



Multipole Mixing Ratio, (E2/M1), and Electric Monopole Strength, (E0/E2), for γ -Transitions in ^{192}Pt

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Abstract: The low-lying collective states in ^{192}Pt are investigated in the frame work of the interacting boson approximation model IBA-2. The contour plot of the potential energy surfaces, $V(\beta, \gamma)$, shows an O(6) character to the nucleus. B(M1), B(E2), $\delta(E2/M1)$ and X(E0/E2) were calculated and reasonable agreement has obtained to the available experimental data. Also, sharp upper bending is observed at $I^{\pi}=10$.

Keywords: Levels Energy, Transition Probability, B(M1), B(E2), Electric Monopole Strength, X(E0/E2) and Mixing Ratio $\delta(E2/M1)$

1. Introduction

The low-lying states of ^{192}Pt have been investigated by many authors experimentally and theoretically since they are lying in the transitional region between the end of the rare earth nuclei and the doubly closed shell of ^{208}Pb .

The collective properties for the low-lying states of ^{192}Pt have been investigated experimentally by means of Coulomb excitation using 3.3–4.8 MeV/nucleon ^{40}Ca , ^{58}Ni , ^{136}Xe and ^{208}Pb beams [1] and also by β^{-} and electron capture decay of ^{192}Ir [2-6]. The level structures of the shape transitional nuclei $^{190,192}\text{Pt}$ have been studied by $(\alpha, xn\gamma)$ reactions on enriched Os targets. The measurements included γ -ray singles using Ge(Li) detectors, prompt and delayed γ - γ coincidences, half-life determinations in the range 1-500 ns, and γ -ray angular distributions. Detailed level scheme for ^{192}Pt , incorporating much new spectroscopic information where they assigned spin, parity to levels and gamma rays multipolarity are reported. Acute back bending is observed at about spin 10 in the positive parity yarest sequences which is attributed to intersection of the ground bands [7].

Theoretically analytical solution for the Davydov-Chaban Hamiltonian [7] lead to simpler special potentials, levels energy and B(E2) for β and γ bands. Also, the calculated levels, J, π of ground, β and γ bands, B(E2), γ branching ratios, probability density contours in (β, γ) plane; deduced

evidence of shape evolution and possible shape coexistence [8-11]. The calculated prolate and oblate levels, J of low-lying positive-parity states, quadrupole moment, quadrupole deformation parameter β_2 , contour diagrams of the ground state energies [12]. The effective IBA Hamiltonian is used [13] to describe the low-lying spectra and electromagnetic properties of several series of even-even isotopes including Pt and Os. They reproduced some of 250 experimental energies with rms deviation of about 100 keV as well as transition probabilities. The sdg boson model has been used [14] and a satisfactory description of E2 and E4 properties is obtained for Pt and Os nuclei. The model has also predicted dynamic shape transitions from O(6) to SU(3) limits for these nuclei. The values of g-factor were evaluated [15] from transient field measurements and the IBA-2 has been used in measuring mass variations of $g(2_1^+)$.

The main purpose of the present study is:

- To see what kind of nuclei is ^{192}Pt ;
- To calculate levels energy, reduced transition probability of gamma ray transitions since very few values are available in literature and back bending;
- To calculate the multipole mixing ratio, $\delta(E2/M1)$, and the electric monopole transitions, X(E0/E2).

2. The Interacting Boson Approximation Model (IBA-2)

2.1. Levels Energy

The IBA-2 model of Arima [16-18] was applied to the positive-parity of the low-lying states in ^{192}Pt . The Hamiltonian employed for the present calculation is given by puddu [19] as:

$$H = \varepsilon_{\pi} (d^{\dagger} \times d)_{\pi} + \varepsilon_{\nu} (d^{\dagger} \times d)_{\nu} + V_{\pi\pi} + V_{\nu\nu} + KQ_{\pi} \cdot Q_{\nu} + M_{\pi\nu} \quad (1)$$

Where e_{π} , e_{ν} are the proton and neutron effective charge energies respectively and they are assumed equal $e_{\pi} = e_{\nu} = \varepsilon$. The third and fourth term of Eq. (1) represent the proton-neutron interactions operator which is given by:

$$V_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2} C_{L\rho} (2L+1)^{1/2} [(d^{\dagger} \times d^{\dagger})_{\rho} \cdot (d \times d)_{\rho}]^{(0)} \quad (2)$$

The fifth term is the quadrupole operator which is given by the usual expression (K is the strength of the proton and neutron bosons quadrupole interactions):

$$Q_{\rho} = [(s^{\dagger} \times d) + (d^{\dagger} \times s)]_{\rho} + \chi_{\rho} (d^{\dagger} \times s)_{\rho} \quad (3)$$

$\rho = \pi \text{ or } \nu$

The sixth term is the Majorana operator and is given by

$$M_{\pi\nu} = 2 \sum_{K=1,3} \zeta_K (d_{\pi}^{\dagger} \times d_{\nu}^{\dagger}) \cdot (d_{\pi} \times d_{\nu}) + (d_{\pi}^{\dagger} s_{\nu}^{\dagger} - s_{\pi}^{\dagger} d_{\nu}^{\dagger}) \cdot (d_{\pi} s_{\nu} - s_{\pi} d_{\nu}) \quad (4)$$

$\rho = \pi \text{ or } \nu$

2.2. Reduced Transition Probabilities, $B(E2)$

The electric quadrupole transition operator employed in this study is defined as

$$T^{(E2)} = e_{\pi} Q_{\pi} + e_{\nu} Q_{\nu} \quad (5)$$

where:

$T^{(E2)}$: Absolute transition probability of the electric quadrupole (E2) transition,

e_{π} and e_{ν} : The proton and neutron effective charges, and

Q_{ρ} : The quadrupole operators which is the same as that in Eq.(3).

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states are given by

$$B(E2, I_i \rightarrow I_f) = [\langle I_f | T^{(E2)} | I_i \rangle]^2 / (2I_i + 1) \quad (6)$$

where

I_i : The initial state of the electric quadrupole transition, and
 I_f : The final state of the electric quadrupole transition.

The proton and neutron boson numbers N_{π} and N_{ν} respectively can be treated as parameters. They are fixed to be half the number of valence fermions and counted from the nearest closed shell. The closed shells considered here in the present calculation are 82 and 126 for protons and neutrons respectively.

The effective charges e_{π} and e_{ν} depend on the total number

of bosons N , number of neutron bosons N_{ν} , number of proton bosons N_{π} and the experimental value of $B(E2, 2_1^+ \rightarrow 0_1^+)$, where

$$N = N_{\pi} + N_{\nu} \quad (7)$$

The effective charges are calculated using Eq.(8) with the help of the experimental value of $B(E2, 2_1^+ \rightarrow 0_1^+)$, [20].

$$1/N_{\pi} [NB(E2, 2_1^+ \rightarrow 0_1^+) / (N+3)]^{1/2} = e_{\pi} + N_{\nu} / N_{\pi} \quad (8)$$

3. Results and Discussion

3.1. The Potential Energy Surfaces

The energy surface, $E(\beta, \gamma)$, for ^{192}Pt nucleus as a function of the deformation parameters β and γ has been calculated using the following expression by Ginocchio [21]:

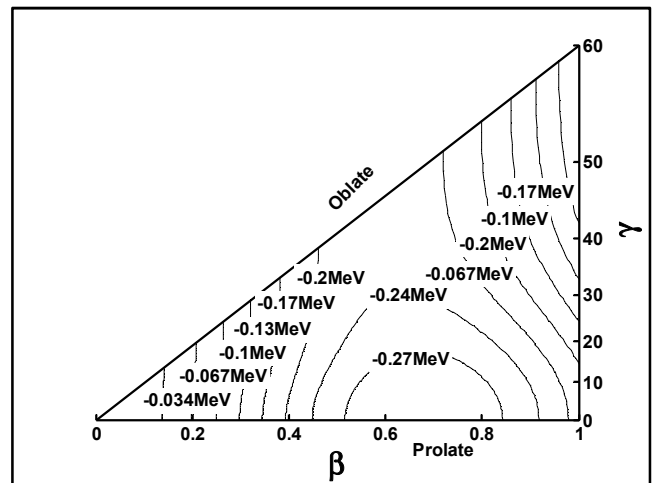
$$E_{N_{\nu} N_{\pi}}(\beta, \gamma) = \langle N_{\pi} N_{\nu}; \beta\gamma | H_{np} | N_{\pi} N_{\nu}; \beta\gamma \rangle$$

$$= \varepsilon_d (N_{\nu} + N_{\pi}) \beta^2 (1 + \beta^2)^{-1} + \beta^2 (1 + \beta^2)^{-2} \{ k N_{\pi} N_{\nu} [4 - (X_{\pi}^- + X_{\nu}^-) \beta \cos 3\gamma + X_{\pi}^- X_{\nu}^- \beta^2] + N_{\nu} (N_{\nu} - 1) (1/10 C_0 + 1/7 C_2) \beta^2 \} \quad (9)$$

where

$$X_{\rho}^- = (2/7)^{0.5} X_{\rho} \quad \rho = \pi \text{ or } \nu \quad (10)$$

The contour plot of the potential energy surfaces is presented in figure 1, where the relation is between the potential energy surfaces and deformation parameter β which are shown at different values of γ . The value of γ varied between 0° and 60° . Also, figure 1 shows the same previous relation but only at $\gamma=0^\circ$ (prolate) and $\gamma=60^\circ$ (oblate). It is clear from the graphs that ^{192}Pt is an O(6) nucleus where the deformation on the prolate and oblate sides are nearly equal. The energy ratios, Table 1., are supporting that assumption.



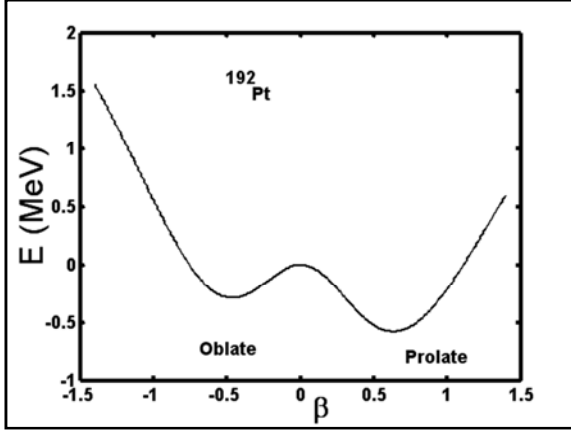


Figure 1. Contour plot of the potential energy surfaces.

Table 1. Energy ratios for ^{192}Pt .

Quantity	Exp*	IBA-2	SU(5)	O(6)	SU(3)
$E_{2_2^+}/E_{2_1^+}$	1.93	2.21	2.00	2.50	3.33
$E_{4_1^+}/E_{2_1^+}$	2.48	2.61	2.00	2.50	3.33
$E_{6_1^+}/E_{2_1^+}$	4.31	4.76	3.00	4.50	7.00
$E_{4_2^+}/E_{2_1^+}$	3.80	4.00	3.00	4.50	7.00
$E_{3_1^+}/E_{2_1^+}$	2.91	3.60	3.00	4.50	7.00
$E_{8_1^+}/E_{2_1^+}$	6.38	7.37	4.00	7.00	12.00
$E_{4_3^+}/E_{2_1^+}$	6.12	5.28	4.00	7.00	12.00

*Ref. [22]

3.2. Levels Energy

The calculated excitation energies of ^{192}Pt are displayed in Figure 2. The agreement between the calculated and observed values by other authors is satisfactory.

The levels are reported experimentally [22] and classified into bands by Sakai [23]. The Hamiltonian parameters of Eq. (1) used in the present calculation are:

$$\varepsilon = 0.180, K = -0.180, X_{\pi} = 0.180, X_{\nu} = 0.100,$$

$C_{L\nu} = -0.280, -0.249, 0.030$ $C_{1\pi} = -0.260, -0.360, -0.090$ all in MeV.

The computer codes NPBOS and NPBEM [24] of the IBA-2 have been used in calculating energy eigenvalues, wave functions and E2 transition matrix elements.

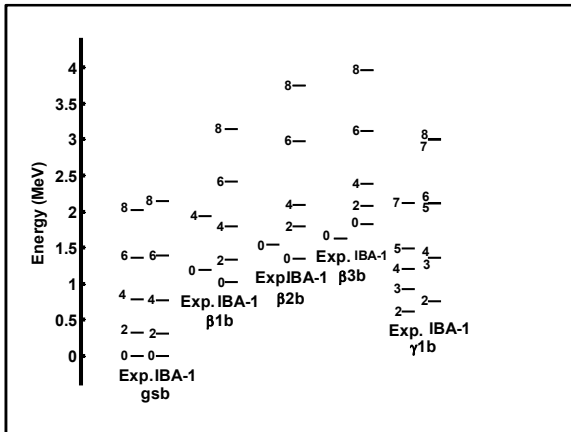


Figure 2. Comparison between Exp. [22] and IBA levels energy.

3.3. Reduced Transition Probabilities, $B(E2)$ and $B(M1)$

The proton and neutron effective charges e_{π} and e_{ν} were determined using the experimental reduced transition probability $B(E2, 2_{1+} \rightarrow 0_{1+}) = 0.3750(6) e^2 b^2$ with the help of Hamilton [20] equations and the calculated values are:

$$e_{\pi} = 0.246 e. \text{ and } e_{\nu} = 0.175 e.b$$

$B(E2)$ values are calculated, then normalized to the experimental data of $B(E2, 2_{1+} \rightarrow 0_{1+}) = 0.3750(6)$ and presented in Table 2. Unfortunately, few experimental data are available [25] where they compared to the present calculation.

To calculate M1 transition probability using IBA-2 model we take $L=1$ and $B(M1)$ can be calculated for various transitions. The calculated values are presented in Table 2 and compared to the available experimental data.

Table 2. Transition probability ratios, $B(E2)$ in $(eb)^2$ and $B(M1)$ in $(\mu_N)^2$,

I_i^+	I_f^+	$B(E2)_{Exp^*}$	$B(E2)_{IBA}$	I_i^+	I_f^+	$B(E1)_{Exp^*}$	$B(E1)_{IBA}$
2_1	0_1	0.3750(6)	0.3999	1_1	0_1	—	0.0083
0_2	2_1	—	0.0661	1_1	0_2	—	0.0013
2_2	0_1	0.0044	0.0042	2_2	2_1	0.0002	0.0006
2_2	0_2	—	0.0002	2_3	2_1	—	0.0054
2_2	2_1	0.4600	0.3882	2_4	2_1	—	0.0022
2_3	2_1	—	0.0010	3_1	2_1	0.0002	0.0002
2_4	2_1	—	0.0007	3_1	2_2	0.0005	0.0009
2_3	2_2	—	0.0078	3_1	4_1	0.0002	0.0008
2_4	2_2	—	0.0017	4_2	3_1	—	0.0000
3_1	2_1	0.0044	0.0043	4_2	4_1	—	0.0014
3_1	2_2	0.6711	0.4980	4_2	3_2	—	0.0003
4_1	2_1	0.5860	0.5541	4_2	3_3	—	0.0046
4_1	2_2	—	0.0043	2_4	2_2	—	0.0017
4_2	2_1	—	0.0013	2_4	2_3	—	0.0150
3_1	4_1	0.2500	0.1728	3_2	2_1	—	0.0089
4_2	4_1	—	0.2317	3_2	2_2	—	0.0002
4_2	2_2	—	0.3074	3_2	2_3	—	0.1072

* Ref [25]

3.4. Mixing Ratios $\delta(E2/M1)$

To calculate the mixing ratios for γ - transitions we need first to calculate the gyromagnetic g- factors which is g_{π} for proton and g_{ν} for neutron. The relation between g - factors and magnetic moment $\mu (2_1^+)$ is:

$$\mu (2_1^+) = 2g (2_1^+) \quad (11)$$

and

$$g = g_{\pi} [N_{\pi}/(N_{\pi}+N_{\nu})] + g_{\nu} [N_{\nu}/(N_{\pi}+N_{\nu})] \quad (12)$$

Then, M1-E2 mixing parameters [26] is given by:

$$\delta(E2/M1) = 0.835 E \gamma \Delta(E2/M1) \quad (13)$$

where

$$\Delta(E2/M1) = \langle I_f \parallel T^{E2} \parallel I_i \rangle / \langle I_f \parallel T^{M1} \parallel I_i \rangle \quad (14)$$

and

$$\langle I_f \parallel T^{E2} \parallel I_i \rangle = e_{\pi} Q_{\pi} + e_{\nu} Q_{\nu}$$

$$= [(2I_i + 1)B(E2, I_i - I_f)]^{1/2}, \quad (15)$$

$$\langle I_f || T^{M1} || I_i \rangle = 0.77 [(d^+ d^-)_{\pi}^{(1)} - (d^+ d^-)_{\nu}^{(1)}] (g_{\pi} - g_{\nu}) \quad (16)$$

Table 3. Multipole mixing ratios $\delta(E2/M1)$ in (eb/ μ_N).

I_i^+	I_f^+	E_{γ}	δ_{exp}^*	IBA-2.
2 ₂	2 ₁	0.295	10.0±32	9.85
2 ₃	2 ₁	1.122	----	0.91
3 ₁	2 ₁	0.604	4.48±5	6.46
3 ₁	2 ₂	0.309	7.42±4	14.11
3 ₁	4 ₁	0.136	3.30	10.50
4 ₂	3 ₁	0.280	5.40	171.05
4 ₂	4 ₁	0.416	2.90	4.63

*Ref. [25]

The sign of $\delta(E2/M1)$ is given from the sign of the matrix elements used in the calculations. The calculated values are compared to the experimental data which show reasonable agreement and displayed in Table 3

3.5. Strength of the Electric Monopole Transitions, $X_{iff}(E0/E2)$

The electric monopole transitions, E0, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. It is a pure penetration effect where it caused by the electromagnetic interaction between the nuclear charge and the atomic electron which penetrates the nucleus. The strength of the electric monopole transition $X_{iff}(E0/E2)$ [27] can be determined by the relation:

$$X_{iff}(E0/E2) = B(E0, I_i - I_f) / B(E2, I_i - I_f) \quad (17)$$

Where

$$I_i = I_f = 0, I_f = 2 \text{ or } I_i = I_f \neq 0, I_f = I_i$$

$$X_{iff}(E0/E2) = 2.54 \times 10^9 A^{3/4} [E^5_{\gamma} \text{ MeV} / \Omega_{KL}] \times \alpha(E2) \times [T_c(E0, I_i - I_f) / T_c(E2, I_i - I_f)] \quad (18)$$

A: Mass number;

I_i : Spin of the initial state where E0 and E2 transitions are depopulating it;

I_f : Spin of the final state of E0 transition;

I_f : Spin of the final state of E2 transition;

E_{γ} : Gamma ray energy;

Ω_{KL} : Electronic factor for K, L shells [28];

$\alpha(E2)$: Conversion coefficient of the E2 transition;

$T_c(E0, I_i - I_f)$: absolute transition probability of the E0 transition

between I_i and I_f states, and

$T_c(E2, I_i - I_f)$: Absolute transition probability of the E2 transition between I_i and I_f states.

The E0 strength can be considered as the ratio between the reduced transition probability of competing E0 and electric quadrupole, E2, transitions de-populating the same level. The calculated values are presented in Table 4 with the available experimental data.

Table 4. Calculated $X_{iff}(E0/E2)$.

I_i^+	I_f^+	$X_{iff}(E0/E2)$	
		Exp*[25]	IBA-2
0 ₂	0 ₁	-----	0.0100
0 ₃	0 ₁	-----	0.5450
0 ₃	0 ₂	-----	0.0007
0 ₄	0 ₁	-----	0.0164
0 ₄	0 ₂	-----	0.0050
0 ₄	0 ₃	-----	0.0389
2 ₂	2 ₁	0.016±0.014	0.0006
2 ₃	2 ₁	-----	0.9107
2 ₄	2 ₁	-----	0.2727
2 ₃	2 ₂	-----	0.0100
2 ₄	2 ₂	-----	0.2857
2 ₄	2 ₃	-----	0.0572

3.6. Back Bending

To study back bending we need a relation between the moment of inertia I and energy parameters $\hbar\omega$ where we used Eq's (19, 20):

$$2J/\hbar^2 = [(4I-2) / \Delta E(I \rightarrow I-2)] \quad (19)$$

$$(\hbar\omega)^2 = (I^2 - I + 1) [\Delta E(I \rightarrow I-2) / (2I-1)] \quad (20)$$

The relation between $2J/\hbar^2$ and I is given in Figure 3 and sharp upper bending at $I^+=10$ is observed which is due to ground state band crossing.

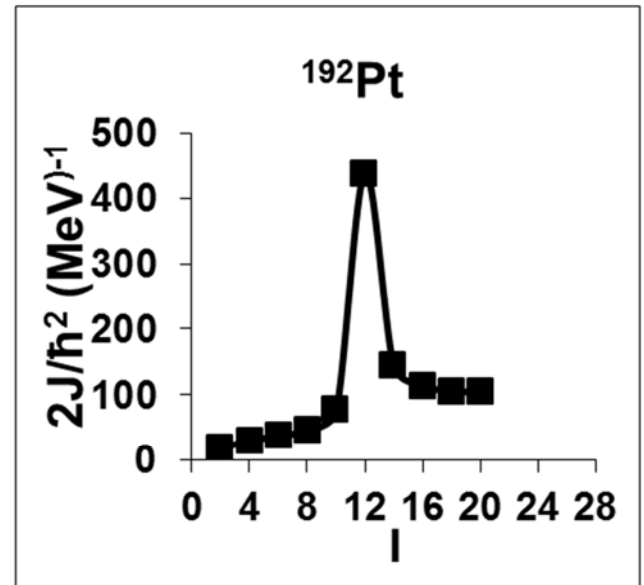


Figure 3. Angular momentum I as a function of $2J/\hbar^2$.

3.7. The Staggering

The presence of the odd-even parity states has encouraged us to study staggering effect for ^{192}Pt . Staggering pattern between the (+ve) parity ground state and the (-ve) parity octupole band states have been calculated, $\Delta I=1$, using staggering function equations (21, 22) with the help of the experimental data[22].

$$\text{Stag.}(I) = 6 \Delta E(I) - 4 \Delta E(I-1) - 4 \Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2) \quad (21)$$

$$\Delta E(I) = E(I+1) - E(I) \quad (22)$$

The calculated staggering pattern is presented in figure 4. We can see the beat pattern which indicate an interaction between the ground state and the octupole band states.

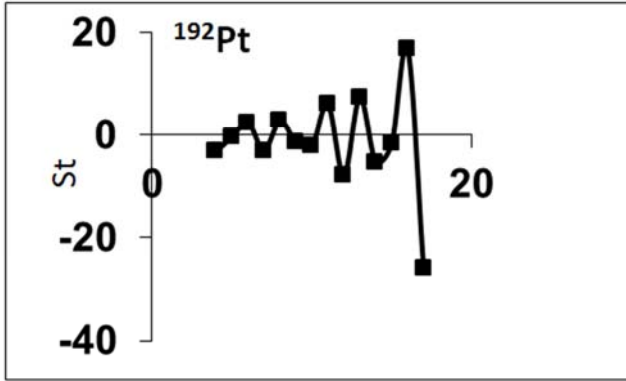


Figure 4. Staggering pattern for ^{192}Pt .

5. Conclusion

1. The contour plots of the potential energy surfaces, $V(\beta, \gamma)$, for $^{192-202}\text{Pt}$ presented in Figure 5. show that ^{192}Pt is an O(6) nucleus and that is supported by γ - rays energy ratios, Table 1.

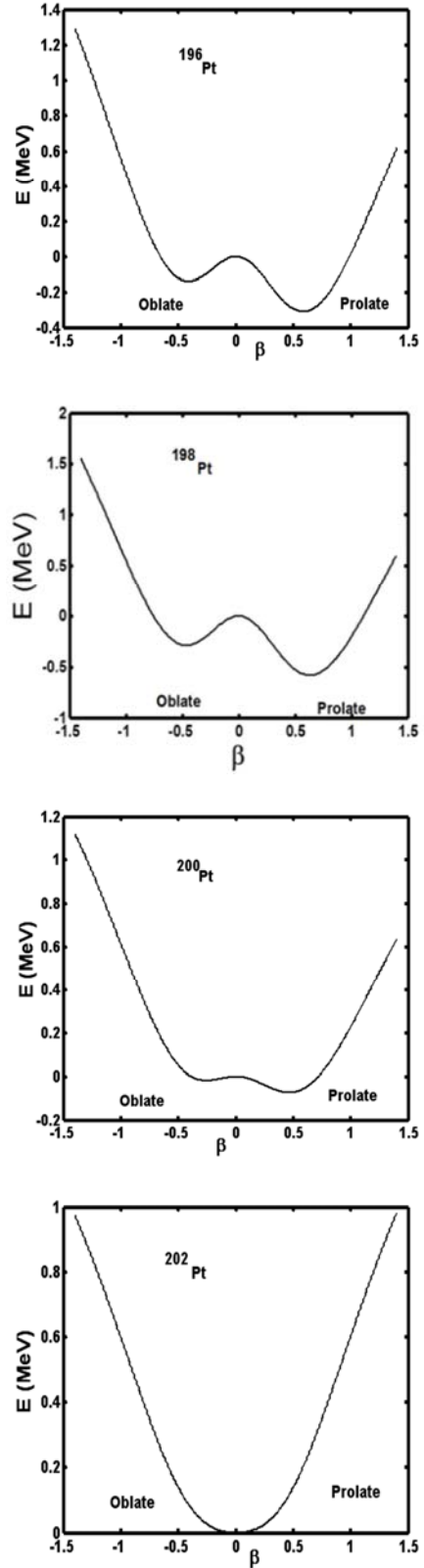
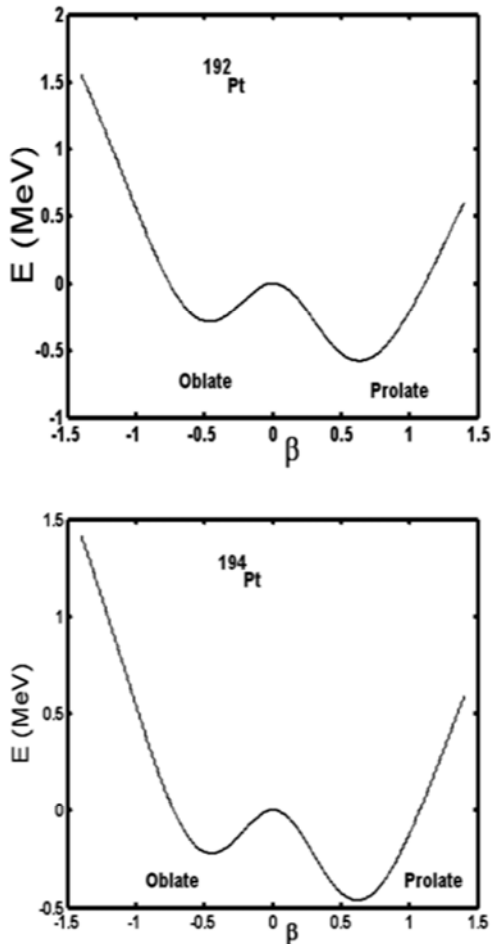


Figure 5. Contour plot of the potential energy surfaces for $^{192-202}\text{Pt}$.

2. The levels energy for ^{192}Pt were calculated using IBA-2 model. The agreement between the calculated and experimental values are fairly good for the low - lying states and slightly poor for high spin states which may be due to

band crossing.

3. The transition probability of gamma transitions were calculated and normalized to $B(E2, 2_1^+ \rightarrow 0_1^+) = 0.3750(6) e^2 b^2$. The agreement between the calculated and experimental values are good.

4. The multipolarity of γ - ray transitions and the electric monopole strength are calculated. They show reasonable agreement to the available measured experimental values.

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