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Lakshmi - Manoj generalized Yang-Fourier transforms to heat-conduction in a semi-infinite fractal bar

Lakshmi Narayan Mishra[1, 2], Manoj Sharma[3], Vishnu Narayan Mishra[4]

[1]Department of Mathematics, National Institute of Technology, Silchar - 788 010, District - Cachar (Assam), India
[2]L. 1627 Awadh Puri Colony Beniganj, Phase – III, Opposite – Industrial Training Institute (I. T. I.), Faizabad - 224 001 (Uttar Pradesh), India
[3]Department of Mathematics, RJIT, BSF Academy, Tekanpur, Gwalior (M.P.), India
[4]Applied Mathematics and Humanities Department, Sardar Vallabhbhai National Institute of Technology, Ichchhanath Mahadev Dumas Road, Surat – 395 007 (Gujarat), India

Email address:
lakshminarayanmishra04@gmail.com (L. N. Mishra), manoj240674@yahoo.co.in (M. Sharma), vishnunarayanmishra@gmail.com (V. N. Mishra)

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Abstract: In the present era, fractional calculus plays an important role in various fields. Fractional Calculus is a field of mathematic study that grows out of the traditional definitions of the calculus integral and derivative operators in much the same way fractional exponents is an outgrowth of exponents with integer value. Based on the wide applications in engineering and sciences such as physics, mechanics, chemistry, and biology, research on fractional ordinary or partial differential equations and other relative topics is active and extensive around the world. In the past few years, the increase of the subject is witnessed by hundreds of research papers, several monographs, and many international conferences.The purpose of present paper to solve 1-D fractal heat-conduction problem in a fractal semi-infinite bar has been developed by local fractional calculus employing the analytical Manoj Generalized Yang-Fourier transforms method.

Keywords: Fractal Bar, Heat-Conduction Equation, Lakshmi-Manoj Generalized Yang-Fourier Transforms, Yang-Fourier Transforms, Local Fractional Calculus

1. Introduction

Manoj Generalized Yang-Fourier transforms which is obtained by authors by generalization of Yang-Fourier transforms is a technique of fractional calculus for solving mathematical, physical and engineering problems. The fractional calculus is continuously growing in last five decades [1-7]. Most of the fractional ordinary differential equations have exact analytic solutions, while others required either analytical approximations or numerical techniques to be applied, among them: fractional Fourier and Laplace transforms [8, 41], heat-balance integral method [9-11], variation iteration method (VIM) [12-14], decomposition method [15, 41], homotopy perturbation method [16, 41] etc.

The problems in fractal media can be successfully solved by local fractional calculus theory with problems for non-differential functions [25-32]. Local fractional differential equations have been applied to model complex systems of fractal physical phenomena [30-41] local fractional Fourier series method [38], Yang-Fourier transform [39, 40, 41]

2. Generalized Yang-Fourier Transform and Its Properties

Let us Consider $f(x)$ is local fractional continuous in $(-\infty,\infty)$ we denote as $f(x)\in C_{\alpha,\beta}(-\infty,\infty)$ [32, 33, 35].

Let $f(x)\in C_{\alpha,\beta}(-\infty,\infty)$ The Generalized Yang-Fourier transform developed by authors is written in the form [30, 31, 39, 40, 41]:

$$F_{\alpha,\beta}\{f(x)\} = f_{\omega}^{F,\alpha,\beta}(\omega) =$$

$$\frac{(\delta)_\beta}{\Gamma(1+\alpha+\beta)}\int_{-\infty}^{\infty} E^{\delta}{}_{\alpha,\beta}\left(-i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right)f(x)(dx)^{\alpha+\beta}. \quad (1)$$

When we put β equal to zero, it converts in to the Yang-Fourier transform [41].

Then, the local fractional integration is given by [30-32, 35-37, 41]:

$$\frac{(\delta)_\beta 1}{\Gamma(1+\alpha+\beta)}\int_a^b f(t)\,(dx)^{\alpha+\beta} = \frac{(\delta)_\beta}{\Gamma(1+\alpha+\beta)}\lim_{\Delta t\to 0}\sum_{j=0}^{j=N-1} f(t_j)\,(\Delta t_j)^{\alpha+\beta} \quad (2)$$

where $\Delta t_j = t_{j+1} - t_j$, $\Delta t = max\{\Delta t_1, \Delta t_2, \Delta t_j, \dots\}$ and $\{t_j, t_{j+1}\}$, $j = 0, \dots, N-1$, $t_0 = a$, $t_N = b$, is a partition of the interval $[a, b]$.

If $F_{\alpha,\beta}\{f(x)\} = f_\omega^{F,\alpha,\beta}(\omega)$, then its inversion formula takes the form [30, 31, 39, 40, 41]

$$f(x) = F_{\alpha,\beta}^{-1}[f_\omega^{F,\alpha,\beta}(\omega)] = \frac{(\delta)_\beta}{(2\pi)^{\alpha+\beta}}\int_{-\infty}^{\infty} E^{\delta}{}_{\alpha,\beta}\left(-i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right)f_\omega^{F,\alpha,\beta}(\omega)(d\omega)^{\alpha+\beta}, \quad (3)$$

when we put β equal to zero, it converts in to the Yang Inverse Fourier transform [41].

Some properties are shown as it follows [30, 31]:

Let $F_{\alpha,\beta}\{f(x)\} = f_\omega^{F,\alpha,\beta}(\omega)$, and $F_{\alpha,\beta}\{g(x)\} = f_\omega^{F,\alpha,\beta}(\omega)$, and let be two constants, $if\ (\delta)_0$. Then we have:

$$F_{\alpha,\beta}\{cf(x) + dg(x)\} = cF_{\alpha,\beta}\{f(x)\} + dF_{\alpha,\beta}\{g(x)\} \quad (4)$$

If $\lim_{|x|\to\infty} f(x) = 0$, then we have:

$$F_{\alpha,\beta}\{f^{\alpha,\beta}(x)\} = i^{\alpha+\beta}\omega^{\alpha+\beta}F_{\alpha,\beta}\{f(x)\} \quad (5)$$

In equation (5) the local fractional derivative is defined as:

$$f^{\alpha,\beta}(x_0) = \left.\frac{d^{\alpha+\beta}f(x)}{dx^{\alpha+\beta}}\right|_{x=x_0} = \lim_{x\to x_0}\frac{\Delta^{\alpha+\beta}[f(x)-f(x_0)]}{(x-x_0)^{\alpha+\beta}} \quad (6)$$

where

$$\Delta^{\alpha+\beta}[f(x) - f(x_0)] \cong \Gamma(1+\alpha+\beta)\Delta[f(x) - f(x_0)],$$

As a direct result, repeating this process, when:

$$f(0) = f^{\alpha,\beta}(0) = \cdots = f^{(k-1)\alpha,(k-1)\beta}(0) = 0 \quad (7)$$

$$F_{\alpha,\beta}\{f^{k\alpha,k\beta}(x)\} = i^{\alpha+\beta}\omega^{\alpha+\beta}F_{\alpha,\beta}\{f(x)\} \quad (8)$$

3. Heat Conduction in a Fractal Semi-Infinite Bar

If a fractal body is subjected to a boundary perturbation, then the heat diffuses in depth modeled by a constitutive relation where the rate of fractal heat flux $\overline{q}(x,y,z,t)$ is proportional to the local fractional gradient of the temperature [32,41], namely:

$$\overline{q}(x,y,z,t) = -K^{2\alpha+2\beta}\nabla^{\alpha+\beta}T(x,y,z,t). \quad (9)$$

Here the pre-factor $K^{2a+2\beta}$ is the thermal conductivity of the fractal material. Therefore, the fractal heat conduction equation without heat generation was suggested in [32] as:

$$K^{2\alpha+2\beta}\frac{d^{2(\alpha+\beta)}T(x,y,z,t)}{dx^{2(\alpha+\beta)}} - \rho_{\alpha+\beta}c_{\alpha+\beta}\frac{d^{2(\alpha+\beta)}T(x,y,z,t)}{dx^{2(\alpha+\beta)}} = 0, \quad (10)$$

where $\rho_{\alpha+\beta}$ and $c_{\alpha+\beta}$ are the density and the specific heat of material, respectively.

The fractal heat-conduction equation with a volumetric heat generation $g(x,y,z,t)$ can be described as [32, 41]:

$$K^{2\alpha+2\beta}\nabla^{2\alpha+2\beta}T(x,y,z,t) + g(x,y,z,t)\rho_{\alpha+\beta}c_{\alpha+\beta}\frac{\partial^{(\alpha+\beta)}T(x,y,z,t)}{\partial t^{(\alpha+\beta)}} \quad (11)$$

The 1-D fractal heat-conduction equation [32, 41] reads as:

$$K^{2\alpha+2\beta}\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}} - \rho_{\alpha+\beta}c_{\alpha+\beta}\frac{\partial^{(\alpha+\beta)}T(x,t)}{\partial t^{(\alpha+\beta)}} = 0,\ 0 < x < \infty,\ t > 0 \quad (12a)$$

with initial and boundary conditions are:

$$\frac{\partial^{(\alpha+\beta)}T(0,t)}{\partial t^{(\alpha+\beta)}} = E^{\delta}{}_{\alpha+\beta}t^{\alpha+\beta}, T(0,t) = 0 \quad (12b)$$

The dimensionless forms of (12a, b) are [35, 41]:

$$\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}} = \frac{\partial^{(\alpha+\beta)}T(x,t)}{\partial x^{(\alpha+\beta)}} = 0 \quad (13a)$$

$$\frac{\partial^{(\alpha+\beta)}T(0,t)}{\partial x^{(\alpha+\beta)}} = E^{\delta}{}_{\alpha+\beta}t^{\alpha+\beta}, T(0,t) = 0. \quad (13b)$$

Based on equation (12a), the local fractional model for 1-D fractal heat-conduction in a fractal semi-infinite bar with a source term $g(x,t)$ is:

$$K^{2\alpha+2\beta}\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}} - \rho_{\alpha+\beta}c_{\alpha+\beta}\frac{\partial^{(\alpha+\beta)}T(x,t)}{\partial t^{(\alpha+\beta)}} = g(x,t),\ -\infty < x < \infty,\ t > 0 \quad (14a)$$

with

$$T(x,0) = f(x), -\infty < x < \infty. \quad (14b)$$

The dimensionless form of the model (14a, b) is:

$$\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}} = \frac{\partial^{(\alpha+\beta)}T(x,t)}{\partial t^{(\alpha+\beta)}} = 0,\ -\infty < x < \infty, t > 0 \quad (15a)$$

$$T(x,0) = f(x), -\infty < x < \infty. \quad (15b)$$

4. Solutions by the Generalized Yang-Fourier Transform Method

Let us consider that $F_{\alpha,\beta}\{T(x,t)\} = T_\omega^{F,\alpha,\beta}(\omega,t)$ is the Generalized Yang-Fourier transform of *T*(*x*, *t*), regarded as a non-differentiable function of *x*. Applying the Yang-Fourier transform to the first term of equation (15a), we obtain:

$$F_{\alpha,\beta}\left\{\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}}\right\} = \left(i^{2(\alpha+\beta)}\omega^{2(\alpha+\beta)}\right)T_\omega^{F,\alpha,\beta}(\omega,t) = \omega^{2(\alpha+\beta)}T_\omega^{F,\alpha,\beta}(\omega,t). \quad (16a)$$

On the other hand, by changing the order of the local fractional differentiation and integration in the second term of equation (15a), we get:

$$F_{\alpha,\beta}\left\{\frac{\partial^{2(\alpha+\beta)}}{\partial t^{2(\alpha+\beta)}}T(x,t)\right\}=\frac{\partial^{(\alpha+\beta)}}{\partial t^{(\alpha+\beta)}}T_{\omega}^{F,\alpha,\beta}(\omega,t). \quad (16b)$$

For the initial value condition, the Yang-Fourier transform provides:

$$F_{\alpha,\beta}\{T(x,0)\}=T_{\omega}^{F,\alpha,\beta}(\omega,0)=F_{\alpha,\beta}\{f(x)\}=f_{\omega}^{F,\alpha,\beta}(\omega). \quad (16c)$$

Thus we get from equation. (16a, b, c):

$$\frac{\partial^{(\alpha+\beta)}}{\partial t^{(\alpha+\beta)}}T_{\omega}^{F,\alpha,\beta}(\omega,t)+\omega^{2(\alpha+\beta)}T_{\omega}^{F,\alpha,\beta}(\omega,t)=0,\ T_{\omega}^{F,\alpha,\beta}(\omega,0)=f_{\omega}^{F,\alpha,\beta}(\omega). \quad (17)$$

This is an initial value problem of a local fractional differential equation with t as independent variable and w as a parameter.

$$T(\omega,t)=f_{\omega}^{F,\alpha,\beta}(\omega)E^{\delta}{}_{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right). \quad (18a)$$

Consequently, using inversion formula, equation (3), we obtain:

$$T(x,t)=\frac{(\delta)_{\beta}}{(2\pi)^{\alpha+\beta}}\int_{-\infty}^{\infty}E^{\delta}{}_{\alpha,\beta}\left(i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right)f_{\omega}^{F,\alpha,\beta}(\omega)E^{\delta}{}_{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right)(d\omega)^{\alpha+\beta} \quad (18b)$$

$$M_{\omega}^{F,\alpha,\beta}(\omega)=\frac{1}{(2\pi)^{\alpha+\beta}}E^{\delta}{}_{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right). \quad (18c)$$

From [30, 32] we obtain,

$$F_{\alpha+\beta}\left\{E^{\delta}{}_{\alpha+\beta}\left(-\frac{\omega^{2(\alpha+\beta)}}{C^{2(\alpha+\beta)}}\right)\right\}=\frac{C^{(\alpha+\beta)}\pi^{\frac{\alpha+\beta}{2}}}{\Gamma(1+\alpha+\beta)}E^{\delta}E_{\alpha,\beta}\left(-\frac{C^{2(\alpha+\beta)}\omega^{2(\alpha+\beta)}}{4^{(\alpha+\beta)}}\right). \quad (19a)$$

Let $C^{2(\alpha+\beta)}/4^{\alpha+\beta}=t^{\alpha+\beta}$. Then we get:

$$F_{\alpha+\beta}\left\{E^{\delta}{}_{\alpha+\beta}\left(-\frac{\omega^{2(\alpha+\beta)}}{4^{\alpha+\beta}\,t^{\alpha+\beta}}\right)\right\}=\frac{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}{\Gamma(1+\alpha+\beta)}E^{\delta}{}_{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right)=(\delta)_{\beta}\frac{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}{\Gamma(1+\alpha+\beta)}(2\pi)^{\alpha+\beta}M_{\omega}^{F,\alpha,\beta}(\omega) \quad (19b)$$

Thus, $M_{\omega}^{F,\alpha,\beta}(\omega)$ have the inverse

$$\frac{(\delta)_{\beta}}{(2\pi)^{\alpha+\beta}}\int_{-\infty}^{\infty}E^{\delta}{}_{\alpha,\beta}\left(i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right)M_{\omega}^{F,\alpha,\beta}(\omega)(d\omega)^{\alpha+\beta}==\frac{\Gamma(1+\alpha+\beta)}{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}\frac{(\delta)_{\beta}1}{(2\pi)^{\alpha+\beta}}E^{\delta}{}_{\alpha+\beta}\left(-\frac{\omega^{2(\alpha+\beta)}}{4^{\alpha+\beta}\,t^{\alpha+\beta}}\right). \quad (19c)$$

Hence, we get:

$$T(x,t)=(Mf)(x)=(\delta)_{\beta}\frac{\Gamma(1+\alpha+\beta)}{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}\int_{-\infty}^{\infty}f(\xi)E^{\delta}{}_{\alpha,\beta}\left(-\frac{(x-\xi)^{2(\alpha+\beta)}}{4^{\alpha+\beta}t^{\alpha+\beta}}\right)(d\xi)^{\alpha+\beta}. \quad (20)$$

Special case:

If we take $\beta=0$ then the results of generalized Yang Fourier Transforms convert in Yang Fourier Transforms results [41].

5. Conclusions

The communication, presented an analytical solution of 1-D heat conduction in fractal semi-infinite bar by the Generalized Yang-Fourier transform of non-differentiable functions. Some important interesting applications can be seen in ([42]-[46]).

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