Generalized Nörlund summability of fuzzy real numbers

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Abstract: Fuzzy set, mathematical modelling in order to some uncertainty in 1965 was described by L. A. Zadeh [7]. In studies on fuzzy sets, fuzzy numbers [5], fuzzy relations [5], fuzzy function [5], fuzzy sequence [4] is defined as concepts. After Nörlund fuzzy and blurry Riez summability have been identified [6]. In this study, fuzzy Generalized Nörlund summability have been defined and Generalized Nörlund summability necessary and sufficient conditions to ensure the regular was investigated.

Keywords: Generalized Nörlund Summability, Nörlund Mean Fuzzy, Fuzzy Mean Riesz, Cesaro Mean Fuzzy

1. Introduction

This section will be the basic concepts of fuzzy sets.

Definition 1.1. A fuzzy set A on the universe X is a set defined by a membership function \( \mu_A \) representing a mapping

\[ \mu_A : X \rightarrow [0,1]. \]

Here the value \( \mu_A(x) \) for the fuzzy set \( A \) is called the membership value or the grade of membership of \( x \in X \). The membership value represents the degree of \( x \) belonging to the fuzzy set \( A \). [5]

Definition 1.2. Let \( D \) denote the set of all closed and bounded intervals \( X = [a_1, a_2] \) on the real line \( R \). For \( X, Y \in D \), we define

\[ d(X,Y) = \max \{|a_1 - b_1|, |a_2 - b_2|\} \]

where \( X = [a_1, a_2] \), \( Y = [b_1, b_2] \). It is known that \( (D,d) \) is a complete metric space [6].

Definition 1.3. A fuzzy real number \( X \) is called convex if \( X(t) \geq X(s) \wedge X(r) = \min\{X(s), X(r)\} \), where \( s < t < r \). If there exists \( t_0 \in IR \), such that \( X(t_0) = 1 \), then the fuzzy real number \( X \) is called normal. A fuzzy real number \( X \) is a fuzzy set on \( R \) and is a mapping

\[ X : IR \rightarrow I([0,1]) \]

associating each real number \( t \) with its grade of membership

\[ X(t) \] [6].

A fuzzy real number \( X \) is said to be upper-semicontinuous if for each \( \epsilon > 0 \), \( X^{-1}([0, a+\epsilon]) \), for all \( a \in I \) is open in the usual topology of \( R \) [6].

The set of all upper-semicontinuous, normal, convex fuzzy number is denoted by \( R(I) \) [6].

Definition 1.4. The \( \alpha \)-level set of a fuzzy real number \( X \), for \( 0 < \alpha \leq 1 \) denoted by \( X^\alpha \) is defined as

\[ \mu_A(x) = \begin{cases} 1, & x \in [a,b] \\ l(x), & x \in (-\infty,a) \\ r(x), & x \in (b,\infty) \end{cases} \]

where \( l \) is a function from \( (-\infty, a) \) to \([0,1]\) that is monotonic increasing, continuous from the right, and such that \( l(x) = 0 \) for \( x \in (-\infty, w_1) \); \( r \) is a function from \((b,\infty)\) to \([0,1]\) that is monotonic decreasing, continuous from the left, and such that \( r(x) = 0 \) for \( x \in (w_2,\infty) \). [3]
Definition 1.5. The set $\mathbb{R}$ of all real numbers can be embedded in $R(I)$. For each $r \in \mathbb{R}$, $r \in R$ is defined by $\mathcal{P}(r) = \begin{cases} 1, t = r & \text{if } 0, t \neq r & [6]. \end{cases}$

Definition 1.6. Let $\tilde{d} : R(I) \times R(I) \to \mathbb{R}$ be defined by
$$\tilde{d}(X,Y) = \sup_{0 \leq r \leq 1} d(X^r, Y^r).$$

Then $\tilde{d}$ defines a metric on $R(I)$. It is well known that $(R(I), \tilde{d})$ is a complete metric space. The additive identity and multiplicative identity in $R(I)$ are denoted by $\tilde{0}$ and $\tilde{1}$ respectively [6].

2. Preliminaries

Fuzzy sequence defined on fuzzy sets in this section will be Reisz and Nörlund averages. A fuzzy set of samples to be explained.

Definition 2.1. A sequence $(\tilde{A}_k)$ of fuzzy numbers is said to be convergent to the fuzzy number $\tilde{A}_0$, written as $\lim_{k \to \infty} \tilde{A}_k = \tilde{A}_0$, if for every $\varepsilon > 0$ there exists a positive integer $N$ such that $d(\tilde{A}_k, \tilde{A}_0) < \varepsilon$ for every $k > N$. [1]

Example 2.1.

$$\tilde{A}_k(x) = \begin{cases} \frac{k}{k+2} x + \frac{2-3k}{k+2}, & x \in \left[\frac{3k-2}{k}, 4\right] \\ -\frac{k}{k+2} x + \frac{5k+2}{k+2}, & x \in \left[4, \frac{5k+2}{k}\right] \\ 0, & x \in \left[\frac{3k-2}{k}, \frac{5k+2}{k}\right] \end{cases}$$

from of $\tilde{A} = (\tilde{A}_k)$ Consider the fuzzy number sequence. Limit of this sequence,
$$\tilde{A}_0(x) = \begin{cases} x-3, & x \in [3,4] \\ x+5, & x \in [4,5] \\ 0, & x \in [3,5] \end{cases}$$

3. Generalized Nörlund Summability of Fuzzy Real Numbers

This section will be defined the fuzzy generalized Nörlund summability and explored regularity conditions on the summability.

Definition 3.1. Let $(\alpha_n), (\gamma_n)$ be a sequence of non-negative real numbers which are not all zero and
$$P_n = p_1 + p_2 + \ldots + p_n$$
for all $n \in \mathbb{N}$. A sequence $(\tilde{\alpha}_n)$ of fuzzy real numbers is said to be summable by Nörlund mean $(\tilde{N}, p)$ to $\tilde{L}$, if
$$\tilde{d}\left(\frac{1}{P_n} \sum_{v=1}^{n} p_{n-v+1} \tilde{\alpha}_v, \tilde{L}\right) \to 0$$
as $n \to \infty$.

Definition 3.2. Let $(\alpha_n)$ be a sequence of non-negative real numbers which are not all zero and
$$P_n = p_1 + p_2 + \ldots + p_n$$
for all $n \in \mathbb{N}$. A sequence $(\tilde{\alpha}_n)$ of fuzzy real numbers is said to be summable by Riesz mean $(\tilde{R}, p)$ to $\tilde{L}$, if
$$\tilde{d}\left(\frac{1}{P_n} \sum_{v=1}^{n} p_{n-v+1} \alpha_v, \tilde{L}\right) \to 0$$
as $n \to \infty$. 

Figure 1.1. Fuzzy number [3]

Figure 1.2. $(\tilde{\alpha}_n)$ Fuzzy number sequence $\tilde{x}_n$, the convergence of the fuzzy number [2]
there exists no such that, for all \( n \geq n_0 \).

Proof: Sufficiency: Let \((\sigma_n)\) be any any convergent sequence of fuzzy real numbers, and \(\lim \sigma_n = \overline{\sigma} \). Without loss of generality, we may assume that \( \overline{\sigma} = \overline{L} \). For a fixed \( \varepsilon > 0 \) there exists no such that, \( \overline{d}(s_n, \overline{0}) < H \) for \( n \geq n_0 \). If \( \lim_{n \to \infty} P_n = 0 \) then for all \( \varepsilon > 0 \) there exists \( n_1 \in \mathbb{N} \) such that,

\[
\frac{P_n}{r_n} < \frac{\varepsilon}{2H \max(n_0, n_1)}
\]

for \( n > n_1 \). Let \( n_2 = \max(n_0, n_1) \) and assume \( \overline{d}(\sigma_n, \overline{0}) < \frac{\varepsilon}{2} \) for all \( n > n_2 \). Then,

\[
\frac{P_n}{r_n} < \frac{\varepsilon}{2H \max(n_0, n_1)}.
\]

So for all \( n > n_2 \) we obtain.

\[
\overline{d}\left(\frac{1}{r_n} \sum_{i=1}^{n} p_{n-i+1} q_{i} \overline{\sigma}_{i}, \overline{0}\right) \leq \overline{d}\left(\frac{1}{r_n} \sum_{i=1}^{n} p_{n-i+1} q_{i} \overline{\sigma}_{i}, \overline{0}\right) + \overline{d}\left(\frac{1}{r_n} \sum_{i=1}^{n} p_{n-i+1} q_{i} \overline{\sigma}_{i}, \overline{0}\right)
\]

\[
= \overline{d}\left(\frac{1}{r_n} (p_{n} q_{1} \overline{\sigma}_{1} + ... + p_{n-n_2} q_{n_2} \overline{\sigma}_{n_2}) \overline{0}\right) + \overline{d}\left(\frac{1}{r_n} \sum_{i=n_n+1}^{n} p_{n-i+1} q_{i} \overline{\sigma}_{i}, \overline{0}\right)
\]

\[
= \frac{P_n q_1}{r_n} \overline{d}(\overline{\sigma}_1, \overline{0}) + ... + \frac{P_n q_{n_2}}{r_n} \overline{d}(\overline{\sigma}_{n_2}, \overline{0}) + \overline{d}\left(\frac{1}{r_n} \sum_{i=n_n+1}^{n} p_{n-i+1} q_{i} \overline{\sigma}_{i}, \overline{0}\right)
\]

\[
\leq \frac{P_n q_1}{r_n} H + ... + \frac{P_n q_{n_2}}{r_n} H + \frac{P_n q_{n_2}}{r_n} \frac{\varepsilon}{2} + ... + \frac{P_n q_n}{r_n} \frac{\varepsilon}{2}
\]

4. Results

In this section, fuzzy generalized summability Nörlund been identified and will be investigated regularity conditions on the summability.

\((\overline{N}, p, q)\) regular if and only if \( \lim_{n \to \infty} \frac{P_n}{r_n} = 0 \) we obtained.

References


