Science Journal of Applied Mathematics and Statistics
2014; 2(4): 71-77
Published online July 30, 2014 (http://www.sciencepublishinggroup.com/j/sjams)
doi: 10.11648/j.sjams.20140204.11

Solution of multi-objective transportation problem via fuzzy programming algorithm

Osuji, George A.[1], Okoli Cecilia N.[2], Opara, Jude[3]

[1]Department of Statistics, Nnamdi Azikiwe University, PMB 5025, Awka Anambra State, Nigeria
[2]Department of Statistics, Anambra State University, PMB 02, Uli Anambra State
[3]Department of Statistics, Imo State University, PMB 2000, Owerri, Nigeria

Email address:
george.osuji99@yahoo.com (Osuji G. A.), cecilia.okoli@yahoo.com (Okoli C. N.), judend88@yahoo.com (Opara J.)

To cite this article:
Osuji, George A. , Okoli Cecilia N. , Opara, Jude. Solution of Multi-Objective Transportation Problem Via Fuzzy Programming Algorithm. *Science Journal of Applied Mathematics and Statistics.* Vol. 2, No. 4, 2014, pp. 71-77. doi: 10.11648/j.sjams.20140204.11

Abstract: This paper is on the solution of multi-objective transportation problem via fuzzy programming algorithm. The data for this paper was collected by an egg dealer in whose main office is located at Orji Owerri Imo State Nigeria, who supplies the product to different wholesalers (destinations) after taking it from different poultry farm (sources), and the time and cost of transportation from source i to destination j were recorded. TORA statistical software was employed in the data analysis, and the results of the analysis revealed that if we use the hyperbolic membership function, then the crisp model becomes linear. The result also revealed that the optimal compromise solution does not alter if we compare it with the solution obtained by the linear membership function. Thus, if we compare it with the solution obtained by the linear membership function, it is shown that the fuzzy optimal values do not depend on the chosen membership function be it linear or non-linear membership function.

Keywords: MOTP, Transportation Problem, Fuzzy Programming Algorithm, Hyperbolic Membership Function, Linear Membership Function, Optimization Problem

1. Introduction

Decision making is the process of identifying and choosing alternatives based on the values and preferences of the decision maker. It is the process of sufficiently reducing uncertainly and doubt about alternatives to allow a reasonable choice to be made from among them. Decision making based solely on a single criterion appears insufficient as soon as the decision–making process deals with the complex organizational environment. So, one must acknowledge the presence of several criteria that lead to the development of multi-criteria decision making.

Optimization is a kind of the decision making, in which decision have to be taken to optimize one or more objectives under some prescribed set of circumstances. These problems may be a single or multi-objective and are to be optimized (maximized or minimized) under a specified set of constraints. The constraints usually are in the form of inequalities or equalities. Such problems which often arise as a result of mathematical modeling of many real life situations are called optimization problems.

1.1. Single-Objective Optimization Problem

In many real life situations problems are modeled and solved as single-objective optimization problems in a deterministic and crisp environment. The general form of single-objective optimization problems is:

Minimize (or Maximize) $f(X),\ X=(x_1,x_2,\ldots,x_n)$

Subject to

$$g_j(X)\le 0, \qquad j=1,2,\ldots,k$$

$$l_j(X)\ge 0, \qquad j=1,2,\ldots,r$$

$$h_j(X)=0, \qquad j=1,2,\ldots,m$$

Where $f, g_1, g_2,\ldots,g_k, l_1, l_2,\ldots,l_r, h_1,\ldots,h_2,\ldots,h_m$ are real valued functions defined on Rn . X= $(x_1,x_2,\ldots,x_n)\in R^n$ is called decision vector and $x_1,x_2,\ldots,x_n$ are called decision or unknown variables. In case all the functions (objective function constraints) are linear then the above problems is called linear programming problem, otherwise it is called non-linear programming problem.

1.2. Transportation Problems

It is a special type of linear programming problem which arises in many practical applications. In the beginning it was founded for determining the optimal shipping patter, so it is called transportation problems. The conventional and very well known transportation problem consists in transporting a certain product from each of m origins $i = 1,2,..., m$ to any of n destination $i = 1,2,...., n$. The origins are production facilities with respective capacities $a_1, a_2,..., a_a$ and the destination are warehouse with required levels of demand $b_1, b_2,..., b_n$. for the transport of a unit of the given product from the i^{th} source to the i^{th} destination a cost c_{ij} is given for which, without loss of generally, we can assume $c_{ij} \geq 0, \forall i, j$. Hence, one must determine the amounts x_{ij} too be transported from all the origins $i = 1,2,..., m$ to all the destination $j = 1,2,..., n$ in such a way that the total cost is minimized. This problem can be suitably modeled as a linear programming problem. Thus the conventional transportation problem can be mathematically expressed as:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad i = 1,2,..., m \quad \text{(Row restrictions)}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad j = 1,2,..., n \quad \text{(Column restrictions)}$$

$$x_{ij} \geq 0 \quad \forall_i \text{ and } j$$

$$\sum_{i=1}^{m} a_i = \sum_{j=i}^{n} b_j \quad \text{(Balanced condition)}$$

1.3. Multi-Objective Optimization Problem

Many real life optimization problems re multi-objective in nature and are to be optimized simultaneously subject to a common set of constrains. The most general mathematical model of a multi-objective in nature and are to be optimized simultaneously subject to a common set of constrains. The most general mathematical model of a multi-objective optimization problem is:

Maximize $F(X) = [f_1(X), f_2(X),..., f_m(X)]$, $X = (x_1, x_2,..., x_n)$

Subject to:

$$g_j(X) \leq 0, \quad j = 1,2,..., k$$
$$h_j(X) = 0, \quad j = 1,2,..., m$$
$$l_j(X) \geq 0, \quad j = 1,2,..., r$$

Where $f_1, f,..., f_m$ are the objective functions, Variables $x_1, x,..., x_n$ are called decision variables and X is called decision vector. This problem is also called multi-objective programming problem.

1.4. Multi-Objective Transportation Problems (MOTP)

In real life situation, all the transportation problems are not single objective. The transportation problems which are characterized by multiple objective functions are considered here. A special type of linear programming problem in which constraint are of equality type and all the objectives are conflicting with each other, are called MOTP. Similar to a typical transportation problem, in a MOTP problem a product is to transported from m sources to n destination and their capacities are $a_1, a_2,..., a_m$ and $b_1, b_2,..., b_n$ respectively. In addition, there is a penalty c_{ij} associated with transporting a unit of product from i^{th} source to j^{th} destination. This penalty may be cost or delivery time or safety of delivery or etc. a variable x_{ij} represents the unknown quantity to be shipped from i^{th} source to j^{th} destination. A mathematical model of MOTP with r objectives, m source and n destinations can be written as:

Minimize $Z_r = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{r} x_{ij}, \; r = 1,2,..., K$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1,2,..., m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad j = 1,2,..., n$$

$$x_{ij} \geq 0 \quad \forall i, j$$

The subscript on Z_r and superscript on c_{ij}^{r} are related to the r^{th} penalty criterion. Without loss of generality, it may be assumed that $a_i \geq 0$ and $b_j \geq 0 \forall i, j$ and the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

1.5. MOTP with Equality and Inequality Constraints

MOTP with equality constraint is of the form:

Minimize $Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij}, k = 1,2,..., K$

Subject to

$$\sum_{j=i}^{n} x_{ij} = a_{i,} \quad i = 1,2,\dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots,n$$

$$x_{ij} \geq 0, \ \forall i, j$$

The above problem is feasible if and only if the condition $\sum_{i=1}^{m} a_i = \sum_{j-1}^{n} b_j$ holds.

MOTP with inequalities both in supply and demand constraints can be presented as:

Minimize $Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, k = 1,2,\dots,K$

Subject to

$$\sum_{j=i}^{n} x_{ij} \leq a_{i,} i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, j = 1,2,\dots,n$$

$$x_{ij} \geq 0, \quad \forall i, j$$

This problem is feasible if and only if $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$.

In this paper, we shall present only fuzzy programming technique to find an optimal compromise solution of a transportation problem with several objectives. Here both linear and non-linear membership functions are used.

1.6. Related Literature Review

Some works have been done in transportation problem which in one way or the other relates to this paper.

Wahed and Lee (2006) proposed an interactive fuzzy goal programming approach to determine the preferred compromise solution for the multi-objective transportation problem. The proposed approach considers the imprecise nature of the input data by implementing the minimum operator and also assumes that each objective function has a fuzzy goal. The approach focuses on minimizing the worst upper bound to obtain an efficient solution which is close to the best lower bound of each objective function. The solution procedure controls the search direction via updating both the membership values and the aspiration levels.

Zangiabadi and Maleki (2007) presented a fuzzy goal programming approach to determine an optimal compromise solution for the multi-objective transportation problem by assuming that each objective function has a fuzzy goal. A special type of non-linear (hyperbolic) membership function is assigned to each objective function to describe each fuzzy goal. The approach focuses on minimizing the negative deviation variables from 1 to obtain a compromise solution of the multi-objective transportation.

Surapati and Roy (2008) presented a priority based fuzzy goal programming approach for solving a multi-objective transportation problem with fuzzy coefficients. Firstly, they defined the membership functions for the fuzzy goals. Subsequently, they transformed the membership functions into membership goals, by assigning the highest degree (unity) of a membership function as the aspiration level and introducing deviational variables to each of them. In the solution process, negative deviational variables are minimized to obtain the most satisfying solution.

Osuji et al (2013) carried out a research on the paradox algorithm application of linear transportation problem. Two numerical examples were used for the study. In their paper, an efficient algorithm for solving a linear programming problem was explicitly discussed, and it was concluded that paradox does not exist in the first set of data, while paradox existed in the second set of data. The Vogel's Approximation Method (VAM) was used to obtain the initial basic feasible solution via the Statistical Software Package known as TORA. The first set of data revealed that paradox does not exist, while the second set of data showed that paradox exists. The method however gave a step by step development of the solution procedure for finding all the paradoxical pair in the second set of data.

Lau et al. (2009) presented an algorithm called the fuzzy logic guided non-dominated sorting genetic algorithm to solve the multi-objective transportation problem that deals with the optimization of the vehicle routing in which multiple depots, multiple customers, and multiple products are considered. Since the total traveling time is not always restrictive as a time constraint, the objective considered compromises not only the total traveling distance, but also the traveling time.

2. Fuzzy Programming Technique to Solve Multi-Objective Transportation Problems

In this section, fuzzy programming technique to solve the MOTP with different type of membership functions is presented.

2.1. Linear Programming Formulation of MOTP

A MOTP can be stated as:

Minimize $Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, k = 1,2,\dots,K$

Subject to

$$\sum_{j=i}^{n} x_{ij} = a_{i,} i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,...,n$$

$$x_{ij} \geq 0, \quad \forall i \text{ and j}$$

The subscription Z_k and superscript on C_{ij}^k denote the K^{th} penalty criterion. We assume that $a_i \geq 0$ for all $i, b_j \geq 0$ for all $j, C_{ij} \geq 0$ for all i and j, and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad \text{(Equilibrium condition)}$$

a_i is the quantity of material available at source $O_i(i = 1,2,...,m)$

b_j is the quantity of material required at destination $D_j(j = 1,2,...,n)$ and

C_{ij} is fuzzy unit cost of transportation from source O_i to destination D_j.

2.2. Fuzzy Programming Technique to Solve MOTP

In fuzzy programming technique, we first find the lower bound as L_k and the upper bound as U_k for the K^{th} objective function $Z_k, k = 1,2,...,k$ where U_k is the highest acceptable level of achievement for objective k, L_k the aspired level of achievement for objective k and $d_k = U_k - L_k$ the degradation allowance for objective k.

When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. The solution of MOTP can be obtained by the following steps:

Step 1. Solve the MOTP as a single-objective transportation problem K times by taking one of the objectives at a time

Step 2. From the above results, determined the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix as follows:

	$Z_1(X)$	$Z_2(X)$	$Z_K(X)$
$X^{(1)}$	Z_{11}	Z_{22}	Z_{1K}
$X^{(2)}$	Z_{21}	Z_{22}	Z_{2K}
$X^{(K)}$	Z_{k1}	Z_{k2}	Z_{kK}

Where $X^{(1)}$, $X^{(2)},...,X^{(k)}$ are the isolated optimal solutions of the K different transportation problems for K different objective function, $Z_{ij} = Z_j (X^i)$, $i=1,2,...,K; j = 1, 2,...,k$ be the i^{th} row and j^{th} column element of the pay-off matrix.

Step 3. From step 2, find for each objective the U^k and the L^k corresponding to the set of

solution, where, U_k = maximum$(Z_{1k},Z_{2k},...Z_{kk})$ and L_k = minimum $(Z_{1k},Z_{2k},...,Z_{kk}$ $k = 1,2,...,K$

An initial fuzzy model of the problem can be;

Obtain X_{ij}, $i=1,2,...,m$, $j = 1,2,...,n$

$$Z_k \leq L_k, \quad \text{k} = 1,2,\ldots,\text{k}$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad \text{i} = 1,2,\ldots,\text{m}$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad j = 1,2,...,n$$

$$X_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

Step 4. Define a membership function $\mu(Z_k)$, for the k^{th} objective function

Step 5. Convert the fuzzy mode of the problem, obtained in step, into the following crisp

Model;

Maximize λ

Subject to $\lambda \leq \mu(Z_k)$

$$\sum_{j=1}^{n} X_{ij} = a_i, \; i = 1,2,..., m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \; j = 1,2,\ldots,n$$

$$X_{ij} \geq 0 \; \forall i \text{ and } j$$

$$\lambda \geq 0$$

Step 6. Solve the crisp model by an appropriate mathematical programming algorithm

Step 7. The solution obtained in step 6 will be the optimal compromise solution of the MOTP

2.3. Fuzzy Programming Technique with Linear Membership Function

A linear membership function is defined as:

$$\mu^L(Z_k) = \begin{cases} 1, & if \quad Z_k \leq L_k \\ 1 - \dfrac{Z_k - L_k}{U_k - L_k}, & if \quad L_k < Z_k < U_k \\ 0, & if \quad Z_k \geq U_k \end{cases}$$

If we use a linear membership function, the crisp model can be simplified as:

Maximize λ

Subject to

$$Z_k + \lambda(U_k - L_k) \leq U_k, k = 1,2,...,k$$

$$\sum_{j=1}^{n} X_{ij} = a_i, i = 1,2,...,m$$

$$\sum_{i=1}^{m} X_{ij} = b_{j,} j = 1,2,...,n$$

$$X_{ij} \geq 0 \quad \forall i, j$$

$$\lambda \geq 0$$

2.4. Fuzzy Programming Technique with Hyperbolic Membership Function

A hyperbolic membership functions is defined by

$$\mu^H(Z_k) = \begin{cases} 1, & if \quad Z_k \leq L_k \\ \frac{1}{2}\frac{e^{\{(U_k+L_k)/2-Z_k(X)\}a_k} - e^{-\{(U_k+L_k)/2-Z_k(X)\}a_k}}{e^{\{(U_k+L_k)/2-Z_k(X)\}a_k} + e^{-\{(U_k+L_k)/2-Z_k(X)\}a_k}} + \frac{1}{2}, & if \quad L_k < Z_k \quad < U_k \\ 0, & if \quad Z_k \geq U_k \end{cases}$$

Where $a_k = \frac{6}{(U_k - L_k)}$

If we will use the hyperbolic membership function then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

Subject to

$$\lambda \leq \frac{1}{2}\frac{e^{\{(U_k+L_k)/2-Z_k(X)\}a_k} e^{-\{(U_k+L_k)/2-Z_k(X)\}a_k}}{e^{\{(U_k+L_k)/2-Z_k(X)\}a_k} + e^{\{(U_kL_k)/2-Z_k(X)\}a_k}} + \frac{1}{2}, \quad k = 1,2,...,K \tag{1}$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1,2,...,m \tag{2}$$

$$\sum_{i=1}^{m} X_{ij} = b_j \quad j = 1,2,...,n$$

$$X_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

$$\lambda \geq 0$$

Constraint (1) can further be simplified as:

$$\lambda \leq \frac{1}{2} tanh\left[\left\{\frac{U_k + L_k}{2} - Z_k(X)\right\}a_k\right] + \frac{1}{2},$$

$$2\lambda \leq tanh\left[\left\{\frac{U_k + L_k}{2} - Z_k(X)\right\}a_k\right] + 1$$

$$tanh^{-1}(2\lambda - 1) \leq \left\{\frac{(U_k + L_k)}{2} - Z_k(X)\right\}a_k,$$

$$a_k Z_k + tanh^{-1}(2\lambda - 1) \leq \frac{(U_k + L_k)a_k}{2}$$

Now, putting $\tanh^{-1}(2\lambda - 1) = X_{mn+1,}$ constraint (1) is converted to

$a_k Z_k(X) + X_{mn+1} \leq \frac{a_k}{2}(U_k + L_k)$. Hence, the given problem is simplified as:

Maximize X_{mn-+1}

Subject to

$$a_k Z_k(X) + X_{mn+1} \leq \frac{a_k}{2}(U_k + L_k), \quad k = 1,2,..., K$$

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1,2,..., m$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1,2,...,n$$

$$X_{ij} \geq 0 \quad \forall i, j$$

$$X_{mn+1} \geq 0 \text{ Where } X_{mn+1}\, tanh^{-1}(2\lambda - 1)$$

3. Data Analysis

This section shall discuss how multi-objective transportation problem is solved using the algorithm discussed in the previous section. The data for this paper were collected by an egg dealer in whose main office is located at Orji Owerri Imo State Nigeria who supplies the product to different wholesalers after taking it from different poultry farm sources. There are four different suppliers named as S_1, S_2, S_3, S_4 and four destinations namely D_1, D_2, D_3, D_4. The researchers liaised with the egg dealer to study the average total cost and time of transportation for two weeks. Data for time and cost of supplying products from sources i to destination j are presented in Tables 1 and 2 respectively.

How much amount of material be supplied from different sources to all other destinations so that total cost of transportation and time of transportation is minimum.

Table 1. *data for time of supplying products from sources: i to destination j*

Sources \ Destination	D_1	D_2	D_3	D_4	Supply
S_1	24	29	18	23	21
S_2	33	20	29	32	24
S_3	21	42	12	20	18
S_4	25	30	19	24	30
Demand	13	22	26	30	93

Table 2. *Data for cost of supplying products from sources i to destinations j*

Sources \ Destination	D_1	D_2	D_3	D_4	Supply
S_1	14	21	18	13	21
S_2	24	13	21	23	24
S_3	12	30	9	11	18
S_4	13	22	19	14	30
Demand	13	22	26	30	93

The developed problem shall be formulated as:

$$\text{Min. } \begin{aligned} Z_1 = {} & 24X_{11} + 29X_{12} + 18X_{13} + 23X_{14} + 33X_{21} + 20X_{22} + 29X_{23} + 32X_{24} + 21X_{31} + \\ & 42X_{32} + 12X_{33} + 20X_{34} + 25X_{41} + 30X_{42} + 19X_{43} + 24X_{44} + 22X_{42} + 19X_{43} + 14X_{44} \end{aligned} \quad (3)$$

$$\text{Min. } \begin{aligned} Z_2 = {} & 14X_{11} + 21X_{12} + 18X_{13} + 13X_{14} + 24X_{21} + 13X_{22} + 21X_{23} + 23X_{24} + \\ & 23X_{24} + 12X_{31} + 30X_{32} + 9X_{33} + 11X_{34} + 13X_{41}13X_{41} + 22X_{42} + 19X_{43} + 14X_{44} \end{aligned} \quad (4)$$

Subject to:

$$\begin{aligned} & \sum_{j=1}^{4} X_{ij} = 21, \sum_{j=1}^{4} X_{2j} = 24, \sum_{j=1}^{4} X_{3j} = 18, \sum_{j=1}^{4} X_{4j} = 30 \\ & \sum_{j=1}^{4} X_{1i} = 13, \sum_{j=1}^{4} X_{i2} = 22, \sum_{j=1}^{4} X_{i3} = 26, \sum_{i=1}^{4} X_{i4} = 30 \\ & X_{ij} \geq 0, i = 1,2,3,4; \ j = 1,2,3,4 \end{aligned} \quad (5)$$

Where Z_1, Z_2 represents total cost and total time of transportation respectively.

Solving equations (3) and (5), we obtain the optimal solution as:

$$X^{(i)} = \begin{cases} X_{13} = 8, X_{14} = 22, X_{24} = 2, X_{33} = 18, X_{41} = 13, X_{44} = 13, \\ X_{11} = X_{12} = Z_{21} = X_{23} = X_{31} = X_{32} = X_{34} = X_{42} = X_{43} = 0 \end{cases}$$

$$Z_1\left(X^{(i)}\right) = 1898, \text{ and } Z_2\left(X^{i}\right) = 1286$$

Solving equation (4) and (5), we obtain the optimal solution as;

$$X^{(2)} = \begin{cases} X_{14} = 21, X_{22} = 22, X_{23} = 2, X_{33} = 18, X_{41} = 15, X_{43} = 6, X_{44} = 9, \\ X_{44} = 9, X_{11} = X_{12} = X_{13} = X_{21} = X_{24} = X_{31} = X_{32} = X_{34} = X_{42} = 0 \end{cases}$$

$Z_2\left(X^{2}\right) = 1272$, and $Z_1\left(X^{i}\right) = 1902$

The outcomes obtained from step 1 give the following pay-off matrix as;

$$\begin{matrix} & Z_1\left(X^{(1)}\right) & Z_2\left(X^{(2)}\right) \\ X^{(1)} & 898 & 1286 \\ X^{(2)} & 1902 & 1272 \end{matrix}$$

From the pay-off matrix, we obtain

$$U_1 = \max imum\{1898, 1902\} = 1902$$
$$L_1 = \min imum\{1898, 1902\} = 1898$$

$$U_2 = \max\ imum\ \{1286, 1272\} = 1286$$
$$L_2 = \min\ imum\ \{1286, 1272\} = 1272$$

If a linear membership function is employed, the crisp model can be presented as follows:

Maximize λ

Subject to:

$$\begin{aligned} & 24X_{11} + 21X_{12} + 18X_{13} + 23X_{14} + 33X_{21} + 20X_{22} + 29X_{23} + 32X_{24} + 21X_{31} \\ & + 42X_{32}12X_{33} + 20X_{34} + 25X_{41} + 30X_{42} + 19X_{43} + 24X_{44} + 41 \leq 1902 \end{aligned}$$

$$\begin{aligned} & 14X_{11} + 21X_{12} + 18X_{13} + 13X_{14} + 24X_{21} + 15X_{23} + 23X_{24} + 12X_{31} + \\ & 30X_{32}9X_{33} + 11X_{34} + 15X_{41} + 22X_{42} + 19X_{43} + 14X_{44} + 14\lambda \leq 1286 \end{aligned}$$

$$\sum_{j=1}^{4} X_{1j} = 21, \sum_{j=1}^{4} X_{2j} = 24, \sum_{j=1}^{4} X_{3j} = 18, \sum_{j=1}^{4} X_{4j} = 30, \sum_{i=1}^{4} X_{1i} = 15, \sum_{i=1}^{4} X_{i2} = 22, \sum_{i=1}^{4} X_{i3} = 26, \sum_{i=1}^{4} X_{i4} = 30$$

$$X_{ij} \geq 0, i = 1,2,3,4; \ j = 1,2,3,4 \, and \, \lambda \geq 0$$

The optimal solution of the above problem is thus presented below as;

$$X^* = \begin{cases} X_{14} = 21, X_{21} = 1, X_{22} = 22, X_{23} = 1, X_{33} = 18, X_{41} = 14, X_{43} = 7, \\ X_{44} = 9, X_{11} = X_{12} = X_{13} = X_{24} = X_{31} = X_{32} = X_{34} = X_{42} = 0 \end{cases}$$

$$Z_i^* = 1900 \quad Z_2^* = 1279 \text{ and } \lambda^* = 0.50$$

If we use the hyperbolic membership functions, an equivalent crisp model can be formulated as:
Maximize X_{10}
Subject to:

$$36X_{11} + 43.5X_{12} + 27X_{13} + 34.5X_{14} + 49.5X_{21} + 30X_{22} + 43.5X_{23} + 48X_{24} + \\ 31.5X_{31} + 63X_{32} + 18X_{33} + 30X_{34} + 37.5X_{41} + 45X_{42} + 28.5X_{43} + 36X_{44} + X_{10} \le 2850$$

$$\sum_{j-1}^{4} X_{1j} = 21, \sum_{j-1}^{4} X_{2j} + 24, \sum_{j-1}^{4} X_{3j} = 18, \sum_{j-1}^{4} X_{4j} = 30$$

$$\sum_{i-1}^{4} X_{i1} = 15, \sum_{i2}^{4} X_{i2} = 22, \sum_{i1}^{4} X_{i3} = 26, \sum_{i1}^{4} X_{i4} = 30$$

$$14X_{11} + 21X_{12} + 18X_{13} + 13X_{14} + 24X_{21} + 15X_{22} + 21X_{23} + 23X_{24} + 12X_{31} + \\ 30X_{32} + 9X_{33} + 11X_{34} + 15X_{41} + 22X_{42} + 19X_{43} + 14X_{44} + X_{10} \le 1918.5$$

$$X_{ij} \ge 0, i = 1,2,3,4; j = 1,2,3,4 \text{, and } X_{10} \ge 0$$

Solving the above problem, the optimal solution is shown a follows:

$$X^* = \begin{cases} X_{14} = 21, X_{22} = 22, X_{24} = 2, X_{33} = 18, X_{41} = 15, X_{45} = 8X_{44} = 7, \\ X_{11} = X_{12} = X_{13}X_{21} = X_{23} = X_{32} = X_{32} = X_{34} = X_{42} = 0 \end{cases}$$

$$X_{10} = 3.00$$

Therefore;
$Z_1^* = 1898, Z_2^* = 1286$, and $\lambda^* = 0.50$

4. Conclusion

In this paper, two special types of membership functions linear and non-linear are used to solve the MOTP. From the results in this paper, it is observed that if we use the hyperbolic membership function, then the crisp model becomes linear. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function. However, if we compare with the solution obtained by the linear membership function, it is shown that the fuzzy optimal values do not depend on the chosen membership function whether linear or non-linear membership function is used.

References

[1] Lau, H. C.W., Chan, T.M., Tsui, W. T., Chan, F. T. S., Ho, G. T. S. and Choy, K. L., A (2009): fuzzy guided multi-objective evolutionary algorithm model for solving transportation problem, Expert System with Application: An International Journal, Vol. 36,2009, pp.8255-8268.

[2] Osuji G. A. Opara J., Nwobi A. C., Onyeze V., Iheagwara A. I.(2013): Paradox Algorithm in Application of a Linear Transportation Problem. American Journal of Applied Mathematics and Statistics, 2014, Vol. 2, No. 1,10-15. DOI:10.12691/ajams-2-1-3

[3] Surapati, P. and Roy, T. K. (2008). Multi-objective transportation model with fuzzy parameters: Priority based fuzzy goal programming approach, Journal of Transportation System Engineering and Informational Technology, Vol. 8, 2008, pp 40-48

[4] Wahed, W. F. and Lee, S. M. (2006). Interactive fuzzy goal programming for multi-objective transportation problems, Omega, Vol. 34, 2006, pp. 158-166.

[5] Zangiabadi, M. and Maleki, H. R. (2007). Fuzzy goal programming for multi-objective transportation problems, Applied Mathematics Computation, Vol. 24-2007, pp. 449-460.