Mathematics Letters
2018; 4(2): 25-33
http://www.sciencepublishinggroup.com/j/ml
doi: 10.11648/j.ml.20180402.11
ISSN: 2575-503X (Print); ISSN: 2575-5056 (Online)

A Proposed New Average Method for Solving Multi-Objective Linear Programming Problem Using Various Kinds of Mean Techniques

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To cite this article:
Zahidul Islam Sohag, Md. Asadujjaman. A Proposed New Average Method for Solving Multi-Objective Linear Programming Problem Using Various Kinds of Mean Techniques. *Mathematics Letters*. Vol. 4, No. 2, 2018, pp. 25-33. doi: 10.11648/j.ml.20180402.11

Received: April 22, 2018; **Accepted**: May 24, 2018; **Published**: June 14, 2018

Abstract: In this paper, we initiated New Average Method with various kinds of mean techniques to solve multi-objective linear programming problem. In this method multi-objective functions are converted into single objective function by using different kinds of mean techniques. Also an algorithm of New Average Method is suggested for solving multi-objective linear programming problem. We illustrate numerical problem using Chandra Sen's Method, Average Method and New Average Method. The numerical result in this paper indicates that New Average Method gives promising result than Chandra Sen's Method and Average Method. Also we observed that, in New Average Method, Harmonic mean technique gives better result than other mean techniques like as Quadratic mean, Arithmetic mean, Identric mean, Logarithmic mean, Geometric mean techniques.

Keywords: Multi-objective Linear Programming Problem, Average Method, New Average Method, Harmonic Mean

1. Introduction

Linear Programming problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. In different sectors like design, construction, maintenance, producing planning, financial and corporate planning and engineering, decision makers have to take decisions and their ultimate goal is to minimize effort or maximize profit. Linear programming problem is formed with a cost or profit function with some constraints conditions, where a single cost or profit function need to be optimized [1]. However, in many situations, decision makers want to optimize several different objective functions at the same time under same constraints conditions. This leads to Multi-Objective concept. It is seen that if the multiple objective functions are not similar to each other, then this problem becomes more critical. There have been many methods suggested for Multi-objective linear programming problem (MOLPP).

A study of multi objective linear programming problem is introduced by Chandra Sen. In which multi- objective function are converted into single objective function with limitation that, individually each objective function optimum value must be greater than zero [2]. Using mean and median solving multi objective programming problem is studied by Sulaiman and Sadiq [3]. Sulaiman and Mustafa also used Harmonic mean to solve MOLPP [4]. A new geometric average technique is studied to solve MOLFPP by Nahar and Alim [5]. They also proposed a Statistical Average Method using Arithmetic, Geometric and Harmonic mean [6].

In order to extend this work, in this paper we propose an algorithm to solve MOLPP with New Average Method with various types of mean techniques and compare the numerical result with Chandra Sen's Method and Average Method. New Average Method gives better result than Chandra Sen's Method and Average Method. Among all New Harmonic mean technique gives the best result.

2. Mathematical Formulation of MOLPP

Mathematical general form of MOLPP is given as:

$$Max\ Z_1 = C_1^t X + \alpha_1$$

$$Max\ Z_2 = C_2^t X + \alpha_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Max\ Z_r = C_r^t X + \alpha_r$$

$$Min\ Z_{r+1} = C_{r+1}^t X + \alpha_{r+1}$$

$$Min\ Z_{r+2} = C_{r+2}^t X + \alpha_{r+2} \qquad (1)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Min\ Z_s = C_s^t X + \alpha_s$$

Subject to:

$$AX = b$$

$$X \geq 0$$

Where, X is n-dimensional and b is m-dimensional vectors. A is $m \times n$ matrix. $\alpha_1, \alpha_2, \dots\dots.., \alpha_s$ are scalars. Here, Z_i is need to be maximized for $i = 1,2, \dots\dots\dots, r$ and need to be minimized for $i = r+1, \dots\dots., s$.

3. Chandra Sen's Method

In this method, firstly all objective functions need to be maximized or minimized individually by Simplex method. By solving each objective function of equation (1) following equations are obtained:

$$Max\ Z_1 = \varphi_1$$

$$Max\ Z_2 = \varphi_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Max\ Z_r = \varphi_r$$

$$Min\ Z_{r+1} = \varphi_{r+1}$$

$$Min\ Z_{r+2} = \varphi_{r+2}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Min\ Z_s = \varphi_s$$

Where, $\varphi_1, \varphi_2, \dots\dots\dots., \varphi_s$ are the optimal values of objective functions.

These values are used to form a single objective function by adding (for maximum) and subtracting (for minimum) of each result of dividing each Z_i by φ_i. Mathematically,

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{Z_i}{|\varphi_i|}$$

Where, $|\varphi_i| \neq 0$.

Subject to the constraints are remain same as equation (1).

Then this single objective linear programming problem is optimized.

4. Average Method of MOLPP

In this method, initially all optimized values of each objective functions are calculated under given constraints. Then a single objective function is constructed by adding all maximization objective functions and subtracting all minimization objective functions and divided them by different kinds of means of maximization objective functions' absolute maximum values and different kinds of means of minimization objective functions' absolute minimum values respectively. Then this single objective function is optimized for the same constraints. For this method, different kinds of mean techniques are discussed.

4.1. Different Kinds of Mean Techniques

Contraharmonic Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{CH.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{CH.M._2}$$

Where, $CH.M._1 = CH.M.(|\varphi_1|, |\varphi_2|, \dots\dots, |\varphi_r|)$ & $CH.M._2 = CH.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots\dots, |\varphi_s|)$ and CH. M. is Contraharmonic mean.

Quadratic Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{Q.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{Q.M._2}$$

Where, $Q.M._1 = Q.M.(|\varphi_1|, |\varphi_2|, \dots\dots, |\varphi_r|)$ & $Q.M._2 = Q.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots\dots, |\varphi_s|)$ and Q. M. is Quadratic mean.

Nueman-Sándor Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{NS.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{NS.M._2}$$

Where, $NS.M._1 = NS.M.(|\varphi_1|, |\varphi_2|, \dots\dots, |\varphi_r|)$ & $NS.M._2 = NS.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots\dots, |\varphi_s|)$ and NS. M. is Nueman-Sándor mean.

Arithmetic Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{A.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{A.M._2}$$

Where, $A.M._1 = A.M.(|\varphi_1|, |\varphi_2|, \dots\dots, |\varphi_r|)$ & $A.M._2 = A.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \dots\dots, |\varphi_s|)$ and A. M. is Arithmetic mean.

Identric Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{I.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{I.M._2}$$

Where, $I.M._1 = I.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $I.M._2 = I.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and I. M. is Identric mean.

Heronian Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{He.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{He.M._2}$$

Where,
$He.M._1 = He.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $He.M._2 = He.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and He. M. is Heronian mean.

Arithmetic-geometric Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{AG.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{AG.M._2}$$

Where,
$AG.M._1 = AG.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $AG.M._2 = AG.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and AG. M. is Arithmetic-geometric mean.

Logarithmic Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{L.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{L.M._2}$$

Where, $L.M._1 = L.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $L.M._2 = L.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and L. M. is Logarithmic mean.

Geometric Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{G.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{G.M._2}$$

Where, $G.M._1 = G.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $G.M._2 = G.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and G. M. is Geometric mean.

Geometric-harmonic Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{GH.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{GH.M._2}$$

Where,
$GH.M._1 = GH.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $GH.M._2 = GH.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and GH. M. is Geometric-harmonic mean.

Harmonic Mean Technique:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{H.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{H.M._2}$$

Where, $H.M._1 = H.M.(|\varphi_1|, |\varphi_2|, \ldots \ldots, |\varphi_r|)$ & $H.M._2 = H.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots \ldots, |\varphi_s|)$ and H. M. is Harmonic mean.

4.2. Algorithm for Average Method of MOLPP

Step 1: Use Simplex method to find the optimal value of each of the objective function.

Step 2: Check the feasibility of step1, if it is feasible then go to step 3 otherwise use dual simplex method to remove infeasibility.

Step 3: Assign a name to each of the optimal value of corresponding objective function. Say $Max\ Z_i = \varphi_i, i = 1,2, \ldots \ldots, r$ and $Min\ Z_i = \varphi_i, i = r+1, r+2, \ldots \ldots, s$.

Step 4: Calculate the values of

$$CH.M._1, CH.M._2, Q.M._1, Q.M._2, NS.M._1, NS.M._2$$
$$, A.M._1, A.M._2, I.M._1, I.M._2, He.M._1, He.M._2,$$
$$AG.M._1, AG.M._2, L.M._1, L.M._2, G.M._1, G.M._2,$$
$$GH.M._1, GH.M._2, H.M._1, H.M._2$$

Step 5: Optimize the combined objective function using same constraints as follow:

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{CH.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{CH.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{Q.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{Q.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{NS.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{NS.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{A.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{A.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{I.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{I.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{He.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{He.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{AG.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{AG.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{L.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{L.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{G.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{G.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{GH.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{GH.M._2}$$

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{H.M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{H.M._2}$$

5. New Average Method of MOLPP

In this method firstly all objective functions are solved individually by using simplex method. Then the result of addition (maximum type) and subtraction (minimum type) of objective functions are divided by different kinds of means to build a single maximum type objective function, where means are calculated by using maximum absolute value of maximum type objective functions and minimum absolute value of minimum type objective functions. Then this single objective function is optimized for the same constraints. For this method, different kinds of mean techniques are discussed.

5.1. Different Kinds of New Mean Techniques

New Contraharmonic Mean Technique:

Let, $M_1 = \min(|\varphi_i|), i = 1,2, \ldots\ldots\ldots, r$ and $M_2 = \max(|\varphi_i|), i = r+1, r+2, \ldots\ldots\ldots, s$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/CHM_{av}$$

Where, $CHM_{av} = \frac{\left(\frac{{M_1}^2+{M_2}^2}{2}\right)}{\left(\frac{M_1+M_2}{2}\right)}$

New Quadratic Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/QM_{av}$$

Where, $QM_{av} = \sqrt{\frac{1}{2}({M_1}^2 + {M_2}^2)}$

New Neuman-Sándor Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/NSM_{av}$$

Where, $NSM_{av} = \frac{M_1 - M_2}{2\ \mathrm{arcsinh}\left(\frac{M_1-M_2}{M_1+M_2}\right)}$

New Arithmetic Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/AM_{av}$$

Where, $AM_{av} = \frac{M_1+M_2}{2}$

New Identric Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/IM_{av}$$

Where, $IM_{av} = \begin{cases} M_1\ if\ M_1 = M_2 \\ \frac{1}{e}^{M_1-M_2}\sqrt{\frac{{M_1}^{M_1}}{{M_2}^{M_2}}} & else \end{cases}$

New Heronian Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/HeM_{av}$$

here, $HeM_{av} = \frac{1}{3}\left(M_1 + \sqrt{M_1 M_2} + M_2\right)$

New Arithmetic-geometric Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/AGM_{av}$$

Where, $AGM_{av} =$ The value of the number where (a_n) & (b_n) converge. Here, $a_0 = M_1$& $b_0 = M_2$ and $a_{n+1} = \frac{1}{2}(a_n + b_n), b_{n+1} = \frac{1}{2}\sqrt{a_n b_n}$.

New Logarithmic Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/LM_{av}$$

Where, $LM_{av} = \begin{cases} M_1\ if\ M_1 = M_2 \\ \frac{M_1-M_2}{\ln M_1 - \ln M_2} & else \end{cases}$

New Geometric Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/GM_{av}$$

Where, $GM_{av} = \sqrt{M_1 \times M_2}$

New Geometric-harmonic Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/GHM_{av}$$

Where, $GHM_{av} =$ The value of the number where (a_n) & (b_n) converge. Here, $a_0 = M_1$& $b_0 = M_2$ and $b_{n+1} = \frac{2}{\frac{1}{a_n}+\frac{1}{b_n}}, a_{n+1} = \frac{1}{2}\sqrt{a_n b_n}$.

New Harmonic Mean Technique:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/HM_{av}$$

Where, $HM_{av} = \frac{2}{\frac{1}{M_1}+\frac{1}{M_2}}$

5.2. Algorithm for New Average Method of MOLPP

Step 1: Use Simplex method to find the optimal value of each of the objective function.

Step 2: Check the feasibility of step1, if it is feasible then go to step 3 otherwise use dual simplex method to remove infeasibility.

Step 3: Assign a name to each of the optimal value of corresponding objective function. Say $Max\ Z_i = \varphi_i, i = 1,2, \ldots\ldots, r$ and $Min\ Z_i = \varphi_i, i = r+1, r+2, \ldots\ldots, s$.

Step 4: Calculate M_1 & M_2 where, $M_1 = \max(|\varphi_i|), i = 1,2, \ldots\ldots\ldots, r$ and $M_2 = \min(|\varphi_i|), i = r+1, r+2, \ldots\ldots\ldots, s$.

Step 5: Calculate the values of

$$CM_{av}, QM_{av}, NSM_{av}, AM_{av}, IM_{av}, HeM_{av}, AGM_{av}, LM_{av}, GM_{av}, GHM_{av}, HM_{av}$$

Step 6: Optimize the combined objective function using same constraints as follow:

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/CM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/QM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/NSM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/AM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/IM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/HeM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/AGM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/LM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/GM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/GHM_{av}$$

$$Max\ Z = \left(\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i\right)/HM_{av}$$

6. Mathematical Example of MOLPP

$$Max\ Z_1 = x_1 + 2x_2$$

$$Max\ Z_2 = x_1$$

$$Min\ Z_3 = -2x_1 - 3x_2$$

$$Min\ Z_4 = -x_2$$

Subject to:

$$6x_1 + 8x_2 \leq 47$$

$$x_1 + x_2 \geq 3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

The optimal values of the each of the objective function are calculated by using simplex method and are given below:

Now for first objective function,

$$Max\ Z_1 = x_1 + 2x_2$$

Subject to:

$$6x_1 + 8x_2 + x_3 = 47$$

$$x_1 + x_2 - x_4 = 3$$

$$x_1 + x_5 = 4$$

$$x_2 + x_6 = 3$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Table 1. *Simplex table for 1st objective function.*

C_B	C_j	1	2	0	0	0	0	b_i
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_3	6	8	1	0	0	0	47
0	x_4	-1	-1	0	1	0	0	-3
0	x_5	1	0	0	0	1	0	4
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	1	2	0	0	0	0	$Z_1 = 0$
0	x_3	-2	0	1	8	0	0	23
2	x_2	1	1	0	-1	0	0	3
0	x_5	1	0	0	0	1	0	4
0	x_6	-1	0	0	1	0	1	0
	$\overline{C_j}$	-1	0	0	2	0	0	$Z_1 =6$
0	x_3	6	0	1	0	0	-8	23
2	x_2	0	1	0	0	0	1	3
0	x_5	1	0	0	0	1	0	4
0	x_4	-1	0	0	1	0	1	0
	$\overline{C_j}$	1	0	0	0	0	-2	$Z_1 = 6$
1	x_1	1	0	1/6	0	0	-8/6	23/6
2	x_2	0	1	0	0	0	1	3
0	x_5	0	0	-1/6	1	0	7/3	1/6
0	x_4	0	0	1/6	1	0	-1/3	23/6
	$\overline{C_j}$	0	0	-1/6	0	0	0	Z_1=9.83333

So, the optimized value, $Max\ Z_1 = 9.8333$

Now for second objective function,

$$Max\ Z_2 = x_1$$

Subject to:

$$6x_1 + 8x_2 + x_3 = 47$$
$$x_1 + x_2 - x_4 = 3$$
$$x_1 + x_5 = 4$$
$$x_2 + x_6 = 3$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Table 2. *Simplex table for 2nd objective function.*

C_B	C_j	1	0	0	0	0	0	b_i
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_3	6	8	1	0	0	0	47
0	x_4	-1	-1	0	1	0	0	-3
0	x_5	1	0	0	0	1	0	4
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	1	0	0	0	0	0	$Z_2 = 0$
0	x_3	0	2	1	6	0	0	23
1	x_1	1	1	0	-1	0	0	3
0	x_5	0	-1	0	1	1	0	1
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	0	-1	0	1	0	0	$Z_2 = 3$
0	x_3	0	8	1	0	-6	0	17
1	x_1	1	0	0	0	1	0	4
0	x_4	0	-1	0	1	1	0	1
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	0	0	0	0	-1	0	$Z_2 = 4$

So, the optimized value, $Max\ Z_2 = 4$

Now for third objective function,

$$Min\ Z_3 = -2x_1 - 3x_2$$

Subject to:

$$6x_1 + 8x_2 + x_3 = 47$$
$$x_1 + x_2 - x_4 = 3$$
$$x_1 + x_5 = 4$$
$$x_2 + x_6 = 3$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Table 3. *Simplex table for 3rd objective function.*

C_B	C_j	-2	-3	0	0	0	0	b_i
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_3	6	8	1	0	0	0	47
0	x_4	-1	-1	0	1	0	0	-3
0	x_5	1	0	0	0	1	0	4
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	-2	-3	0	0	0	0	$Z_3 = 0$
0	x_3	-2	0	1	8	0	0	23
-3	x_2	1	1	0	-1	0	0	3
0	x_5	1	0	0	0	1	0	4
0	x_6	-1	0	0	1	0	1	0
	$\overline{C_j}$	1	0	0	-4	0	0	$Z_3 = -9$
0	x_3	6	0	1	0	0	-8	23
-3	x_2	0	1	0	0	0	1	3
0	x_5	1	0	0	0	1	0	4
0	x_4	-1	0	0	1	0	1	0
	$\overline{C_j}$	-2	0	0	0	0	3	$Z_3 = -9$
-2	x_2	1	0	1/6	0	0	-8/6	23/6
-3	x_2	0	1	0	0	0	1	3
0	x_5	0	0	-1/6	0	1	8/6	1/6
0	x_4	0	0	1/6	1	0	-1/3	23/6
	$\overline{C_j}$	0	0	1/3	0	0	1/3	Z_3=-16.66667

So, the optimized value, $Min\ Z_3 = -16.66667$

Now for fourth objective function,

$$Min\ Z_4 = -x_2$$

Subject to:

$$6x_1 + 8x_2 + x_3 = 47$$
$$x_1 + x_2 - x_4 = 3$$
$$x_1 + x_5 = 4$$
$$x_2 + x_6 = 3$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Table 4. *Simplex table for 4th objective function.*

C_B	C_j	0	-1	0	0	0	0	b_i
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_3	6	8	1	0	0	0	47
0	x_4	-1	-1	0	1	0	0	-3
0	x_5	1	0	0	0	1	0	4
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	0	-1	0	0	0	0	Z_4=0
0	x_3	-2	0	1	8	0	0	23
-1	x_2	1	1	0	-1	0	0	3
0	x_5	1	0	0	0	1	0	4
0	x_6	-1	0	0	1	0	1	0
	$\overline{C_j}$	1	0	0	-1	0	0	Z_4=-3
0	x_3	6	0	1	0	0	-8	23
-1	x_2	0	1	0	0	0	1	3
0	x_5	1	0	0	0	1	0	4
0	x_4	-1	0	0	1	0	1	0
	$\overline{C_j}$	0	0	0	0	0	1	Z_4=-3

So, the optimized value, $Min\ Z_4 = -16.66667$

Table 5. *Initial Table.*

i	φ_i	$\lvert\varphi_i\rvert$	Values of M_1 & M_2
1	9.83333	9.83333	$M_1 = 9.83333$
2	4	4	
3	-16.66667	16.66667	$M_2 = 3$
4	-3	3	

Now using the values of φ_i, M_1 & M_2 different types of means are calculated and given below:

Table 6. *Values of Means.*

$C.M._1$	8.14658	$C.M._2$	14.5819	CM_{av}	8.2359
$Q.M._1$	7.5064	$Q.M._2$	11.9745	QM_{av}	7.2696
$NS.M._1$	7.1122	$NS.M._2$	10.5372	NSM_{av}	6.6985
$A.M._1$	6.9160	$A.M._2$	9.83333	AM_{av}	6.4166
$I.M._1$	6.7031	$I.M._2$	8.9337	IM_{av}	6.0919
$He.M._1$	6.7016	$He.M._2$	8.9125	HeM_{av}	6.0882
$AG.M._1$	6.5901	$AG.M._2$	8.3953	AGM_{av}	5.9137
$L.M._1$	6.4852	$L.M._2$	7.9698	LM_{av}	5.7560
$G.M._1$	6.2716	$G.M._2$	7.0711	GM_{av}	5.4314
$GH.M._1$	5.9692	$GH.M._2$	5.9556	GHM_{av}	4.9883
$H.M._1$	5.6867	$H.M._2$	5.0841	HM_{av}	4.5974

Now for New Harmonic mean technique,
Harmonic mean, $HM_{av} = \frac{2}{\frac{1}{9.83333}+\frac{1}{3}} = 4.5974$

$$Max\ Z = \{(Z_1 + Z_2) - (Z_3 + Z_4)\}/HM_{av}$$

$$= \{(x_1 + 2x_2 + x_1) - (-2x_1 - 3x_2 - x_2)\}/4.5974$$

$$= (4x_1 + 6x_2)/4.5974$$

$$= 0.87\ x_1 + 1.305\ x_2$$

Subject to:

$$6x_1 + 8x_2 + x_3 = 47$$

$$x_1 + x_2 - x_4 = 3$$

$$x_1 + x_5 = 4$$

$$x_2 + x_6 = 3$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Table 7. *Simplex table for New Harmonic mean techniques objective function.*

C_B	C_j	**0.87**	**1.305**	**0**	**0**	**0**	**6**	b_i
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	
0	x_3	6	8	1	0	0	0	47
0	x_4	-1	-1	0	1	0	0	-3
0	x_5	1	0	0	0	1	0	4
0	x_6	0	1	0	0	0	1	3
	$\overline{C_j}$	0.5517	0.8275	0	0	0	0	Z=0
0	x_3	-2	0	1	8	0	0	23
1.305	x_2	1	1	0	-1	0	0	3
0	x_5	1	0	0	0	1	0	4
0	x_6	-1	0	0	1	0	1	0
	$\overline{C_j}$	-0.2276	0	0	0.8725	0	0	Z=0
0	x_3	6	0	1	0	0	-8	23
1.305	x_2	0	1	0	0	0	1	3
0	x_5	1	0	0	0	1	0	4
0	x_4	-1	0	0	1	0	1	0
	$\overline{C_j}$	0.5517	0	0	0	0	-0.8275	Z=3.915
0.87	x_1	1	0	1/6	0	0	-8/6	23/6
1.305	x_2	0	1	0	0	0	1	3
0	x_5	0	0	-1/6	1	0	7/3	1/6
0	x_4	0	0	1/6	1	0	-1/3	23/6
	$\overline{C_j}$	0	0	-.145	0	0	-.145	Z=7.2504

So, the optimized value, $Max\ Z = 7.2504$

Now using Simplex method, Z is optimized for different mean techniques and result are shown below:

Table 8. *Final Table.*

Techniques	x_i	**Value of Z**	
Chandra Sen	(3.83333, 3)	3.9534	
		Average Method	New Average Method
Contraharmonic Mean	(3.83333, 3)	3.0263	4.0473
Quadratic Mean	(3.83333, 3)	3.4631	4.5853
Neuman-Sándor Mean	(3.83333, 3)	3.7879	4.9762
Arithmetic Mean	(3.83333, 3)	3.9761	5.1948
Identric Mean	(3.83333, 3)	4.2402	5.4717
Heronian Mean	(3.83333, 3)	4.2459	5.4750
Arithmetic-geometric Mean	(3.83333, 3)	4.4164	5.6366
Logarithmic Mean	(3.83333, 3)	4.5750	5.7911
Geometric Mean	(3.83333, 3)	4.9604	6.1372
Geometric-harmonic Mean	(3.83333, 3)	5.5917	6.6823
Harmonic Mean	(3.83333, 3)	6.2715	7.2504

It is seen that new proposed method is better than Chandra Sen's Method and Average Method, and new Harmonic Mean technique gives the best result.

7. Conclusion

In this paper, we have used different types of method for solving MOLPP such as Chandra Sen's Method, Average Method and New Average Method. New Average Method is compared with Average Method and Chandra Sen's Method. It is seen that Chandra Sen's Method is better than some mean (Contraharmonic mean, Quadratic mean, Neuman-Sándor mean) techniques in Average Method and New Average Method is better than Chandra Sen's Method and Average Method for all mean techniques. Among all the techniques, new Harmonic mean technique gives the best result.

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