



A Model-Based Approach for The Demonstration of Fermat's Last Theorem

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To cite this article:

Xu Hong. A Model-Based Approach for The Demonstration of Fermat's Last Theorem. *Pure and Applied Mathematics Journal*.

Vol. 6, No. 5, 2017, pp. 144-147. doi: 10.11648/j.pamj.20170605.12

Received: May 20, 2017; **Accepted:** August 2, 2017; **Published:** October 26, 2017

Abstract: Model-based approaches have many applications in bioinformatics studies. Fermat's last theorem is a theorem in number theory. This paper presented a model-based approach for the demonstration of Fermat's Last Theorem (FLT). Through the establishment of mathematical model, Fermat's Last Theorem (FLT) is converted into a problem for feasible solutions in linear programming. By prompting assumptions, and refuting the assumptions, it is demonstrated that Fermat's Last Theorem is right. The model-based approach for the demonstration of Fermat's Last Theorem (FLT) can assist scientific researchers in biological researches.

Keywords: Model-Based, Fermat's Last Theorem (FLT), Feasible Solutions, Linear Programming

1. Introduction

Fermat's Last Theorem is the name of the statement in number theory that:

It is impossible to separate any power higher than the second into two like powers, or, more precisely: If an integer n is greater than 2, then the equation $x^n + y^n = z^n$ has no solutions in non-zero integers x , y , and z .

In 1637, Pierre de Fermat wrote, in his copy of Claude-Gaspar Bachet's translation of the famous *Arithmetica* of Diophantus, "I have a truly marvellous proof of this proposition which this margin is too narrow to contain." (Original Latin: "Cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.")

Previous studies show that, Dr Andrew Wiles, a British mathematician and a former professor at Princeton University, has demonstrated Fermat's Last Theorem in 1990s using advanced modern mathematics (https://en.wikipedia.org/wiki/Fermat's_Last_Theorem).

However, how many people have fully understood Dr Andrew Wiles' thesis which is more than one hundred pages is still questionable.

One potential solution is provided by Fermat himself. However, Fermat's proof has never been found and the question is still open.

Model-based approaches have been widely used in many

bioinformatics studies [1-9].

Based on the understanding of Fermat's Last Theorem, this paper presented a model-based approach for the demonstration of Fermat's Last Theorem. Through the establishment of mathematical model, Fermat's Last Theorem is converted into a problem for feasible solutions in linear programming [10-14]. Results from the model-based analysis of Fermat's Last Theorem show that Fermat's Last Theorem is right.

2. Methods

A Model-Based Approach for the Demonstration of Fermat's Last Theorem Algorithm Details

Fermat's Last Theorem: If an integer n is greater than 2, then the equation $x^n + y^n = z^n$ has no solutions in non-zero integers x , y , and z .

Proof:

We only need to prove if $n > 2$ and n is a prime number and $x^n + y^n = z^n$ has no solutions in non-zero integers x , y , and z , then we can say Fermat's Last Theorem is right. (When $n > 2$ and n is a non-prime number, it is easy reduced to the circumstance for $n > 2$ and n is a prime number, Lemma 1.)

Lemma 1.

If $n > 2$ and n is a prime number and $x^n + y^n = z^n$ has no solutions in nonzero integers $x, y,$ and z ; then for $n > 2$ and n is a non-prime number, $x^n + y^n = z^n$ has no solutions in nonzero integers $x, y,$ and z .

Proof.

Suppose $p > 2$ and p is a prime number, $x^p + y^p = z^p$ has no solutions in nonzero integers $x, y,$ and z .

If $n > 2$ and n is a non-prime number, then n is the product of a prime number p_0 and a positive integer $q, n = q \times p_0$ ($q \in N^+, p_0$ is a prime number).

Because $a^{p_0} + b^{p_0} = c^{p_0}$ has no solutions in nonzero integers $a, b,$ and $c,$ we can conclude that $(x^q)^{p_0} + (y^q)^{p_0} = (z^q)^{p_0}$ has no solutions in nonzero integers $x, y,$ and z . That is to say $x^n + y^n = z^n$ has no solutions in nonzero integers $x, y,$ and z ($n > 2$ and n is a non-prime number).

Step 1 (Model Assumptions).

Assume $n > 2, n$ is a prime number and $x^n + y^n = z^n$ has solutions in nonzero integers $x, y,$ and z .

Step 2.

Since $y \neq x, y > x$ or $y < x,$ might as well set $y > x$ (Else, the same method can be applied to $y < x$); and because $x^n + y^n = z^n > y^n, z > y$.

Step 3.

Might as well set

$$z - y = b \quad (b \in N^+) \quad (1)$$

Since

$$x, b \in N^+, \exists a \ (a \in N^+), a = x + b \quad (a, b, x \in N^+) \quad (2)$$

Step 4.

From (1) and (2),

$$a - x = z - y \wedge z - a = y - x \quad (a, b, x \in N^+) \quad (3)$$

Step 5.

From Step 4, we can conclude that $y = a$ is a solution to the linear system of equations $a - x = z - y \wedge z - a = y - x$ in (3), $y = a$ is bound to be a feasible solution to the equation $x^n + y^n = z^n$ ($n > 2, n$ is a prime number).

Step 6.

Since $z - y = b$ and $a = x + b \ (a, b, x \in N^+),$ the equation $x^n + y^n = z^n$ ($n > 2, n$ is a prime number) can be rewritten as

$$(a - b)^n + y^n = (y + b)^n \quad (4)$$

Step 7.

Since $y = a$ is a feasible solution to the equation $x^n + y^n = z^n$ ($n > 2, n$ is a prime number), it can be concluded that $y = a$ is also bound to be a feasible solution to the equation $(a - b)^n + y^n = (y + b)^n \in (4).$

Step 8.

On the other hand, it can be demonstrated that $y = a$ is not a feasible solution to the equation $(a - b)^n + y^n = (y + b)^n$ in (4) (Lemma 2), and it can be concluded that $y = a$ is not a feasible solution to the equation $x^n + y^n = z^n$ ($n > 2, n$ is a prime number). This is a contradiction to the assumptions in Step 1. (Since according to the assumptions in Step 1, $y = a$ is bound to be a feasible solution to the equation $x^n + y^n = z^n$ ($n >$

$2, n$ is a prime number).

Lemma 2.

$y = a$ is not a feasible solution to the equation $(a - b)^n + y^n = (y + b)^n$ ($n > 2, n$ is a prime number).

Proof.

Suppose $y = a \ (a \in N^+)$ is a feasible solution to the equation $(a - b)^n + y^n = (y + b)^n$ ($n > 2$ and n is a prime number).

There are two circumstances:

(1). a, b are prime to each other, $(a, b) = 1.$

Suppose a, b are a solution to the equation $(a - b)^n + a^n = (a + b)^n.$ Equation $(a - b)^n + a^n = (a + b)^n$ can be rewritten as: $a^n = (a + b)^n - (a - b)^n.$

Because a^n is divisible by $a, (a + b)^n - (a - b)^n$ ($n > 2$ and n is a prime number) $= 2 \times (C_n^0 \times b^n + C_n^2 \times a^2 \times b^{n-2} + \dots + C_n^{n-1} \times a^{n-1} \times b) = 2 \times b^n + 2 \times a \times (C_n^2 \times a \times b^{n-2} + \dots + C_n^{n-1} \times a^{n-2} \times b)$ ($n > 2$ and n is a prime number) is not divisible by $a, a^n \neq (a + b)^n - (a - b)^n.$

We can conclude that a, b are not a solution to the equation $(a - b)^n + a^n = (a + b)^n,$ and $y = a \ (a \in N^+)$ is not a feasible solution to the equation $(a - b)^n + y^n = (y + b)^n$ ($n > 2$ and n is a prime number).

(2). a, b are not prime to each other.

Suppose $(a, b) = k. \exists a_1, \exists b_1, a = k \times a_1, b = k \times b_1,$ and a_1, b_1 are prime to each other, $(a_1, b_1) = 1.$

(1). a_1, b_1 are not a solution to the equation $(a - b)^n + a^n = (a + b)^n,$ we can conclude that $a, b,$ are also not a solution to the equation $(a - b)^n + a^n = (a + b)^n.$

So when a, b are not prime to each other, $y = a \ (a \in N^+)$ is not a feasible solution to the equation $(a - b)^n + y^n = (y + b)^n$ ($n > 2$ and n is a prime number).

The assumptions in Step 1 are not right, and Fermat's Last Theorem can be deduced.

3. Result and Discussion

Biologists usually sought to mathematic theories and methods to answer biological questions. Demonstration of Fermat's Last Theorem [15, 16] may assist scientific researchers in biological researches.

This paper presented a model-based approach for the demonstration of Fermat's Last Theorem for the general case $n > 2.$

Previous studies show that Fermat's Last Theorem has some connections with Fibonacci numbers [17]. Fibonacci numbers [18-21] have many implications in biological studies [22-27].

Fermat himself demonstrated the equation $x^n + y^n = z^n$ has no solutions in non-zero integers $x, y,$ and z for $n = 4.$ However, he didn't show us his proof of the theorem for the general case $n > 2.$

Model-based approaches have been widely adopted in many bioinformatics studies.

In early studies of protein-DNA interactions, Chip-on-chip technology was used [1]. Johnson et al build up a mathematic model for ChIP-on-chip studies [3]. Though it is primarily designed for the Affymetrix platform, it can also be extended to other chip-chip platforms [3].

ChIP-Seq is an improvement of ChIP-on-chip [5]. Zhang et al built up a mathematical model called MACS for ChIP-Seq data analysis [4, 9]. The model used a dynamic Poisson distribution to effectively capture local biases in the genome, allowing for more robust predictions.

Xu et al further divided ChIP-Seq signal into signal and noise, and they developed a signal-noise model for significance analysis of ChIP-Seq with negative control [8]. Compared with Zhang et al's model [4, 9], in addition to finding the strong protein-DNA binding sites in the genome, Xu et al's model can also find weak signals which Zhang et al's model cannot find [8].

Feng et al proposed a poisson-mixture model (PMM) for the identification of changes in RNA polymerase II binding quantity from PolII ChIP-Seq data [2].

White et al used a model-based approach for the study of the role of nonspecific interactions in molecular chaperones [7].

Recently, Si et al used a mixture model-based approach for the clustering of RNA-Seq data [6].

A recent study provided an ingenious proof of Fermat's Last Theorem (FLT) by associating Fermat's Last Theorem with the Beal Conjecture [28].

4. Conclusion

In this paper, a model-based approach for the demonstration of Fermat's Last Theorem for the general case $n > 2$ is proposed. Through the establishment of mathematical model, Fermat's Last Theorem is converted into a problem for feasible solutions in linear programming. By prompting assumptions, and refuting the assumptions, it is demonstrated that Fermat's Last Theorem is right.

Acknowledgements

The author acknowledges Systems Biology Division, Zhejiang-California International Nano Systems Institute, Zhejiang University, Hangzhou, Zhejiang 310058, China for providing financial support for this study.

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