define and analyse many topological properties. The aim of this paper is to study Πgβ-connectedness and the notions of Intuitionistic Fuzzy Πgβ-separated sets, Intuitionistic Fuzzy ΠgβM-connectedness, and Intuitionistic Fuzzy Πgβ-disconnectedness is dealt with in detail. Some of their types and their characterizations in Intuitionistic fuzzy topological spaces is studied. The problem focuses on the results when connectedness is replaced with Πgβ–connectedness in intuitionistic fuzzy topological spaces.

Keywords: Intuitionistic Fuzzy Connected, Intuitionistic Fuzzy Πgβ-connected, Intuitionistic Fuzzy ΠgβC5-connectedness, Intuitionistic Fuzzy ΠgβC5-connectedness, Intuitionistic Fuzzy ΠgβM-connectedness, Intuitionistic Fuzzy Πgβ-Super Connected Sets, and Intuitionistic Fuzzy Πgβ-strongly Connected

1. Introduction

A predominant characteristic of a topological space is the concept of connectedness and disconnectedness. The former is one of the topological properties that is used to distinguish topological spaces. Connectedness [3] is a powerful tool in topology. Many researchers have investigated the basic properties of connectedness. The first attempt to give a precise definition of these spaces was made by Weierstrass who in fact instigated the notion of arc wise connectedness. However, the notion of connectedness which is used today was introduced by Cantor (1883) in general topology. Later on Zadeh [12] introduced the notion of fuzzy sets. Fuzzy topological space was further developed by Chang [5]. Coker [6] introduced the intuitionistic fuzzy topological spaces. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Ooscag and Coker [6]. Several types of fuzzy connectedness in intuitionistic fuzzy topological spaces were defined by Tursali and Coker [11] and studies these spaces very extensively and also delved into various generalization too of these spaces. Recently Jenitha Premalatha and Jothimani [7] proposed herald into a new class of sets called Πgβ-closed sets in intuitionistic fuzzy topological space, and these concepts have been used to define and analyse many topological properties. The aim of this paper is to study Πgβ-connectedness and the notions of Intuitionistic Fuzzy Πgβ-separated sets, Intuitionistic Fuzzy Πgβ-connectedness, and Intuitionistic fuzzy Πgβ-disconnectedness is dealt with in detail. Some of their types and their characterizations in Intuitionistic fuzzy topological spaces is studied. The problem focuses on the results when connectedness is replaced with Πgβ–connectedness in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1: [2] An intuitionistic fuzzy (IF) set A in X is an object having the form A=<x, μA(x), νA(x)>/x∈X where μA(x) and νA(x) denote the degree of membership and non-membership respectively, and 0≤μA(x)+νA(x)≤1

Definition 2.2: [2] Let A and B be IFSs of the form A
= \{< x, \mu_A(x), \nu_A(x) >/ x \in X \} \text{ and } B = \{< x, \mu_B(x), \nu_B(x) >/ x \in X \}. \text{ Then (i) } A \subset B \text{ if and only if } \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \text{ for all } x \in X.

(i) \ A = B \text{ if and only if } A \subset B \text{ and } B \subset A

(ii) \ A \cap B = \{< x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x) >/ x \in X \}

(iii) \ A \supset B = \{< x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x) >/ x \in X \}.

Definition 2.3: \[4\] An intuitionistic fuzzy topology (IFT) for short on X is a family τ of IFSs in X satisfying the following axioms.

(i) \ O, 1 \in \tau

(ii) G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau

(iii) \cup \{G_i / i \in I \} \in \tau.

Definition 2.4: \[4\] Let (X, τ) be an IFTS and A = < x, \mu, \nu > be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

Int(A) = \bigcup \{G / G \text{ is an IFOS in } X \text{ and } G \subset A \}

Cl(A) = \bigcap \{K / K \text{ is an IFCS in } X \text{ and } A \subset K \}

Definition 2.5: An IF subset A is said to be IF regular open [8] if A = Int(Cl(A)). The finite union of IF regular open sets is said to be IF\pi-open [8].

The complement of IF\pi-open set is said to be IF\pi-closed [8].

Definition 2.6: A is said to be IF\beta-open [1] if A = Int(Cl(Cl(A))). The family of all IF\beta-open sets of X is denoted by IF\betaO(X).

The complement of a IF\beta-open set is said to be IF\beta-closed [1].

The intersection of all IF\beta-closed sets containing A is called IF\beta-closure [2] of A, and is denoted by IF\beta-Cl(A).

The IF\beta-Interior [2] of A, denoted by IF\beta-Int(A), is defined as union of all IF\beta-open sets contained in A.

It is well known IF\beta-Cl(A) = A \cup Int(Cl(Cl(A))) and IF\beta-Int(A) = A \cap Cl(Cl(Cl(A))).

Definition 2.7 (f\beta): A mapping f: (X, \tau) \rightarrow (Y, \sigma) is called an intuitionistic fuzzy \text{ f\beta}-continuous mapping f\beta(V) is an IF\text{ f\beta}-closed Set in (X, \tau) for every IFCS V of (Y, \sigma).

Definition 2.8 (f\beta): A mapping f: (X, \tau) \rightarrow (Y, \sigma) is called an intuitionistic fuzzy \text{ f\beta}-irresolute if f\beta(V) is an IF-\text{f\beta} closed Setin (X, \tau) for every IF-\text{f\beta}-closed set V of (Y, \sigma).

Definition 2.9 (\beta): Two IF\beta sets A and B in X are said to be q-coincident (AQ\beta) if and only if there exists an element x in X such that \mu_A(x) \land \nu_A(x) \lor \nu_B(x) \land \mu_B(x)

Definition 2.10 (\beta): Two IF\beta sets A and B in X are said to be not q-coincident (AQ\beta B) if and only if A \subset B.

3. Intuitionistic Fuzzy \pi\G\beta Connected Spaces and Its Types

Definition 3.1: \[9\] Two subsets A and B in a IF space (X, \tau) are said to be IF\pi\G\beta-separated if and only if

A \cap \text{Cl}(B) = \emptyset \text{ and } \text{Cl}(A) \cap B = \emptyset.

Remark: 3.1: Each two IF\pi\G\beta-seperated sets are always disjoint, since A \cap B \subset A \cap \text{Cl}(B) = \emptyset.

Theorem 3.1 Let A and B be nonempty sets in an IF space (X, \tau). The following statements hold:

(i) If A and B are IF\pi\G\beta-separated and A \subset A \cap B \subset B then A and B are also IF\pi\G\beta-separated.

(ii) If A \cap B \neq \emptyset \text{ such that each of A and B are both } \pi\G\beta-closed then A and B are IF\pi\G\beta separated.

(iii) Each of A and B are both IF\pi\G\beta-closed (\pi\G\beta-open) and if \text{H=Cl}(X-B) \text{ and } G = \text{Cl}(X-A), then Hand G are IF\pi\G\beta-separated.

Proof:

(i) Since A \subset A \cap B \subset B \text{ then } \text{Cl}(A) \subset \text{Cl}(A) \cap B= \emptyset \text{ and } B \subset \text{Cl}(B).

Hence A and B are IF\pi\G\beta-separated.

(ii) Since A = \text{Cl}(A) \cap B and B = \text{Cl}(B) \cap A \text{ and } A \cap B = \emptyset, then \text{Cl}(A) \cap B \neq \emptyset \text{ and } B \subset \text{Cl}(B) \cap A = \emptyset.

Hence A and B are IF\pi\G\beta-separated. If A and B are IF\pi\G\beta-open then their complements are IF\pi\G\beta-closed.

(iii) If A and B are IF\pi\G\beta-open, then X \setminus A \text{ and } X \setminus B \text{ are IF\pi\G\beta-closed. Since } H \subset X-B, \text{Cl}(H) \subset \text{Cl}(X-B) = X-B \text{ and } G \subset \text{Cl}(H) \subset X-B.

Thus G \cap \text{Cl}(H) = \emptyset. Similarly, H \cap \text{Cl}(G) \subset \text{Cl}(H) \subset X-B.

Thus \text{Cl}(A) \cap \text{Cl}(B) \subset \text{Cl}(H) \subset X-B.

Definition 3.2: A point x \in \text{Cl}(X) is called an IF\pi\G\beta-limit point of a set A \subset X, if every IF\pi\G\beta-open set \text{U \subset X} containing x contains a point of A other than x.

Theorem 3.3: Let A and B be nonempty disjoint subsets of a space E and A \setminus B \subset A \subset E. Then A and B are IF\pi\G\beta-separated if and only if each of A and B is IF\pi\G\beta-closed (IF\pi\G\beta-open) in E.

Proof: Let A and B be IF\pi\G\beta-separated sets. By Definition 3.1, A contains no IF\pi\G\beta-limit points of B. Then B contains all IF\pi\G\beta-limit points of B which are in A \setminus B and B is IF\pi\G\beta-closed in A \setminus B.

Therefore B is IF\pi\G\beta-closed in E. Similarly A is IF\pi\G\beta-closed in E.

Definition 3.3: A subset S of a space X is said to be IF\pi\G\beta-connected relative to X if theredoes not exist two IF\pi\G\beta-separated subsets A and B relative to X and S = \text{A \setminus B}.

Otherwise S is said to be IF\pi\G\beta disconnected.

Definition 3.4: An intuitionistic fuzzy topological space (X, \tau) issaid to be intuitionistic fuzzy \text{f\beta}-disconnected if there exists an intuitionistic fuzzy \text{f\beta}-open set A, B in X, A \cap B = \emptyset.

If X is not IF\pi\G\beta-connected then it is said to be intuitionistic fuzzy \text{f\beta}-connected.

Theorem 3.4: Let A \subset B \subset C such that A be a nonempty
By intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed sets. Therefore there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B such that A=B. The space is called intuitionistic fuzzy πgβ-connected if and only if there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that A=B. Let A and B be two intuitionistic fuzzy πgβ-open sets in (X, τ) such that A=B. Therefore B is an intuitionistic fuzzy πgβ-open set. Since A=B implies B=B. This implies B is a proper IFS which is both intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed (X, τ). Hence (X, τ) is not an intuitionistic fuzzy πgβ-connected space. But this is a contradiction to our hypothesis that there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B.

Sufficiency: Let A be both intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed in (X, τ) such that A=0. Now let B=A. Then B is an intuitionistic fuzzy πgβ-open set and A=1. This implies B=A≠0, which is a contradiction to our hypothesis. Therefore, (X, τ) is an intuitionistic fuzzy πgβ-connected space.

Theorem 3.10: An IFTS (X, τ) is an intuitionistic fuzzy πgβ-connected space if and only if there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that A=B, B=(β-Cl(A))^c and A=(β-Cl(B))^c.

Proof: Necessity: Assume that there exist IFS sets A and B such that A=B≠0, A=B, B=(β-Cl(A))^c and A=(β-Cl(B))^c. Since B=(β-Cl(A))^c and A=(β-Cl(B))^c are intuitionistic fuzzy πgβ-open sets in (X, τ), A and B are intuitionistic fuzzy πgβ-open sets in (X, τ). This implies that (X, τ) is not an intuitionistic fuzzy πgβ-connected space, which is a contradiction.

Therefore there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that A=B, B=(β-Cl(A))^c and A=(β-Cl(B))^c. Sufficiency: Let A be both intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-connected in (X, τ) such that A=0. Now by taking B=A, will lead to the contradiction to our hypothesis. Hence (X, τ) is an intuitionistic fuzzy πgβ-connected space.

Definition 3.9: An IFTS (X, τ) is said to be an intuitionistic fuzzy πgβ-connected space if and only if every intuitionistic fuzzy πgβ-closed set in a space X and B, C are intuitionistic fuzzy πgβ-separated. Then one of the following conditions holds:

(i) \( A \subseteq B \) and \( A \cap C = 0 \).
(ii) \( A \subseteq C \) and \( A \cap B = 0 \).

Proof: Suppose \((A \subseteq B) \) and \( A \cap C = 0 \). Therefore there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that A=B. The space is called intuitionistic fuzzy πgβ-connected if and only if there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B in (X, τ) such that A=B. Let A and B be two intuitionistic fuzzy πgβ-open sets in (X, τ) such that A=B. Therefore B is an intuitionistic fuzzy πgβ-open set. Since A=0 implies B=0. This implies B is a proper IFS which is both intuitionistic fuzzy πgβ-open and intuitionistic fuzzy πgβ-closed (X, τ). Hence (X, τ) is not an intuitionistic fuzzy πgβ-connected space. But this is a contradiction to our hypothesis that there exist no non-zero intuitionistic fuzzy πgβ-open sets A and B.
(ii) \((X, \tau)\) is an intuitionistic fuzzy GO-connected space.
(iii) \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space.

**Proof:** (i)\(\rightarrow\)(ii) is obvious from Theorem 3.8
(ii)\(\rightarrow\)(iii) is obvious.
(iii)\(\rightarrow\)(i) Let \((X, \tau)\) be an intuitionistic fuzzy \(\pi\beta\)-connected space. Suppose \((X, \tau)\) is not an intuitionistic fuzzy \(\pi\beta\)-connected space, then there exists a proper IFS \(A\) in \((X, \tau)\) which is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). Since \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space, it is both intuitionistic fuzzy open and intuitionistic fuzzy closed in \((X, \tau)\). This implies that \((X, \tau)\) is not an intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space, which is a contradiction to our hypothesis. Therefore \((X, \tau)\) is intuitionistic fuzzy \(\pi\beta\)-connected space.

**Theorem 3.12:** If \(f: (X, \tau)\rightarrow(Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-continuous surjection and \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space, then \((Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space.

**Proof:** Let \((X, \tau)\) be intuitionistic fuzzy \(\pi\beta\)-connected space. Suppose \((Y, \sigma)\) is not an intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space, then there exists a proper IFS \(A\) which is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((Y, \sigma)\). Since \(f\) is an intuitionistic fuzzy \(\pi\beta\)-continuous mapping, \(f^{-1}(A)\) is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). But this is a contradiction to hypothesis. Hence \((Y, \sigma)\) is intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space.

**Theorem 3.13:** If \(f: (X, \tau)\rightarrow(Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-irresolute surjection and \((X, \tau)\) is an intuitionistic fuzzy \(\pi\beta\)-connected space, then \((Y, \sigma)\) is an intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space.

**Proof:** Suppose \((Y, \sigma)\) is not an intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space, then there exists a proper IFS \(A\) which is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((Y, \sigma)\). Since \(f\) is an intuitionistic fuzzy \(\pi\beta\)-irresolutesumming, \(f^{-1}(A)\) is both intuitionistic fuzzy \(\pi\beta\)-open and intuitionistic fuzzy \(\pi\beta\)-closed in \((X, \tau)\). But this is a contradiction to hypothesis. Hence \((Y, \sigma)\) is intuitionistic fuzzy \(\pi\beta\)-\(\frac{1}{2}\)space.

**Definition 3.10:** An IFTS \((X, \tau)\) is called intuitionistic fuzzy \(\pi\beta\)-open set if \(A\subseteq\pi\beta\text{Int}(\pi\beta\text{Cl}(A))\). The complement of an intuitionistic fuzzy regular \(\pi\beta\)-open set is called an intuitionistic fuzzy regular \(\pi\beta\)-closed set.
**Definition 3.13:** An IFTS($X, \tau$) is called an intuitionistic fuzzy $\pi g\beta$-super connected space if there exists no intuitionistic fuzzy regular $\pi g\beta$ open set in ($X, \tau$).

**Theorem 3.18:** Let ($X, \tau$) be an IFTS, then the following are equivalent.

(i)(X, $\tau$) is an intuitionistic fuzzy $\pi g\beta$-superconnected space.

(ii) For every non-zero intuitionistic fuzzy regular $\pi g\beta$ open set $A$, $\pi g\beta-Cl(A) = 1$.

(iii) For every intuitionistic fuzzy regular $\pi g\beta$ closed set $A$ with $A = 1$, $\pi g\beta-Int(A) = 0$.

(iv) There exists no intuitionistic fuzzy regular $\pi g\beta$ open sets $A$ and $B$ in ($X, \tau$) such that $A = 0 \Rightarrow B = \pi g\beta-Cl(A)$, $A = \pi g\beta-Int(B)$.

(v) There exists no intuitionistic fuzzy regular $\pi g\beta$ closed sets $A$ and $B$ in ($X, \tau$) such that $A = 1 \Rightarrow B = \pi g\beta-Int(A)$, $A = \pi g\beta-Int(B)$.

**Proof:** (i)$\Rightarrow$(ii) Assume that there exists an intuitionistic fuzzy regular $\pi g\beta$-open set $A$ in ($X, \tau$) such that $A = 0$ and $\pi g\beta-Cl(A) = 1$. Now let $B = \pi g\beta-Int(\pi g\beta-Cl(A))$. Then $B$ is a proper intuitionistic fuzzy regular $\pi g\beta$-open set in ($X, \tau$).

This is a contradiction to the fact that ($X, \tau$) is an intuitionistic fuzzy $\pi g\beta$-super connected space. Therefore $\pi g\beta-Cl(A) = 1$.

(ii)$\Rightarrow$(iii) Let $A = 1$, be an intuitionistic fuzzy regular $\pi g\beta$-closed set in ($X, \tau$). If $B = A^c$, then $B$ is an intuitionistic fuzzy regular $\pi g\beta$-open set in ($X, \tau$) with $B = 0$. That is $\pi g\beta-Int(B) = 0$. Hence $\pi g\beta-Int(A) = 0$.

(iii)$\Rightarrow$(iv) Let $A$ and $B$ be two intuitionistic fuzzy regular $\pi g\beta$ open sets in ($X, \tau$) such that $A = 0$ and $\pi g\beta-Cl(A) = 1$. Now let $B = \pi g\beta-Int(\pi g\beta-Cl(A))$. Then $B$ is a proper intuitionistic fuzzy regular $\pi g\beta$-open set in ($X, \tau$).

But this is a contradiction to (ii). Hence ($X, \tau$) is an intuitionistic fuzzy $\pi g\beta$-super connected space.

**Definition 3.14:** An IFTS ($X, \tau$) is IF $\pi g\beta$-strongly connected if there exists no nonempty IF $\pi g\beta$-closed sets $A$ and $B$ in $X$ such that $\mu_A + \mu_B \geq 1$. In other words, an IFTS ($X, \tau$) is IF $\pi g\beta$-strongly connected if there exists no nonempty IF $\pi g\beta$-closed sets $A$ and $B$ in $X$ such that $A \cap B = 0$.

**Definition 3.15:** An IFTS ($X, \tau$) is IF $\pi g\beta$-strongly connected if there exists if no IF $\pi g\beta$-open sets $A$ and $B$ in $X$, $A \cap B = 0$ such that $\mu_A + \mu_B \geq 1$. In other words, an IFTS ($X, \tau$) is IF $\pi g\beta$-strongly connected if there exists if no IF $\pi g\beta$-open sets $A$ and $B$ in $X$, $A \cap B = 0$.

**Theorem 3.19:** Let $f(X, \tau)$ $\longrightarrow (Y, \sigma)$ be a IF $\pi g\beta$-irresolute surjection. If $X$ is an IF $\pi g\beta$-strongly connected, then is also IF $\pi g\beta$-strongly connected.

**Proof:** Suppose that $Y$ is not IF $\pi g\beta$-strongly connected then there exists IF $\pi g\beta$-Closed sets $C$ and $D$ in $Y$ such that $C \cap D = 0$. Since $f$ is IF $\pi g\beta$- irresolute, $f^{-1}(C)$, $f^{-1}(D)$ are IF $\pi g\beta$-Closed sets in $X$ and $f^{-1}(C) \cap f^{-1}(D) = 0$. If $f^{-1}(C) = 0$, then $f^{-1}(D) = C$ which implies $f(0) = C$. So $0 = C$ is a contradiction, Hence $X$ is IF $\pi g\beta$-strongly disconnected is a contradiction. Thus ($Y, \sigma$) is IF $\pi g\beta$-strongly connected.

**Definition 3.16:** A and $B$ are non-zero intuitionistic fuzzy sets in ($X, \tau$). Then $A$ and $B$ are said to be IF $\pi g\beta$-weakly separated if $\pi g\beta-Cl(A) \subseteq B$ and $\pi g\beta-Cl(B) \subseteq A^c$.

(ii) IF $\pi g\beta$-q-separated if ($\pi g\beta-Cl(A)) \cap B = 0 \Rightarrow (\pi g\beta-Cl(B)) \cap A = 0$.

**Definition 3.17:** An IFTS ($X, \tau$) is said to be IF $\pi g\beta$ $C_2$-disconnected if there exists IF $\pi g\beta$-weakly separated non-zero intuitionistic fuzzy sets $A$ and $B$ in ($X, \tau$) such that $A \cup B = 1$.

**Definition 3.18:** An IFTS ($X, \tau$) is said to be IF $C_M$ - disconnected if there exists IF $\pi g\beta$-q-separated non-zero IF's $A$ and $B$ in ($X, \tau$) such that $A \cup B = 1$.

**Remark 3.2:** An IFTS ($X, \tau$) is be IF $\pi g\beta$ $C_3$-connected if and only if ($X, \tau$) is IF $\pi g\beta$ $C_M$ - connected.

**Definition 3.19:** An IFTS($X, \tau$) is said to be an
intuitionistic fuzzy πgβ extremally disconnected if the πgβ closure of every intuitionistic fuzzy πgβ open set in (X, τ) is an intuitionistic fuzzy πgβ open set.

Theorem 3.20: Let (X, τ) be an intuitionistic fuzzy πgβ T_1/2 space, then the following are equivalent.

(i) (X, τ) is an intuitionistic fuzzy πgβ extremally disconnected space.

(ii) For each intuitionistic fuzzy πgβ closed set A, πgβ-Int(A) is an intuitionistic fuzzy πgβ closed set.

(iii) For each intuitionistic fuzzy πgβ open set A, πgβcl(A)=(πgβcl(πgβcl(A))^C)

(iv) For each intuitionistic fuzzy πgβ open sets A and B with πgβ-Cl(A)=B^C, πgβ-Cl(A)=πgβ-Cl(B)^C

Proof: (i) → (ii) Let A be any intuitionistic fuzzy πgβ closed set. Then A^C is an intuitionistic fuzzy πgβ open set. So (i) implies that πgβ -Cl(A^C)=(πgβ-Int(A))^C is an intuitionistic fuzzy πgβ open set. Thus πgβ-Cl(A) is an intuitionistic fuzzy πgβ closed set in (X, τ).

(ii) →(iii) Let A be an intuitionistic fuzzy πgβ open set. Then we have πgβ-Cl(πgβ-Cl(A))=πgβ-Cl(πgβ-Int(A)^C). Therefore (πgβ-Cl(πgβ-Cl(A))^C=(πgβ-Int(πgβ-Cl(A)^C))^C. Since A is an intuitionistic fuzzy πgβ open set, A^C is an intuitionistic fuzzy πgβ closed set. So by(ii) πgβ-Int(A^C) is an intuitionistic fuzzy πgβ closed set. That is πgβ-Cl(πgβ-Int(A^C))=πgβ-Int(A^C).

(iii)→(iv) Let A and B be any two intuitionistic fuzzy πgβ-open sets in (X, τ) such that πgβ-Cl(A)=B^C. (iii) implies πgβ-Cl(A)=(πgβ-Cl(πgβ-Cl(A))^C=πgβ-Cl(πgβ-Cl(B)^C)^C. Hence by (iv), πgβ-Cl(A) = (πgβ-Cl(B))^C. Since πgβ-Cl(B) is an intuitionistic fuzzy πgβ-closed set as the space is an intuitionistic fuzzy πgβT_1-space, it follows that πgβ-Cl(A) is an intuitionistic fuzzy πgβ open set. This implies that (X, τ) is an intuitionistic fuzzy πgβ extremely disconnected space.

4. Applications

While focusing on some applications of connectedness, fixed point theorems in connection with application of connectedness. Fixed point theorems are useful in obtaining the (unique) solutions of differential and integral equations. In robotic motion planning, the connectedness of the configuration space conveys that one can reach the desired arrangement of solid objects from any initial arrangement.

5. Conclusion

The πgβ closed sets are used to introduce the concepts πgβ-connected spaces. Also, the characterization and the types of πgβ-connected spaces have been framed and analyzed. In general, the entire content will be a successful tool for the researchers for finding the path to obtain the results in the context of connected spaces in bi topology and can be extended to Group theory. Also it is believed that this approach will prove useful for studying structures in the phase space of dynamical systems.

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