1-Quasi Total Fuzzy Graph and Its Total Coloring

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Abstract: The fuzzy graph theory, its properties, total coloring and applications are currently climbing up. With this concept of fuzzy graph, total fuzzy graph is defined and its properties as well as fuzzy total colorings have been well discussed and studied. Similarly the theory of crisp graph, its properties, applications and colorings are well considered. Moreover, 1-quasi total graphs for crisp graphs, their properties and colorings were discussed by some researchers and the bounds for its total coloring have been established. In this manuscript, from the concept of fuzzy graph we introduced the definition of 1-quasi total graph for fuzzy graphs. To elaborate the definition we provide practical example of fuzzy graph and from this graph we construct the 1-quasi total fuzzy graph of the given fuzzy graph, so that the definition to be meaning full and their relationships can be easily observed from the sketched graphs. In addition some theorems related to the properties of 1-quasi total fuzzy graphs are stated and proved. The results of these theorems are compared with the results obtained from total fuzzy graphs, so that the differences and similarities that 1-quasi total fuzzy graph can have with that of total fuzzy graphs are revealed. Moreover, we define 1-quasi total coloring of fuzzy total graphs and give an example of total coloring of 1-quasi total graphs.

Keywords: Fuzzy Graph, Total Fuzzy Graph, 1-Quasi Total Fuzzy Graph, Total Coloring

1. Introduction

After its emerging, the graph theory rapidly moved in to the main stream of mathematics as it has application in diverse fields of science and computer [1-2]. The total coloring of graphs were introduced by Behazad (1965) after Harry (1972) made chops on the concept of total graphs [3-4]. D. Muthuramakrishnan and G. Jayaraman (2018) studied the total chromatic number of total graphs [5]. The definition of quasi-total graph is given on the paper by D. V. S. Sastry (1984) and R. V. N. SirnivasaRao, etal has introduced 1-quasi total graphs and bounds for its total chromatic number [6-7]. After Zadeh's paper on fuzzy sets, Rosenfeld (1975) introduced fuzzy graphs [8-9].

Later on, Bhattacharya [10] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson J. N. and Peng C. S. [11]. As an advancement fuzzy coloring of fuzzy graph was defined by Eslahchi and Onagh in 2004, and later developed by them as Fuzzy vertex coloring in 2006 [12]. Lavanay. S and Sattanathan extended the concept of fuzzy vertex coloring in to a family of fuzzy sets [13]. S. Kavitha and S. Lavanya defined the total fuzzy graph and studied total chromatic number of total graphs of fuzzy graphs [14].

Total coloring of 1-quasi total graph for crisp graph was studied and total fuzzy graphs and its total chromatic number has been already established. This article addresses the following:

i. Define 1-quasi total fuzzy graph and elaborate it by examples.
ii. Discuss and proof some properties of 1-quasitotal fuzzy graphs and compare the result with the properties of total fuzzy graphs.
iii. Determine the total chromatic number of 1-quasi total fuzzy graph and justify it by example.

2. Preliminaries

Under this topic we give the definition of some concepts which help us as a prerequisite to present the main body of the manuscript. Most of the concepts are from [1-18].

Definition 2.1: A fuzzy graph is defined as an ordered triple $G = (V, \sigma, \mu)$, where V is the set of vertices

$\{v_1, v_2, \ldots, v_n\}, \sigma$ is a fuzzy subset of V, such that $\sigma: V \to [0,1]$ and μ be a fuzzy relation on σ with $\mu: V \to [0,1]$ and that $\mu: V \times V \to [0,1]$ such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v) \ \forall \ u, v \in V$.

Definition 2.2: The underlying crisp graph of the fuzzy graph $G = (V, \sigma, \mu)$ is denoted by $G^* = (V, E)$, where $E \subseteq V \times V$. The crisp graph (V, E) is a special fuzzy graph G with each vertex and each edges of G has the same degree of membership equal to 1.

Definition 2.3: Let $G = (V, \sigma, \mu)$ be a fuzzy graph with the underlying set V. Then, the order of G denoted by $Order(G)$ is defined as:

$$Order(G) = \sum_{u \in V} \sigma(u)$$

and size of G denoted by $Size(G)$ and defined as:

$$Size(u) = \sum_{u,v \in V} \mu(u,v)$$

Definition 2.4: Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The degree of a vertex $u \in V$ is defined as

$$d_G(u) = \sum_{v \ne u, v \in V} \mu(u,v)$$

Definition 2.5: Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The busy value of the vertex v in G is $D(v) = \sum_i \sigma(v) \wedge \sigma(v_i)$ where v_i are neighbors of v and the busy value of G is

$$D(G) = \sum_i D(v_i)$$

where v_i are the vertexes of G.

Definition 2.6: If $\mu(u,v) > 0$, then u and v are said to be adjacent to each other and lie on the edge, $e = (u,v)$. A path ρ in a fuzzy graph $G = (V, \sigma, \mu)$ is a sequence of distinct nodes $v_0, v_1, v_2, \ldots, v_n$ such that $\mu(v_{i-1}, v_i) > 0, 1 \le i \le n$. Here n is called the length of the path.

Definition 2.7: If u, v are vertices in G and if they are connected by means of a path, then the strength of that path is defined as $\wedge_{i=1}^n \mu(v_{i-1}, v_i)$. If u, v are connected by means of paths of length k, then

$$\mu^k(u,v) = \sup\{ \mu(u,v_1) \wedge \mu(v_1,v_2) \wedge \mu(v_2,v_3)$$

$$\wedge \ldots \wedge \mu(v_{k-1},v): u, v_1, v_2, \ldots, v_{k-1}, v \in V\}.$$

If $u, v \in V$, then the strength of connectedness between u and v is,

$$\mu^\infty(u,v) = \sup\{ \mu^k(u,v): k = 1,2,\ldots \}$$

Definition 2.8: Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then, G is said to be connected if $\mu^\infty(u,v) > 0 \ for \ all \ u,v \in \sigma^*$. An arc (u,v) is said to be a strong arc if $\mu(u,v) \ge \mu^\infty(u,v)$ and a node u, is said to be an isolated node, if $\mu(u,v) = 0 \ for \ all \ u \ne v$.

Definition 2.9: $G = (V, \sigma, \mu)$ is a fuzzy cycle if and only if (σ^*, μ^*) is cycle and there does not exist a unique $(x,y) \in \mu^*$

such that $\mu(x,y) = \wedge\{\mu(u,v): (u,v) \in \mu^*\}$.

Definition 2.10: A family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k\}$ of fuzzy sets on $V \cup E$ is called a $k - $fuzzy total coloring of $G = (V, \sigma, \mu)$, if:
a) $Max\{\gamma_i(v)\} = \sigma(v)$ for all $v \in V$ and
b) $Max\{\gamma_i(u,v)\} = \mu(u,v)$ for all edges $(u,v) \in E$
c) $\gamma_i \wedge \gamma_j = 0$
d) For every adjacent vertices u, v of G, $Min\{\gamma_i(u), \gamma_i(v)\} = 0$.

The least value of k for which there exists a $k - $fuzzycoloring is called the fuzzy total chromatic number of G and is denoted by $\chi_T^f(G)$.

3. 1-Quasi Fuzzy Total Graph

In this section we introduce the definition of 1-quasi fuzzy total graph and draw the 1-quasi fuzzy total graph of a given fuzzy graph.

Definition 3.1: Let $G = (V, \sigma, \mu)$ be a fuzzy graph with its underlying set V and crisp graph $G^* = (\sigma^*, \mu^*)$. The pair $Q_1 T_f(G) = (\sigma_{Q_1 T_f}, \mu_{Q_1 T_f})$ of the fuzzy graph G is defined as follows:

Let the node set of $Q_1 T_f(G)$ be $V \cup E$, where V is the vertex set and E is the edge set of the underlying crisp graph. The fuzzy subset $\sigma_{Q_1 T_f}$ is defined on $V \cup E$ as:

$$\sigma_{Q_1 T_f}(u) = \sigma(u), if \ u \in V$$

$$\sigma_{Q_1 T_f}(e) = \mu(e), if \ e \in E$$

The fuzzy relation $\mu_{Q_1 T_f}$ is defined on $(V \cup E) \times (V \cup E)$, called edges of $Q_1 T_f(G)$ as:

$$\mu_{Q_1 T_f}(u,v) = \mu(u,v), if \ u, v \in V$$

$$\mu_{Q_1 T_f}(e_i, e_j) = \mu(e_i) \wedge \mu(e_j), \text{if} e_i \text{ and } e_j \text{ have a node in com} \atop \text{mon between them} = 0, \text{Otherwise} \qquad (1)$$

By definition; $\mu_{Q_1 T_f}(u,v) \le \sigma_{Q_1 T_f}(u) \wedge \sigma_{Q_1 T_f}(v)$ for all $u, v \in V \cup E$. Hence, $\mu_{Q_1 T_f}$ is a fuzzy relation on the fuzzy subset $\sigma_{Q_1 T_f}$. Thus, the pair $Q_1 T_f \left(\sigma_{Q_1 T_f}, \mu_{Q_1 T_f}\right)$ is a fuzzy graph, and it is termed as 1-Quasi total fuzzy graph of G.

Example 3.1: Consider the fuzzy graph $G = (V, \sigma, \mu)$ where its underlying crisp graph $G^* = (V, E)$ has vertex set $V = \{v_1, v_2, v_3\}$ and edge set $E = \{v_1 v_2, v_2 v_3, v_3 v_1\}$. Let the fuzzy vertex set defined on V be as follows:
$\sigma(v_1) = 0.4, \sigma(v_2) = 0.5, \sigma(v_3) = 0.7$.
Let the fuzzy relation defined on the fuzzy edge set be follows:

$$\mu(v_1, v_2) = 0.2, \mu(v_2, v_3) = 0.4, \mu(v_3, v_1) = 0.4.$$

Since, $\mu(v_i, v_j) \le \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in V$, the graph $G = (V, \sigma, \mu)$ is a fuzzy graph and its graph is as shown in the figure 1.
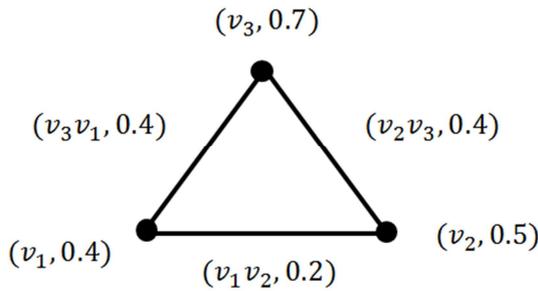
Figure 1. Fuzzy Graph.

Now, let us construct the 1-quasi total fuzzy graph of the fuzzy graph in the example as follows:

That is, $Q_1 T_f \left(\sigma_{Q_1 T_f}, \mu_{Q_1 T_f} \right)$ of the fuzzy graph G, where the node set of $Q_1 T_f$ is $V \cup E$, which is the set $\{v_1, v_2, v_3, v_1 v_2, v_2 v_3, v_3 v_1\}$. We define the fuzzy subset $\delta_{Q_1 T_f}$ as follows:

$$\sigma_{Q_1 T_f}(u) = \sigma(u), if\ u \in V$$

$$\sigma_{Q_1 T_f}(e) = \mu(e), if\ e \in E.$$

Hence, we have the following fuzzy subsets $\sigma_{Q_1 T}$:

$$\sigma_{Q_1 T_f}(v_1) = \sigma(v_1) = 0.4, \sigma_{Q_1 T_f}(v_2) = \sigma(v_2) = 0.5,$$
$$\sigma_{Q_1 T_f}(v_3) = \sigma(v_3) = 0.7$$

$$\sigma_{Q_1 T_f}(v_1 v_2) = \mu(v_1, v_2) = 0.2, \sigma_{Q_1 T_f}(v_2 v_3)$$

$$= \mu(v_2, v_3) = 0.4, \sigma_{Q_1 T_f}(v_3 v_1) = \mu(v_3, v_1) = 0.4$$

The fuzzy relations $\mu_{Q_1 T_f}$ will be as follows:

$$\mu_{Q_1 T_f}(u, v) = \mu(u, v), if\ u, v \in V$$

$\mu_{Q_1 T_f}(e_i, e_j) = \mu(e_i) \wedge \mu(e_j)$, if e_i and e_j have a node in common between them$= 0$, otherwise

Hence,

$$\mu_{Q_1 T_f}(v_1, v_2) = \mu(v_1, v_2) = 0.2,$$

$$\mu_{Q_1 T_f}(v_2, v_3) = \mu(v_2, v_3) = 0.4,$$

$$\mu_{Q_1 T_f}(v_3, v_1) = \mu(v_3, v_1) = 0.4$$

$$\mu_{Q_1 T_f}(v_1 v_2, v_2 v_3,) = \mu(v_1 v_2) \wedge \mu(v_2 v_3) = 0.2$$

$$\mu_{Q_1 T_f}(v_1 v_2, v_3 v_1,) = \mu(v_1 v_2) \wedge \mu(v_3 v_1) = 0.2$$

$$\mu_{Q_1 T_f}(v_2 v_3, v_3 v_1,) = \mu(v_2 v_3) \wedge \mu(v_3 v_1) = 0.4$$

Since, $\mu_{Q_1 T_f}(v_i, v_j) \leq \sigma_{Q_1 T_f}(v_i) \wedge \sigma_{Q_1 T_f}(v_j)$ for all $v_i, v_j \in V \cup E$, the graph $Q_1 T_f \left(\sigma_{Q_1 T_f}, \mu_{Q_1 T_f} \right)$ is a fuzzy graph and from the above node sets $V \cup E$, fuzzy subsets $\sigma_{Q_1 T_f}$ and fuzzy relations $\mu_{Q_1 T_f}$, the graph of 1-quasi total fuzzy graph of G is as shown in figure 2.
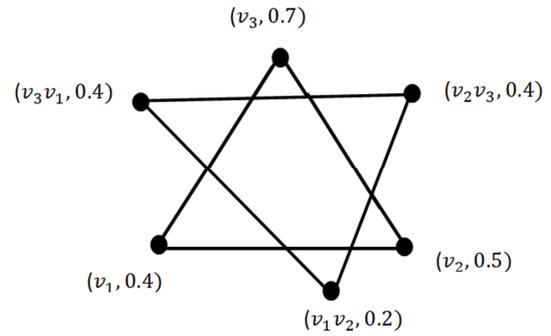

Figure 2. 1-Quasi fuzzy total graph.

Example 3.2: Consider the fuzzy graph $G = (V, \sigma, \mu)$ with the fuzzy vertex set;

$$\sigma(v_1) = 0.3, \sigma(v_2) = 0.4, \sigma(v_3) = 0.6, \sigma(v_4) = 0.8$$

and fuzzy edge set:

$$\mu(v_1, v_2) = 0.2, \mu(v_2, v_3) = 0.3,$$

$$\mu(v_3, v_4) = 0.5, \mu(v_4, v_1) = 0.1$$

Since, $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in V$, the graph $G = (V, \sigma, \mu)$ is a fuzzy graph and its graph is as shown in the figure 3 below.
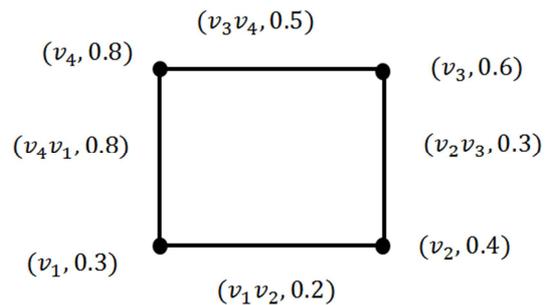

Figure 3. Fuzzy Graph.

Now, from the fuzzy graph in example 3 the 1-quasi total fuzzy graph of G, $Q_1 T_f \left(\sigma_{Q_1 T_f}, \mu_{Q_1 T_f} \right)$ will be defined as follows:

i. Fuzzy vertex set $\sigma_{Q_1 T_f}$ is as follows:

$$\sigma_{Q_1 T_f}(u) = \delta(u), if\ u \in V$$

$$\sigma_{Q_1 T_f}(e) = \mu(e), if\ e \in E.$$

Hence;

$$\sigma_{Q_1 T_f}(v_1) = \sigma(v_1) = 0.3,$$

$$\sigma_{Q_1 T_f}(v_2) = \sigma(v_2) = 0.4,$$

$$\sigma_{Q_1 T_f}(v_3) = \sigma(v_3) = 0.6,$$

$$\sigma_{Q_1 T_f}(v_4) = \sigma(v_4) = 0.8$$

$$\sigma_{Q_1 T_f}(v_1 v_2) = \mu(v_1, v_2) = 0.2,$$

$$\sigma_{Q_1 T_f}(v_2 v_3) = \mu(v_2, v_3) = 0.3,$$

$$\sigma_{Q_1 T_f}(v_3 v_4) = \mu(v_3, v_4) = 0.5,$$

$$\sigma_{Q_1 T_f}(v_4 v_1) = \mu(v_4, v_1) = 0.1$$

ii. The fuzzy edge set $\mu_{Q_1 T_f}$ is as follows:

$$\mu_{Q_1 T_f}(u, v) = \mu(u, v), if\ u, v \in V$$

$\mu_{Q_1 T_f}(e_i, e_j) = \mu(e_i) \wedge \mu(e_j)$, if e_i and e_j have a node in common between them$= 0$, Otherwise

Hence,

$$\mu_{Q_1 T_f}(v_1, v_2) = \mu(v_1, v_2) = 0.2, \mu_{Q_1 T_f}(v_2, v_3) = \mu(v_2, v_3) = 0.3,$$

$$\mu_{Q_1 T_f}(v_3, v_4) = \mu(v_3, v_4) = 0.,$$

$$\mu_{Q_1 T_f}(v_4, v_1) = \mu(v_4, v_1) = 0.1$$

$$\mu_{Q_1 T_f}(v_1 v_2, v_2 v_3,) = \mu(v_1 v_2) \wedge \mu(v_2 v_3) = 0.2 \wedge 0.3 = 0.2$$

$$\mu_{Q_1 T_f}(v_1 v_2, v_3 v_4,) = \mu(v_1 v_2) \wedge \mu(v_3 v_4) = 0$$

$$\mu_{Q_1 T_f}(v_1 v_2, v_4 v_1,) = \mu(v_1 v_2) \wedge \mu(v_4 v_1) = 0.2 \wedge 0.1 = 0.1$$

$$\mu_{Q_1 T_f}(v_2 v_3, v_3 v_4,) = \mu(v_2 v_3) \wedge \mu(v_3 v_4) = 0.3 \wedge 0.5 = 0.3$$

$$\mu_{Q_1 T_f}(v_2 v_3, v_4 v_1,) = 0$$

$$\mu_{Q_1 T_f}(v_3 v_4, v_4 v_1,) = \mu(v_3 v_4) \wedge \mu(v_4 v_1) = 0.5 \wedge 0.1 = 0.1$$

Clearly, $\mu_{Q_1 T_f}(v_i, v_j) \leq \sigma_{Q_1 T_f}(v_i) \wedge \sigma_{Q_1 T_f}(v_j)$ for all $v_i, v_j \in V$ and hence the graph $Q_1 T_f\left(\delta_{Q_1 T_f}, \mu_{Q_1 T_f}\right)$ is a fuzzy graph.

Th e graph $Q_1 T_f = \left(\sigma_{Q_1 T_f}, \mu_{Q_1 T_f}\right)$ is a 1-quasi total fuzzy graph and its graph is as shown in figure 4.

$(v_3 v_4, 0.5)$

$(v_4, 0.8)$ $(v_3, 0.6)$

$(v_4 v_1, 0.1)$ $(v_2 v_3, 0.3)$

$(v_1, 0.3)$ $(v_2, 0.4)$

$(v_1 v_2, 0.2)$

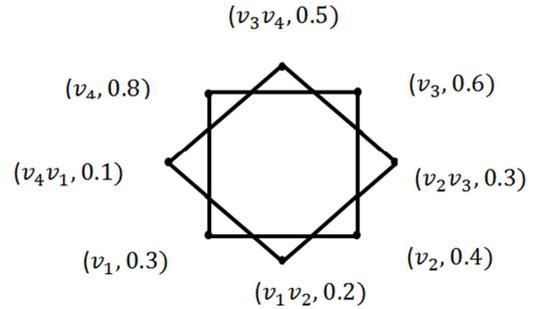*Figure 4. 1-Quasi fuzzy total graph.*

4. Properties of 1-Quasi Fuzzy Total Graph

Theorem 4.1: Let $G = (V, \sigma, \mu)$ be a fuzzy graph.

$$Order\left(Q_{1 T_f}(G)\right) = Order(G) + Size(G).$$

Proof: By the definition of 1-quasi total fuzzy graph, the node set of $Q_1 T_f(G)$ is $V \cup E$ and the fuzzy subset $\sigma_{Q_1 T_f}(u) = \sigma(u), if\ u \in V$ and $\sigma_{Q_1 T_f}(e) = \mu(e), if\ e \in E$.

Now,

$$Order\left(Q_1 T_f(G)\right) = \sum_{u \in V \cup E} \sigma_{Q_1 T_f}(u), by\ definition = \sum_{u \in V} \sigma_{Q_1 T_f}(u) + \sum_{u \in E} \sigma_{Q_1 T_f}(u) =$$

$$\sum_{u \in V} \sigma(u) + \sum_{u \in E} \sigma(u), by\ definition\ of\ Q_1 T_f(u) = Order(G) + Size(G).$$

Note 1: For any fuzzy graph $G = (V, \sigma, \mu)$, $Order(T(G)) = Order(G) + Size(G)$. Where $T(G)$ is fuzzy total graph.

Theorem 4.2: Let $G = (V, \sigma, \mu)$ be a fuzzy graph, then

$$Size(Q_1 T_f) = \sum_{u, v \in V \cup E} \mu_{Q_1 T_f}(u, v) = \sum_{u, v \in V} \mu_{Q_1 T_f}(u, v) + \sum_{u \in V, e \in E} \mu_{Q_1 T_f}(u, e) + \sum_{e_i, e_j \in E} \mu_{Q_1 T}(e_i, e_j)$$

$$= \sum_{u, v \in V} \mu_{Q_1 T_f}(u, v) + 0 + \sum_{e_i, e_j \in E} \mu_{Q_1 T_f}(e_i, e_j)$$

(The second summation is zero, since there is no fuzzy relation between $u \in V$ and $e \in E$ in 1-quasi total fuzzy graph)

$$= \sum_{u, v \in V} \mu(u, v) + \sum_{e_i, e_j \in E} \mu(e_i, e_j)$$

$$Size\left(Q_1 T_f(G)\right) = Size(G) + \sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j)$$

Proof: By definition of size of a fuzzy graph, we have;

$$= Size(G) + \sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j)$$

Note 2: For any fuzzy graph

$$G = (V, \sigma, \mu), Size(T(G)) =$$

$3 Size(G) + \sum_{e_i, e_j \in E} \mu(e_i) \wedge \mu(e_j).$

Theorem 4.3: Let $G = (V, \sigma, \mu)$ be a fuzzy graph, then

$$d\left(Q_1 T_f(u)\right) = d_G(u), if \ u \in V$$

$$d\left(Q_1 T_{f(G)}(u)\right) = \sum_{u,v \in V} \mu_{Q_1 T_{f(G)}}(u,v) + \sum_{u \in V, e \in E} \mu_{Q_1 T_{f(G)}}(u,e),$$

(where u lies on the edge of e in the second summation).

$$= \sum_{u,v \in V} \mu_{Q_1 T_{f(G)}}(u,v) + 0$$

(The second summation is zero, since there is no fuzzy relation between $u \in V$ and $e \in E$ in 1-quasi total fuzzy graph)

$$= \sum_{u,v \in V} \mu(u,v) = d_G(u).$$

Case 2: Let $e_i \in E$, then

$$d\left(Q_1 T_{f(G)}(e_i)\right) = \sum_{u \in V} \mu_{Q_1 T_{f(G)}}(e_i, v) +$$

$$\sum_{e_j \in E} \mu_{Q_1 T_{f(G)}}(e_i, e_j) = 0 + \sum_{e_j \in E} \mu_{Q_1 T_{f(G)}}(e_i) \wedge \mu_{Q_1 T_{f(G)}}(e_j)$$

(The first summation is zero, since there is no fuzzy relation between $u \in V$ and $e \in E$ in 1-quasi total fuzzy graph)$= bussy$ value of e_i in $Q_1 T_{f(G)}$.

Note 3: For any fuzzy graph, $G = (V, \sigma, \mu)$,

$$d\left(T_G(u)\right) = 2 d_G(u), if \ u \in V$$

$$= busy \ value \ of \ e_i \ in \ T(G), if \ u \in E$$

Theorem 4.4: 1-quasi total fuzzy graph of any fuzzy graph is disconnected graph.

Proof: Let, $G = (V, \sigma, \mu)$ be a fuzzy graph.

The fuzzy vertex set of $Q_1 T_f(G)$ consists of $V \cup E$ of G and the fuzzy vertex relation is defined only between $u, v \in V$ and $e_i, e_j \in E$. Since there is no fuzzy relation between $u \in V$ and $e \in E$ of elements in the vertex set of $Q_1 T_f(G)$, then there is no path that connects u and e in $Q_1 T_f(G)$ and $\mu^\infty(u,v) = 0$.

Hence, $Q_1 T_f(G)$ is disconnected graph.

5. 1-Quasi Fuzzy Total Coloring

In this section we introduce the concept of 1-quasi fuzzy total coloring and discuss some of its properties.

Definition 5. (2): A family $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$, of a fuzzy set on $V \cup E$ is called a 1-quasi k-fuzzy total coloring of fuzzy graph, $G = (V, \sigma, \mu)$, if the following three conditions met.

i. $Max\{\gamma_i(v)\} = \sigma(v) \ for \ all \ v \in V$ and

$= busy \ value \ of \ e_i \ in \ Q_1 T_f(G), if \ u \in E$

Proof: By definition of degree of a vertex of a fuzzy graph given in definition 5, we have the following two cases to prove the theorem.

Case 1: Let $u \in V$. Then,

$Max\{\gamma_i(u,v)\} = \mu(u,v) \ for \ all \ edges \ (u,v) \in E.$
ii. $\gamma_i \wedge \gamma_j = 0$
iii. For every adjacent vertices u, v of $Q_1 T_f(G)$, $Min\{\gamma_i(u), \gamma_i(v)\} = 0$.

The least number of colors possible is called 1-quasi fuzzy total chromatic number of $Q_1 T_f(G)$ and it is denoted by $\chi^f_{Q_1}(G)$.

Example 5.1: Consider a fuzzy graph $G = (V, \sigma, \mu)$ with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set $E = \{v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_5, v_5 v_6, v_6 v_1\}$, whose membership functions are defined as follows:

$$\sigma(v_i) = \begin{cases} 0.2, for \ i = 1 \\ 0.7, for \ i = 2 \\ 0.5, for \ i = 3 \\ 0.4, for \ i = 4 \\ 0.6, for \ i = 5 \\ 0.3, for \ i = 6 \end{cases}$$

$$\mu(v_i, v_j) = \begin{cases} 0.2, for \ (i,j) = (1,2) \\ 0.5, for \ (i,j) = (2,3) \\ 0.1, for \ (i,j) = (3,4) \\ 0.4, for \ (i,j) = (4,5) \\ 0.3, for \ (i,j) = (5,6) \\ 0.1, for \ (i,j) = (6,1) \end{cases}$$

Let us define a family of fuzzy sets $\Gamma = \{\gamma_1, \gamma_2\}$ on $V \cup E$ as follows:

$$\gamma_1(v_i) = \begin{cases} 0.2, for \ i = 1 \\ 0.5, for \ i = 3 \\ 0.6, for \ i = 5 \\ 0, Otherwise \end{cases}$$

$$\gamma_2(v_i) = \begin{cases} 0.7, for \ i = 2 \\ 0.4, for \ i = 4 \\ 0.3, for \ i = 6 \\ 0, Otherwise \end{cases}$$

$$\gamma_1(v_i, v_j) = \begin{cases} 0.2, for \ (i,j) = (1,2) \\ 0.1, for \ (i,j) = (3,4) \\ 0.3, for \ (i,j) = (5,6) \\ 0, \quad Otherwise \end{cases}$$

$$\gamma_2(v_i, v_j) = \begin{cases} 0.5, for \ (i,j) = (2,3) \\ 0.4, for \ (i,j) = (4,5) \\ 0.1, for \ (i,j) = (6,1) \\ 0, \quad Otherwise \end{cases}$$

Clearly, a family of fuzzy sets $\Gamma = \{\gamma_1, \gamma_2\}$ defined as

above satisfies the definition of total coloring of fuzzy graphs and hence, $\chi_T^f(G) = 2$.

When we come to our point of concern, we need to determine the chromatic number of 1-quasi total fuzzy graph of the fuzzy graph in the example 5.1.

Now, to construct a 1-quasi total fuzzy graph $Q_1(G) = (V \cup E, \sigma_{Q_1 T_f}, \mu_{Q_1 T_f})$, where

$$V \cup E = \{v_1, v_2, v_3, v_4, v_5, v_6, v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_5, v_5 v_6, v_6 v_1\}.$$

The fuzzy subset of $Q_1 T_f(G)$ will be as follows;

$$\sigma_{Q_1 T_f}(v_i) = \begin{cases} 0.2, for\ i = 1 \\ 0.7, for\ i = 2 \\ 0.5, for\ i = 3 \\ 0.4, for\ i = 4 \\ 0.6, for\ i = 5 \\ 0.3, for\ i = 6 \end{cases}$$

$$\sigma_{Q_1 T_f}(v_i, v_j) = \begin{cases} 0.2, for\ (i,j) = (1,2) \\ 0.5, for\ (i,j) = (2,3) \\ 0.1, for\ (i,j) = (3,4) \\ 0.4, for\ (i,j) = (4,5) \\ 0.3, for\ (i,j) = (5,6) \\ 0.1, for\ (i,j) = (6,1) \end{cases}$$

The fuzzy relation will be:

$$\mu_{Q_1 T_f}(v_i, v_j) = \begin{cases} 0.2, for\ (i,j) = (1,2) \\ 0.5, for\ (i,j) = (2,3) \\ 0.1, for\ (i,j) = (3,4) \\ 0.4, for\ (i,j) = (4,5) \\ 0.3, for\ (i,j) = (5,6) \\ 0.1, for\ (i,j) = (6,1) \end{cases}$$

$$\mu_{Q_1 T_f}(v_i v_j, v_j v_k) = \begin{cases} 0.2, for\ (ij, jk) = (12,23) \\ 0.1, for\ (ij, jk) = (23,34) \\ 0.1, for\ (ij, jk) = (34,45) \\ 0.3, for\ (ij, jk) = (45,56) \\ 0.1, for\ (ij, jk) = (56,61) \\ 0.1, for\ (ij, jk) = (61,12) \end{cases}$$

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of fuzzy subset defined on $V \cup E$ as follows:
For the vertex set;

$$\gamma_1(v_i) = \begin{cases} 0.2, for\ i = 1 \\ 0.5, for\ i = 3 \\ 0.6, for\ i = 5 \\ 0, Otherwise \end{cases}$$

$$\gamma_1(v_i v_j) = \begin{cases} 0.2, for\ ij = 12 \\ 0.1, for\ ij = 34 \\ 0.3, for\ ij = 56 \\ 0, Otherwise \end{cases}$$

$$\gamma_2(v_i) = \begin{cases} 0.7, for\ i = 2 \\ 0.4, for\ i = 4 \\ 0.3, for\ i = 6 \\ 0, Otherwise \end{cases}$$

$$\gamma_2(v_i v_j) = \begin{cases} 0.5, for\ (i,j) = (2,3) \\ 0.4, for\ (i,j) = (4,5) \\ 0.1, for\ (i,j) = (6,1) \\ 0,\quad Otherwise \end{cases}$$

For the edge set;

$$\gamma_1(v_i, v_j) = \begin{cases} 0.2, for\ (i,j) = (1,2) \\ 0.1, for\ (i,j) = (3,4) \\ 0.3, for\ (i,j) = (5,6) \\ 0,\quad Otherwise \end{cases}$$

$$\gamma_1(v_i v_j, v_j v_k) = \begin{cases} 0.2, for\ (ij, jk) = (12,23) \\ 0.1, for\ (ij, jk) = (34,45) \\ 0.1, for\ (ij, jk) = (56,61) \\ 0, Otherwise \end{cases}$$

$$\gamma_2(v_i v_j) = \begin{cases} 0.5, for\ (i,j) = (2,3) \\ 0.4, for\ (i,j) = (4,5) \\ 0.1, for\ (i,j) = (6,1) \\ 0,\quad Otherwise \end{cases}$$

$$\gamma_2(v_i v_j, v_j v_k) = \begin{cases} 0.1, for\ (ij, jk) = (23,34) \\ 0.3, for\ (ij, jk) = (45,56) \\ 0.1, for\ (ij, jk) = (61,12) \\ 0,\quad Otherwise \end{cases}$$

Using table 1 below we can check whether Γ satisfies the definition of 1-quasi total coloring of G.

Table 1. *Example of 1-quasi total coloring of a fuzzy graph $G = (V, \sigma, \mu)$.*

v and e	γ_1	γ_2	Max.	$\gamma_1 \wedge \gamma_2$	Min.
v_1	0.2	0	0.2	0	$Min\{\gamma_i(v_1), \gamma_i(v_2)\} = 0$
v_2	0	0.7	0.7	0	$Min\{\gamma_i(v_2), \gamma_i(v_3)\} = 0$
v_3	0.5	0	0.5	0	$Min\{\gamma_i(v_3), \gamma_i(v_4)\} = 0$
v_4	0	0.4	0.4	0	$Min\{\gamma_i(v_4), \gamma_i(v_5)\} = 0$
v_5	0.6	0	0.6	0	$Min\{\gamma_i(v_5), \gamma_i(v_6)\} = 0$
v_6	0	0.3	0.3	0	$Min\{\gamma_i(v_6), \gamma_i(v_1)\} = 0$
$v_1 v_2$	0.2	0	0.2	0	$Min\{\gamma_i(v_1 v_2), \gamma_i(v_2 v_3)\} = 0$
$v_2 v_3$	0	0.5	0.5	0	$Min\{\gamma_i(v_2 v_3), \gamma_i(v_3 v_4)\} = 0$
$v_3 v_4$	0.1	0	0.1	0	$Min\{\gamma_i(v_3 v_4), \gamma_i(v_4 v_5)\} = 0$
$v_4 v_5$	0	0.4	0.4	0	$Min\{\gamma_i(v_4 v_5), \gamma_i(v_5 v_6)\} = 0$

v and e	γ_1	γ_2	Max.	$\gamma_1 \wedge \gamma_2$	Min.
$v_5 v_6$	0.3	0	0.3	0	$Min\{\gamma_i(v_5 v_6), \gamma_i(v_6 v_1)\} = 0$
$v_6 v_1$	0	0.1	0.1	0	$Min\{\gamma_i(v_6 v_1), \gamma_i(v_1 v_2)\} = 0$
(v_1, v_2)	0.2	0	0.2	0	
(v_2, v_3)	0	0.5	0.5	0	
(v_3, v_4)	0.1	0	0.1	0	
(v_4, v_5)	0	0.4	0.4	0	
(v_5, v_6)	0.3	0	0.3	0	
(v_6, v_1)	0	0.1	0.1	0	
$(v_1 v_2, v_2 v_3)$	0.2	0	0.2	0	
$(v_2 v_3, v_3 v_4)$	0	0.1	0.1	0	
$(v_3 v_4, v_4 v_5)$	0.1	0	0.1	0	
$(v_4 v_5, v_5 v_6)$	0	0.3	0.3	0	
$(v_5 v_6, v_6 v_1)$	0.1	0	0.1	0	
$(v_6 v_1, v_1 v_2)$	0	0.1	0.1	0	

As shown in the table 1 $\Gamma = \{\gamma_1, \gamma_2\}$ satisfies the definition of 1-quasi fuzzy total coloring of fuzzy graph G.

Therefore, $\chi^f_{Q_1 T}(G) = 2$.

6. Conclusion

In this article we have defined 1-quasi total fuzzy graph for a given fuzzy graph. For this concept to be clear we constructed a fuzzy graph and from this graph 1-quasi total fuzzy graph is developed and its graph is sketched. For easy understanding the graph of a given fuzzy graph and its 1-quasi total fuzzy graph is given. For the sake of making the study to be complete, theorems regarding properties of total fuzzy graph are established for 1-quasi total fuzzy graph and the results obtained from the proof of the theorem are compared against the results for total fuzzy graphs. Moreover, we have defined 1-quasi total coloring for fuzzy graph and its total coloring is exemplified.

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