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Oscillation of Second Order Nonlinear Neutral Differential Equations

Hussain Ali Mohamad[1], Intidhar Zamil Mushtt[2]

[1]University of Baghdad, College of Science for Women, Baghdad, Iraq
[2]Al Mustansiriyah University, College of Education, Baghdad, Iraq

Email address:
hussainmohamad22@gmail.com (H. A. Mohamad), intidharalbayaty@yahoo.com (I. Z. Mushtt)

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Abstract: The oscillation criteria are investigated for all solutions of second order nonlinear neutral delay differential equations. Our results extend and improve some results well known in the literature see ([14] theorem 3.2.1 and theorem 3.2.2 pp.385-388). Some examples are given to illustrate our main results.

Keywords: Oscillation, Neutral Differential Equations

1. Introduction

In this paper we consider the second order neutral differential equations of the form

$$[y(t)+\ \delta p(t)y(\tau(t))]''+q(t)f\left(y(\sigma(t))\right)=0, t\geq t_0, \delta=\pm 1 \quad (1.1)$$

Under the following assumptions:

(A_1) $p(t), q(t)\in C([t_0,\infty);R^+), p(t)>0$.

(A_2) $\tau(t),\sigma(t)\in C([t_0,\infty);R), \lim_{t\to\infty}\tau(t)=\infty$, $\lim_{t\to\infty}\sigma(t)=\infty$, $\tau(t)<t$ is increasing function. and $\tau^k(t)=\tau^{k-1}(\tau(t)), k=1,2,\dots,n,\ \tau^{k-1}(t)\geq t_0$.

(A_3) $f(\omega)\in C(R;R), \omega f(\omega)>0$ for $\omega\neq 0$, $f(uv)\geq f(u)f(v)$, $uv>0$ and $|f(u)|\geq\beta|u|, \beta>0$.

By a solution of (1.1), we mean a function $y(t)$ such that $y(t)+\ \delta p(t)y(\tau(t))$ is twice continuously differentiable and $y(t)$ satisfies (1.1) on $[t_0,\infty)$.

A solution is said to be oscillatory if it has arbitrarily large zeros, otherwise it is said to be nonoscillatory, that is a solution $y(t)$ is nonoscillatory if and only if $y(t)$ is eventually positive or eventually negative. Equation (1.1) is said to be oscillatory if every solution of (1.1) is oscillatory.

There has been a considerable investigation of the oscillations of second order neutral differential equations. For typical results we refer to the papers [1,3,5-7,10,12-14]. The results in this paper extend and improve some results well known in ([14], theorem 3.2.1 and theorem 3.2.2 pp.385-388). Also, we show that (A_3) implies that $f(t)$ is nondecreasing while in [13] they assume (A_3) holds in addition to $f(t)$ is nondecreasing. Some examples are given to illustrate the obtained results.

2. Main Results

In this section we present our main results, the following lemma shows that (A_3) implies $f(u)$ is nondecreasing.

Lemma 2.1 Suppose that $f\in C(R,R), uf(u)\geq 0$ for $u\neq 0, \frac{f(u)}{u}\geq\beta>0$ and $f(uv)\geq f(u)f(v)$ for $uv>0$. Then f is nondecreasing function.

Proof: Assume that $u,v>0$ then $f(u),f(v)>0$. We have $f(uv)\geq f(u)f(v)$, $f(u)\geq\beta u$.Then

$$f(uv)\geq\beta uf(v)$$

$$f(uv)\geq\beta vf(u)$$

$$0\geq\beta[uf(v)-vf(u)]$$

$$vf(u)\geq uf(v)$$

$$\frac{vf(u)}{uf(u)}\geq\frac{uf(v)}{uf(u)}$$

$$\frac{f(v)}{f(u)}\leq\frac{v}{u} \quad (2.1)$$

If $v\leq u$ then (2.1) implies that $f(v)\leq f(u)$

Hence f is nondecreasing function. □

Lemma 2.2. [7] Suppose that $f(t);\ g(t):R^+\to R^+$; $g(t)<t, \lim_{t\to\infty}g(t)=\infty$ and

$$\lim\ \inf_{t\to\infty}\int_{g(t)}^{t} f(s)ds > \frac{1}{e}$$

Then the inequality $y'(t) + f(t)y(g(t)) \le 0$ cannot has eventually positive solution, and the inequality $y'(t) + f(t)y(g(t)) \ge 0$ cannot has eventually negative solution.

In the following theorem we investigate eq. (1.1) when $\delta = 1$.

Theorem 2.3 Assume that $(A_1)-(A_3)$ hold, $\sigma(t) \ge t$, and

$$\lim\ \sup_{t\to\infty}\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(\sigma(t))\right) < 1 \quad (2.2)$$

For some positive odd integer n and there exists a function $h(t)\epsilon C^1[t_0,\infty), R^+]$ such that

$$\lim\sup_{t\to\infty}\int_T^t\left[\beta h(s)q(s)f(\gamma(s)) - \frac{[h'(s)]^2}{4h(s)}\right]ds = \infty,\ T \ge t_0 \quad (2.3)$$

where $\gamma(t) = 1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p(\tau^{k-1}(\sigma(t)))$. Then every solution of (1.1) oscillates.

Proof. Assume for the sake of contradiction that (1.1) has a nonoscillatory solution, let $y(t)$ be eventually positive solution of (1.1) and there exists a positive odd integer n such that $y\left(\tau^i(t)\right) > 0, i = 0,1,2,\dots,n, y(\sigma(t)) > 0$, for $t \ge t_1 \ge t_0$. Let

$$z(t) = y(t) + p(t)y(\tau(t)) \quad (2.4)$$

From (1.1) and (2.4) it follows that

$$z''(t) = -q(t)f\left(y(\sigma(t))\right) \le 0, t \ge t_1, \quad (2.5)$$

Since $z(t) > 0$ then it follows that $z'(t) > 0, t \ge t_2 \ge t_1$

Form (2.4) we obtain

$$y(t) = z(t) - p(t)y(\tau(t))$$
$$= z(t) - p(t)[z(\tau(t)) - p(\tau(t))y(\tau^2(t))]$$
$$= z(t) - p(t)\left[z(\tau(t)) - p(\tau(t))[z(\tau^2(t)) - p(\tau^2(t))y(\tau^3(t))]\right]$$
$$= z(t) + \prod_{i=0}^{n} p\left(\tau^i(t)\right)y(\tau^{n+1}(t)) + \sum_{i=1}^{\frac{n-1}{2}}\prod_{k=0}^{2i-1} p(\tau^k(t))z(\tau^{2i}(t)) - \sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(t)\right)z\left(\tau^{2i-1}(t)\right)$$

$$y(t) \ge z(t) - \sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(t)\right)z\left(\tau^{2i-1}(t)\right)$$
$$\ge \left[1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(t)\right)\right]z(t)$$

$$y(\sigma(t)) \ge \left[1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(\sigma(t))\right)\right]z(\sigma(t)) \quad (2.6)$$

By (A_3) and Lemma 2.1 we get

$$f\left(y(\sigma(t))\right) \ge f(1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p(\tau^{k-1}(\sigma(t)))\,z(\sigma(t)))$$
$$\ge f(1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(\sigma(t))\right)f(z(\sigma(t)))$$

$$f\left(y(\sigma(t))\right) \ge f(1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(\sigma(t))\right)\beta z(\sigma(t)) \quad (2.7)$$

Substituting (2.7) into (2.5) we obtain

$$z''(t) + \beta q(t)\left[f\left(1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(\sigma(t))\right)\right)\right]z(\sigma(t)) \le 0 \quad (2.8)$$

Define

$$w(t) = \frac{h(t)z'(t)}{z(\sigma(t))} \quad (2.9)$$

It is obvious that $w(t) > 0$. Let $\gamma(t) = 1-\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p(\tau^{k-1}(\sigma(t)))$ then differentiate (2.9) and using (2.8) it follows that

$$\frac{h(t)w'(t)-w(t)h'(t)}{(h(t))^2}z(t) + \frac{w(t)}{h(t)}z'(t) + \beta q(t)f(\gamma(t))z(\sigma(t)) \le 0 \quad (2.8)$$

$$w'(t) - \frac{h'(t)}{h(t)}w(t) + \frac{z'(t)}{z(t)}w(t) + \beta h(t)q(t)f(\gamma(t)) \le 0$$

Then for $t \ge t_3(=T) \ge t_2$

$$w'(t) \le \frac{h'(t)}{h(t)}w(t) - \frac{z'(t)}{z(t)}w(t) - \beta h(t)q(t)f(\gamma(t))$$
$$= \frac{h'(t)}{h(t)}w(t) - \frac{1}{h(t)}w^2(t) - \beta h(t)q(t)f(\gamma(t))$$
$$= -\beta h(t)q(t)f(\gamma(t)) - \left[\frac{1}{\sqrt{h(t)}}w(t) - \frac{h'(t)}{2\sqrt{h(t)}}\right]^2 + \frac{\left(h'(t)\right)^2}{4h(t)}$$
$$\le -\beta h(t)q(t)f(\gamma(t)) + \frac{\left(h'(t)\right)^2}{4h(t)}$$

$$w'(t) \le -\left[\beta h(t)q(t)f(\gamma(t)) - \frac{\left(h'(t)\right)^2}{4h(t)}\right]$$

Integrating the last inequality from T to t, we get

$$w(t) \le w(T) - \int_T^t\left[\beta h(s)q(s)f(\gamma(s)) - \frac{\left(h'(s)\right)^2}{4h(s)}\right]ds,$$

as $t\to\infty$ the last inequality leads to $w(t)\to-\infty$, which is a contradiction. □

Example 1 Consider the neutral differential equation

$$\left[y(t) + e^{-t}y\left(t-\frac{3\pi}{2}\right)\right]'' + e^{-\frac{\pi}{2}}f\left(y(t+2\pi)\right) = 0, t \ge 1, \text{(E1)}$$

$$\tau(t) = t-\frac{3\pi}{2}, \sigma(t) = t+2\pi, p(t) = e^{-t},$$

$$f\left(y(t+2\pi)\right) = y(t+2\pi)\left[e^{\frac{\pi}{2}} - 2e^{-t+\frac{\pi}{2}}\right], q(t) = e^{-\frac{\pi}{2}}$$

one can find that all conditions of theorem 2.3 are hold

Let $n = 3, \beta = 3, h(t) = c > 0$,

$$\lim\ \sup_{n\to\infty}\sum_{i=1}^{\frac{n+1}{2}}\prod_{k=1}^{2i-1} p\left(\tau^{k-1}(\sigma(t))\right) =$$
$$= \lim_{t\to\infty}\left[e^{-t-2\pi} + e^{-3t-2\pi}\right] = 0$$

$$\limsup_{t\to\infty}\int_T^t\left[\beta h(s)q(s)f(\gamma(s))-\frac{[h'(s)]^2}{4\alpha h(s)}\right]ds=$$

$$=\lim_{t\to\infty}\int_T^t\frac{3c}{2}e^{-\frac{\pi}{2}}f(\gamma(s))ds=\infty$$

so every solution of eq.(E1) oscillates, for instance $y(t)=\sin t$ is such solution.

The following theorem investigate equation (1.1) when $\delta=-1$.

Theorem 2.4 Assume $(A_1)-(A_3)$ hold, $p(t)\le 1$, $\tau^n(\sigma(t))\ge t, \tau^{-n}(\sigma(\alpha(t)))\le t, \alpha(t)>t$ and there exists a function $h(t)\epsilon C^1[t_0,\infty), R^+]$ such that

$$\int_T^\infty[\beta h(s)q(s)f(\gamma_1(s))-\frac{(h'(s))^2}{4h(s)}ds]=\infty, T\ge t_0, \quad (2.10)$$

$$\liminf_{t\to\infty}\int_{\tau^{-n}(\sigma(\alpha(t)))}^t\int_s^{\alpha(s)}q(v)f(\theta(v))\,dvds>\frac{1}{\beta e} \quad (2.11)$$

where $\gamma_1(t)=1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(\sigma(t)))$, $\theta(t)=\sum_{i=1}^n\prod_{k=0}^{i-1}\frac{1}{p(\tau^{-k}(\sigma(t)))}$. Then every solution of (1.1) is oscillatory.

Proof : Assume for the sake of contradiction that (1.1) has a eventually positive solution $y(t)$, then there exists $t_1\ge t_0$ such that for $t\ge t_1$, $y\left(\tau^i(t)\right)>0, i=1,2,\dots,n$, where n is positive integer and $y(\sigma(t))>0$. Let

$$z(t)=y(t)-p(t)y(\tau(t) \quad (2.13)$$

From equation (1.1) and (2.13) we get

$$z''(t)=-q(t)f\left(y(\sigma(t))\right)\le 0, t\ge t_1 \quad (2.14)$$

hence $z'(t), z(t)$ are monotone functions, we claim that $z'(t)>0, for\ t\ge t_2\ge t_1$ otherwise $z'(t)<0$ leads to $z(t)<0$ for $t\ge t_2\ge t_1$ and $\lim_{t\to\infty}z(t)=-\infty$ it follows that $y(t)<p(t)y(\tau(t))\le y(\tau(t))$ then $y(t)$ is bounded.

On the other hand $z(t)>-p(t)y(\tau(t)\ge -y(\tau(t))$

Then $\lim_{t\to\infty}y(t)=\infty$, leads to a contradiction. Therefore $z'(t)>0$ for $t\ge t_2\ge t_1$, there are two cases for $z(t)$:

(a) $z(t)>0$ for $t\ge t_2\ge t_1$; (b) $z(t)<0$ for $t\ge t_2\ge t_1$

Case (a) $z(t)>0$, $z'(t)>0$ and $z''(t)\le 0$

From (2.13) it follows that

$$y(t)=z(t)+p(t)y(\tau(t))=$$

$$=z(t)+p(t)[z(\tau(t))+p(\tau(t))y(\tau^2(t))]$$

$$=z(t)+p(t)z(\tau(t))+p(t)p(\tau(t))[z(\tau^2(t))+p(\tau^2(t))y(\tau^3(t))]$$

$$y(t)=z(t)+\sum_{i=1}^n\prod_{k=0}^{i-1}p\left(\tau^k(t)\right)z\left(\tau^i(t)\right)+\prod_{i=0}^n p\left(\tau^i(t)\right)y(\tau^{n+1}(t))$$

$$\ge z(t)+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(t))z\left(\tau^i(t)\right)$$

$$\ge[1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(t))]z(\tau^n(t))$$

Then there exists $t_3(=T)\ge t_2$ such that for $t\ge T$

$$y(\sigma(t))\ge[1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(\sigma(t)))]z(\tau^n(\sigma(t))) \quad (2.15)$$

Then by using (A_3) and Lemma 2.1 we have

$$f(y(\sigma(t)))\ge f[1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(\sigma(t)))]f(z(\tau^n(\sigma(t))))$$

$$\ge\beta f(1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(\sigma(t))))z(\tau^n(\sigma(t))) \quad (2.16)$$

By substituting (2.16) in (1.1) we obtain

$$z''(t)+\beta q(t)f(1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(\sigma(t))))z(\tau^n(\sigma(t)))\le 0 \quad (2.17)$$

Define

$$w(t)=\frac{h(t)z'(t)}{z(t)}$$

Then $w(t)>0$, moreover

$$w'(t)+\frac{z'(t)}{z(t)}w(t)-\frac{h'(t)}{h(t)}w(t)+\beta h(t)q(t)f(\gamma_1(t))\frac{z(\tau^n(\sigma(t)))}{z(t)}\le 0$$

Where $\gamma_1(t)=1+\sum_{i=1}^n\prod_{k=0}^{i-1}p(\tau^k(\sigma(t)))$.

Since $z(t)\le z\left(\tau^n(\sigma(t))\right)$ then

$$w'(t)\le-\frac{z'(t)}{z(t)}w(t)+\frac{h'(t)}{h(t)}w(t)-\beta h(t)q(t)f(\gamma_1(t))$$

$$=-\frac{1}{h(t)}w^2(t)+\frac{h'(t)}{h(t)}w(t)-\beta h(t)q(t)f(\gamma_1(t))$$

$$=-\left[\frac{1}{\sqrt{h(t)}}w(t)-\frac{h'(t)}{2\sqrt{h(t)}}\right]^2+\frac{(h'(t))^2}{4h(t)}-\beta h(t)q(t)f(\gamma_1(t))$$

Thus

$$w'(t)\le\frac{(h'(t))^2}{4h(t)}-\beta h(t)q(t)f(\gamma_1(t))$$

Integrating the last inequality from T to t we get

$$w(t)\le w(T)-\int_T^t[\beta h(s)q(s)f(\gamma_1(s)-\frac{(h'(s))^2}{4h(s)}]ds$$

as $t\to\infty$, and by (2.11) we deduce that $w(t)\to-\infty$ this is a contradiction.

Case (b) $z(t)<0, z'(t)>0$ and $z''(t)\le 0$

From (2.13) it follows that $y(\tau(t))=\frac{1}{p(t)}(y(t)-z(t))$

Then $y(t)=\frac{1}{p(\tau^{-1}(t))}y(\tau^{-1}(t))-\frac{1}{p(\tau^{-1}(t))}z(\tau^{-1}(t))$

$$y(t)=\frac{1}{p(\tau^{-1}(t))}[\frac{1}{p(\tau^{-2}(t)}y(\tau^{-2}(t))-\frac{1}{p(\tau^{-2}(t))}z(\tau^{-2}(t))]-\frac{1}{p(\tau^{-1}(t))}z(\tau^{-1}(t))$$

$$y(t)=\frac{1}{\prod_{i=1}^n p(\tau^{-i}(t))}y(\tau^{-n}(t))-\sum_{i=1}^n\prod_{k=1}^i\frac{1}{p(\tau^{-k}(t))}z(\tau^{-i}(t))$$

$$\ge-\sum_{i=1}^n\prod_{k=1}^i\frac{1}{p(\tau^{-k}(t))}z(\tau^{-i}(t))$$

$\geq -\sum_{i=1}^{n}\prod_{k=1}^{i}\frac{1}{p\left(\tau^{-k}(t)\right)}z(\tau^{-n}(t))$

Then there exists $t_3(=T) \geq t_2$ such that for $t \geq T$

$$y(\sigma(t)) \geq -\sum_{i=1}^{n}\prod_{k=1}^{i}\frac{1}{p(\tau^{-k}(\sigma(t)))}z(\tau^{-n}(\sigma(t))) \quad (2.18)$$

Then by using (A_3) and Lemma 2.1 we have

$$f(y(\sigma(t))) \geq f(\sum_{i=1}^{n}\prod_{k=1}^{i}\frac{1}{p(\tau^{-k}(\sigma(t)))})f(-z(\tau^{-n}(\sigma(t))))$$

$$\geq -\beta f\left(\sum_{i=1}^{n}\prod_{k=1}^{i}\frac{1}{p(\tau^{-k}(\sigma(t)))}\right)z(\tau^{-n}(\sigma(t))) \quad (2.19)$$

By substituting (2.19) in (2.14) we obtain

$$z''(t) - \beta q(t) f\left(\sum_{i=1}^{n}\prod_{k=1}^{i}\frac{1}{p(\tau^{-k}(\sigma(t)))}\right)z(\tau^{-n}(\sigma(t))) \leq 0 \quad (2.20)$$

Integrating (2.20) from t to $\alpha(t)$ where $\alpha(t) > t$ is continuous function, we get

$$z'(t) + \beta z(\tau^{-n}(\sigma(\alpha(t)))) \int_{t}^{\alpha(t)} q(s) f(\theta(s)) \geq 0 \quad (2.21)$$

where $\theta(t) = \sum_{i=1}^{n}\prod_{k=0}^{i-1}\frac{1}{p(\tau^{-k}(\sigma(t)))}$. By lemma 2.2 and condition (2.13) it follows that eq(2.21) cannot has eventually negative solution, which is a contradiction.

Then every solutions of (1.1) is oscillatory. □

Example 2 Consider the neutral differential equation

$$[y(t) - e^{-2\pi}(3 - \sin t)\, y(t - 2\pi)]''$$

$$+2e^{-\frac{9\pi}{2}}(\cos t + 2)\, y\left(t - \frac{9\pi}{2}\right) = 0, t \geq T \quad (E2)$$

one can find that all conditions of theorem 2.4(or corollary 2.5) are hold, to see that:

Let $n = 2, \beta = \frac{1}{2}, h(t) = c > 0,$

$$f(\gamma_1(t)) = 1 + e^{-2\pi}(3 + \cos t) + e^{-2\pi}(3 + \cos t)^2$$

$$f(\theta(t)) = \frac{1}{e^{-2\pi}(3 + \cos t)} + \frac{1}{e^{-4\pi}(3 + \cos t)^2}$$

$$\lim_{t\to\infty}\int_{T}^{t} ce^{-\frac{9\pi}{2}}(\cos s + 2) f(\gamma_1(s))\ ds = \infty$$

$$\lim_{t\to\infty}\int_{T}^{t} ce^{-\frac{9\pi}{2}}(\cos s + 2) f(\theta(s))\ ds = \infty$$

so every solution of eq.(E2) oscillate for instance $y(t) = e^{t}\cos t$ is such a solution.

3. Conclusions

As mention in the abstract the results in this paper extended and improved theorem 3.2.1 and theorem 3.2.2 [4]

The authors in[13] assume (A_3) holds in addition to $f(u)$ is nondecreasing while we show that (A_3) implies $f(u)$ is nondecreasing. Moreover we can establish other results by assuming $f(uv) \leq f(u)f(v)$ and $f(u) \leq \beta u$ also implies that $f(u)$ is nondecreasing in similar way.

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