Topological BCL^+ -algebras

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To cite this article:

Yonghong Liu. Topological BCL^+ -Algebras. *Pure and Applied Mathematics Journal*. Vol. 3, No. 1, 2014, pp. 11-13.

doi: 10.11648/j.pamj.20140301.13

Abstract: BCL/BCL^+ -algebras [1, 2] are more extensive class than $BCK/BCH/BCI$ -algebras [5-8], introduced by Yonghong Liu. In this paper, we aim to embed topological structure in BCL^+ -algebras, thus we should be introduced to the notion of “good taste”, which is both topological space and metric space. Thereupon, some fundamental properties of topological BCL^+ -algebras are obtained.

Keywords: BCL -Algebra, BCL^+ -Algebra, Topological BCL^+ -Algebra

1. Introduction

In [1], Yonghong Liu introduced a new class of abstract algebra: BCL -algebra. Recently, Yonghong Liu introduced a wide class of abstract algebras: BCL^+ -algebra (see [2, 4]). In [3], Deena Al-Kadi and Rodyna Hosny introduced the deformation of BCL -algebra. They continue to investigate the relation between BCL -algebra and $d/BCH/BCI$ and BCK -algebra.

In this paper we are going to introduce topological spaces for the BCL^+ -algebras, which is derived the fundamental properties, which we will need the following definitions and theorems:

Definition 1.1 [2]

An algebra $(X; *, 1)$ is called a BCL^+ -algebra if it satisfies the following laws hold: for any $x, y, z \in X$,

$BCL^+-1) \quad x * x = 1.$

$BCL^+-2) \quad x * y = 1 \text{ and } y * x = 1 \text{ imply } x = y.$

$BCL^+-3) \quad ((x * y) * z) * ((x * z) * y) = ((z * y) * x).$

Theorem 1.1 [2]

Assume that $(X; *, 1)$ is any a BCL^+ -algebra. Then the following hold: for any $x, y, z \in X$

i) $(x * (x * y)) * y = 1.$

ii) $x * 1 = x \text{ imply } x = 1.$

iii) $((x * y) * (x * z)) * (z * y) = 1.$

iv) $BCL^+-2) \quad x * y = 1 \text{ and } y * x = 1 \text{ imply } x = y.$

Theorem 1.2 [2]

An algebra $(X; *, 1)$ is a BCL^+ -algebra if and only if it satisfies the following conditions: for all $x, y, z \in X$

i) $BCL^+-1) \quad x * x = 1.$

ii) $BCL^+-2) \quad x * y = 1 \text{ and } y * x = 1 \text{ imply } x = y.$

iii) $((x * y) * z) * ((x * z) * y) * ((z * y) * x) = 1.$

iv) $x * (1 * y) = x.$

Definition 1.2 [2]

Suppose that $(X; *, 1)$ is a BCL^+ -algebra, the ordered relation if $x \leq y$ if and only if $x * y = 1$, for all $x, y \in X$, then $(X; \leq)$ is partially ordered set and $(X; *, 1)$ is an algebra of partially ordered relation.

Corollary 1.1 [2]

Let every $x \in X$. Then 1(one) is maximal element in a BCL^+ -Algebra $(X; *, 1)$ such that integral $1 \leq x$ imply $x = 1$.

2. Main Results

For a $g(x, y, z) = (x * y) * z$ -algebra, the topological space is just a basic concept in nature. We have the following definitions.

Definition 2.1

Let $(X; \circ, *, 1)$ be a BCL_D^+ -algebra with two binary

operations $\circ$ and $*$ that satisfies the following properties: for any $x, y, z \in X$.

BCL_D^+ -1) An algebra $D(X) = (X; \circ)$ is a distributive algebra.

BCL_D^+ -2) An algebra $P(X) = (X; *, 1)$ is a $g(x, y, z) = (x * y) * z$ -algebra.

BCL_D^+ -3) (right weakly distribution)
 $x * (y \circ z) = (x * y) \circ (x * z)$.

BCL_D^+ -4) (left weakly distribution)
 $(y \circ z) * x = (y * x) \circ (z * x)$.

Definition 2.2

Let $(X; *, 1)$ be a BCL^+ -algebra and let $(X, \mathfrak{S})$ be a topological space. If

$$\circ, *: X \times X \rightarrow X \text{ and } (x, y) \mapsto x * y \quad (2.1)$$

be a continuous mapping from the product space $(X \times X, \mathfrak{R})$ to the topological space $(X, \mathfrak{S})$, where $\mathfrak{R}$ is product topology of X , then we say that $(X; \mathfrak{S}, *, 1)$ is a topological BCL^+ -algebra.

Theorem 2.1

The product of any tow topological BCL^+ -algebra is again a topological BCL^+ -algebra.

Proof: Let $(X_1; \mathfrak{S}_1, *, 1_1)$ and $(X_2; \mathfrak{S}_2, *, 1_2)$ be topological BCL^+ -algebra, let (x_1, y_1) and (x_2, y_2) is any tow element in X , and defines:

$$\begin{cases} X = X_1 \times X_2 \\ \circ : (x_1, y_1) \circ (x_2, y_2) = (x_1 *_1 x_2, y_1 *_2 y_2) \\ 1 = (1_1, 1_2) \end{cases} \quad (2.2)$$

Now suppose that U is a open neighborhood of $\mathfrak{R}$. By Definition 2.2, $\mathfrak{R}$ be product topology, so we conclude that B_1 is an open neighborhood of $x_1 *_1 x_2$, and B_2 is an open neighborhood of $y_1 *_2 y_2$ in X , we have $V_1 \times V_2 \subseteq U$. Since B_1 is an open neighborhood of x_1 , and B_2 is an open neighborhood of x_2 . Also, B_3 is a open neighborhood of y_1 , and B_4 is a open neighborhood of y_2 , we have

$$B_1 * B_2 \subseteq V_1 \text{ and } B_3 * B_4 \subseteq V_2. \quad (2.3)$$

Then

$$(B_1 * B_2) \circ (B_3 * B_4) \subseteq V_1 \times V_2 \subseteq U, \quad (2.4)$$

and we claim that in fact

$$(B_1 * B_2) \times (B_3 * B_4) = (B_1 \times B_3) \circ (B_2 \times B_4) \subseteq U, \quad (2.5)$$

where $B_1 \times B_3$ is a open neighborhood of (x_1, y_1) in X , and $B_2 \times B_4$ is a open neighborhood of (x_2, y_2) in X . Thus $\circ$ is a continuous and $(X; \mathfrak{S}, \circ, 1)$ is a topological BCL^+ -algebra. ■

Corollary 2.1

The product of any finite topological BCL^+ -algebra is again a topological BCL^+ -algebra.

Corollary 2.2

Any finite product of a topological BCL^+ -algebra is again a topological BCL^+ -algebra.

Example 2.1

Let X be a topological BCL^+ -algebra and let $f(x, y, z)$ and $g(x, y, z)$ be ternary polynomial. Assume that for all $x, y, z \in X$. Suppose the following conditions hold:

- i) $f(x, y, z) = ((x * y) * z) * ((x * z) * y)$, and
- ii) $g(x, y, z) = (z * y) * x$.

If $(f(x, y, z), g(x, y, z)) = 1$, then we say that $f(x, y, z)$ and $g(x, y, z)$ are coprime topological BCL^+ -algebras.

Example 2.2

Let X be a topological BCL^+ -algebra and let $f(x, y, z)$, $g(x, y, z)$ and $h(x, y, z)$ be ternary polynomial. Assume that for all $x, y, z \in X$. Suppose the following conditions hold:

- i) $f(x, y, z) = (z * y) * x$.
- ii) $g(x, y, z) = (x * y) * z$.
- iii) $h(x, y, z) = (x * z) * y$.

If $(f(x, y, z), g(x, y, z)) = 1$ and $(f(x, y, z), h(x, y, z)) = 1$. Then

$$(f(x, y, z), g(x, y, z) * h(x, y, z)) = 1. \quad (2.6)$$

Definition 2.3

If BCL_D^+ -algebra $(X; \circ, *, 1)$ has the property of distribution, let $\mathfrak{S}$ be topology of X and let $(X; \mathfrak{S}, \circ, *, 1)$ be a topological BCL_D^+ -algebra. Then topological BCL_D^+ -algebra $(X; \mathfrak{S}, \circ, *, 1)$ has also the property of distribution.

Theorem 2.2

Let $(X; \circ, *, 1)$ be a BCL_D^+ -algebra and let $\mathfrak{S}$ be topology of X , then $(X; \mathfrak{S}, \circ, *, 1)$ be a topological BCL_D^+ -algebra.

Proof: Let the operations $\circ$

$$\circ : (x_1, y_1) \circ (x_2, y_2) = (x_1 * x_2, y_1 * y_2). \quad (2.7)$$

identically, element $1 = (1, 1)$. By Definition 2.1 and Definition 1.3, we have continuous mapping

$$\circ, * : (X \times X, \mathfrak{R}) \rightarrow (X, \mathfrak{S}). \quad (2.8)$$

where $\mathfrak{R}$ is product topology of X . We deduce that $(X; \mathfrak{S}, \circ, *, 1)$ be a topological BCL_D^+ -algebra. ■

Definition 2.4

Let $(X; \mathfrak{S}, \circ, *, 1)$ be a topological BCL_D^+ -algebra. Suppose the following conditions hold: for any $x, y, z \in X$.

- i) $\mathfrak{S}(x, y) \geq 1$ and $\mathfrak{S}(x, y) = 1$ if and only if $x = y$.
- ii) $\mathfrak{S}(x, y) = \mathfrak{S}(y, x)$.
- iii) $\mathfrak{S}(x, z) \leq \mathfrak{S}(x, y) + \mathfrak{S}(y, z)$.

Then $(X, \mathfrak{S})$ be a metric space of identically element 1.

Example 2.3

Let X be a set and let $\mathfrak{S} : X \times X \rightarrow \mathbb{R}$, $x, y \in X$, by Definition 2.3, we now have

$$\mathfrak{S}(x, y) = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{if } x \neq y. \end{cases} \quad (2.9)$$

Then $\mathfrak{S}$ be a discrete metric and $(X, \mathfrak{S})$ be a discrete metric space.

Theorem 2.3

Let $(X; \circ, *, 1)$ be a BCL_D^+ -algebra and let $\mathfrak{S}$ a discrete topology of X , then $(X, \mathfrak{S})$ be a topological space.

Proof: By Theorem 2.1, $(X, \mathfrak{S})$ be a discrete metric space by Example 2.1, where discrete metric of X is induced by $\mathfrak{S}$. Thus $(X, \mathfrak{S})$ be a topological space. ■

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