An alternative estimator for estimating the finite population mean in presence of measurement errors with the view to financial modelling

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Abstract: This article presents the problem of estimating the population mean using auxiliary information in the presence of measurement errors. We have compared the three proposed estimators being the exponential ratio-type estimator, Solanki et al. (2012) estimator, and the mean per unit estimator in the presence of measurement errors. Financial Model by Gujrati and Sangeetha (2007) has been employed in our empirical analysis. In that, our investigation has indicated that our proposed general class of estimator t_4 is the most suitable estimator with a smaller MSE relative to other estimators under measurement errors.

Keywords: Population Mean, Study Variate, Auxiliary Variates, Mean Squared Error, Measurement Errors, Efficiency, Financial Model

1. Introduction

In survey sampling, the properties of the estimators based on data usually presuppose that the observations are the correct measurements on characteristics being studied. Unfortunately, this ideal is not met in practice for a variety of reasons, such as non response errors, reporting errors, and computing errors, and sensitivity errors. When the measurement errors are negligible small, the statistical inferences based on observed data continue to remain valid. On the contrary, when they are not appreciably small and negligible, the inferences may not be simply invalid and inaccurate but may often lead to unexpected, undesirable and unfortunate consequences (Shalabh, 2001). Some authors including Allen et al. (2003), Manisha and Singh (2001, 2002), Shalabh (1997), Bahl, S. and Tuteja, R. K. (1991), Koyuncu, N. and Kadilar, C. (2010), Singh and Karpe (2008, 2009), Kumar et al. (2011a,b) and Singh et al. (2011) have paid their attention towards the estimation of population mean μ_y of the study variable y using auxiliary information in the presence of measurement errors.

For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i, Y_i) on two

characteristics (x, y) respectively for the i^{th} ($i=1,2,\dots,n$) unit in the sample of size n .

Let the measurement errors be

$$u_i = y_i - Y_i \quad (1.1)$$

$$v_i = x_i - X_i \quad (1.2)$$

which are stochastic in nature with mean zero and variances σ_u^2 and σ_v^2 respectively, and are independent. Further, let the population means of (x, y) be (μ_x, μ_y) , population variances of (x, y) be (σ_x^2, σ_y^2) and σ_{xy} and ρ be the population covariance and the population correlation coefficient between x and y respectively (see Manisha and Singh (2002)).

Let

$$k_1 = \bar{y} - \mu_y = \frac{1}{\sqrt{n}}(w_y - w_u)$$

and,

$$k_2 = \bar{x} - \mu_x = \frac{1}{\sqrt{n}}(w_x - w_v), \text{ var}(\bar{y}) = \frac{\sigma_y^2}{n} \left[1 + \frac{\sigma_u^2}{\sigma_y^2} \right]$$

and

$$E(k_1) = E(k_2) = 0$$

$$E(k_1^2) = \frac{\sigma_y^2}{n} \left(1 + \frac{\sigma_u^2}{\sigma_y^2} \right) = V_{ym}$$

$$E(k_2^2) = \frac{\sigma_x^2}{n} \left(1 + \frac{\sigma_v^2}{\sigma_x^2} \right) = V_{xm}$$

$$E(k_1 k_2) = \frac{\rho \sigma_y \sigma_x}{n} = V_{yxm}$$

In this paper, we have studied the behaviour of some estimators in presence of measurement error.

2. Estimators in Literature

Singh et al. (2011) suggested an exponential ratio type and a difference type estimator under measurement error for estimating $\bar{y}$ as

$$t_1 = \bar{y} \exp \left(\frac{\mu_x - \bar{x}}{\mu_x + \bar{x}} \right) \quad (2.1)$$

$$t_2 = \omega_1 \bar{y} + \omega_2 (\mu_x - \bar{x}) \quad (2.2)$$

The biases and MSE's of the estimators are respectively given by

$$Bias(t_1) = \frac{1}{\mu_x} \left(\frac{3}{8} R_m V_{xm} - \frac{1}{2} V_{yxm} \right) \quad (2.3)$$

$$Bias(t_2) = \mu_y (\omega_1 - 1) \quad (2.4)$$

$$MSE(t_1) = \frac{\sigma_y^2}{n} \left[1 - \frac{C_x}{C_y} \left(\rho - \frac{C_x}{4C_y} \right) \right] + \frac{1}{n} \left[\frac{\mu_y^2}{4\mu_x^2} \sigma_v^2 + \sigma_u^2 \right] \quad (2.5)$$

$$MSE(t_2) = (\omega_1 - 1)^2 \mu_y^2 + \omega_1^2 (a_1) + \omega_2^2 a_2 + 2\omega_1 \omega_2 (-a_3) \quad (2.6)$$

where,

$$a_1 = (V_{ym}), a_2 = (V_{xm}), \text{ and } a_3 = (V_{yxm})$$

Taking expectation of both sides of (3.2), we get the bias of the estimator t_3 to the order $O(n^{-1})$ as

$$Bias(t_3) = \mu_y \left\{ \frac{V_{xm} A}{\mu_x^2} \right\} - \left\{ \frac{B}{\mu_x} V_{xm} \right\} \quad (3.3)$$

Now, optimising MSE of the estimator t_2 with respect to ω_1 and ω_2 , we get

$$\omega_1^* = \frac{b_3 b_4}{b_1 b_3 - b_2^2} \text{ and } \omega_2^* = -\frac{b_2 b_4}{b_1 b_3 - b_2^2} \quad (2.7)$$

where,

$$b_1 = \mu_y^2 + a_1, b_2 = -a_3, b_3 = a_2 \text{ and } b_4 = \mu_y^2.$$

Using these optimum values of ω_1^* and ω_2^* from equation (2.7) into equation (2.6), we get the minimum MSE of the estimator t_2 as

$$MSE(t_2)_{\min} = \left[\mu_y^2 - \frac{b_3 b_4^2}{b_1 b_3 - b_2^2} \right] \quad (2.8)$$

3. Proposed Estimators

Solanki et al. (2012) estimator under measurement error is given by

$$t_3 = \bar{y} \left\{ 2 - \left(\frac{\bar{x}}{\mu_x} \right)^\alpha \exp \left[\frac{\beta (\bar{x} - \mu_x)}{(\bar{x} + \mu_x)} \right] \right\} \quad (3.1)$$

where α and β are suitably chosen scalars.

Let

$$w_u = \frac{1}{\sqrt{n}} \sum_{i=1}^n u_i, w_y = \frac{1}{\sqrt{n}} \sum_{i=1}^n (y_i - \mu_y)$$

$$w_v = \frac{1}{\sqrt{n}} \sum_{i=1}^n v_i, w_x = \frac{1}{\sqrt{n}} \sum_{i=1}^n (x_i - \mu_x)$$

$$C_x = \frac{\sigma_x}{\mu_x} \text{ and } C_y = \frac{\sigma_y}{\mu_y}$$

Expanding equation (3.1) and subtracting μ_y from both sides, we get

$$(t_3 - \mu_y) = (\mu_y + k_1) \left[1 - \frac{k_2}{\mu_x} \left(\alpha + \frac{\beta}{2} \right) - \frac{k_2^2}{\mu_x^2} \left(\alpha(\alpha - 1) + \frac{\beta(\beta - 2)}{8} + \frac{\alpha\beta}{2} \right) \right] \quad (3.2)$$

where,

$$A = \left[\alpha(\alpha - 1) + \frac{\beta(\beta - 2)}{8} + \frac{\alpha\beta}{2} \right], \text{ and } B = \left(\alpha + \frac{\beta}{2} \right).$$

Squaring both sides of (3.2) and taking expectations, the

MSE of t_3 to the order $O(n^{-1})$ is given by

$$MSE(t_3) = E(t_3 - \mu_y)^2 = V_{yxm} + V_{xm} R_m^2 B^2 - 2R_m V_{yxm} B \quad (3.4)$$

where $R_m = \frac{\mu_y}{\mu_x}$.

Following Solanki et al. (2012), we propose a general class of estimator t_4 as

$$(t_4 - \mu_y) = \left\{ (m_1 - 1)\mu_y - m_1\mu_y \left\{ B \frac{k_2}{\mu_x} + \frac{k_2^2 A}{\mu_x^2} \right\} + m_1 k_1 \left\{ 1 - \frac{Bk_2}{\mu_x} \right\} - m_2 k_2 \left\{ 1 - \frac{Bk_2}{\mu_x} \right\} \right\} \quad (3.5)$$

Expanding equation (3.4) and subtracting μ_y from both sides, we get

On taking expectation of both sides of (3.5), we get the bias of the estimator t_4 to the order $O(n^{-1})$ as

$$Bias(t_4) = (m_1 - 1)\mu_y - m_1\mu_y \left\{ \frac{V_{xm} A}{\mu_x^2} \right\} - m_1 \left\{ \frac{B}{\mu_x} V_{yxm} \right\} + m_2 \left\{ \frac{B}{\mu_x} V_{xm} \right\} \quad (3.6)$$

Squaring both sides of (3.5) and taking expectations, the MSE of t_4 to the order $O(n^{-1})$ is

$$MSE(t_4) = E(t_4 - \mu_y)^2 = (m_1 - 1)^2 \mu_y^2 + m_1^2 R_m^2 B^2 V_{xm} + m_1^2 V_{yxm} - 2m_1^2 R_m B V_{yxm} - 2m_1 (m_1 - 1) R_m^2 A V_{xm} + m_2^2 V_{xm} + 2 \left\{ m_2 (m_1 - 1) B R_m V_{xm} + m_1 m_2 R_m V_{xm} - m_1 m_2 V_{yxm} + m_1 (m_1 - 1) B R_m V_{yxm} \right\} \quad (3.7)$$

The MSE of the estimator t_4 can also be written as

$$MSE(t_4) = (m_1 - 1)^2 \mu_y^2 + m_1^2 P_1 + m_2^2 P_2 + 2m_1 m_2 P_3 - 2m_1 P_3 - 2m P A_5 \quad (3.8)$$

where,

or

$$P_1 = (V_{ym} + B^2 R_m^2 V_{xm} - 2R_m^2 A V_{xm}), \quad P_2 = (V_{xm}),$$

$$R_m^2 \frac{V_{xm}}{V_{yxm}} \leq 4 \quad (4.1)$$

$$P_3 = (2B R_m V_{xm} - V_{yxm}), \quad P_4 = (R_m B V_{yxm} - A V_{xm} R_m^2),$$

$$P_5 = (B R_m V_{xm}).$$

Now, optimising MSE t_4 with respect to, m_1 and m_2 , we get the optimum values as -

$$m_1^* = \frac{B_1 P_2 - P_3 P_5}{B_2 P_2 - P_3^2} \quad \text{and} \quad m_2^* = \frac{B_2 P_5 - B_1 P_3}{B_2 P_2 - P_3^2}$$

where,

$$B_1 = \mu_y^2 + P_4, \quad B_2 = \mu_y^2 + P_1.$$

4. Theoretical Efficiency Comparisons

The MSE of the proposed estimator t_4 proposed in (3.4) will be smaller than usual estimator under measurement error case if the following condition is satisfied by the data set

$$\frac{\sigma_y^2}{n} \left[1 - \frac{C_x}{C_y} \left(\rho - \frac{C_x}{4C_y} \right) \right] + \frac{1}{n} \left[\frac{\mu_y^2}{4\mu_x^2} \sigma_v^2 + \sigma_u^2 \right] \leq \frac{\sigma_y^2}{n} \left(1 + \frac{\sigma_u^2}{\sigma_y^2} \right)$$

As we know that the estimators t_3 defined in (3.1) is the particular member of the generalised estimator t_4 so, if the above condition is satisfied for different values of α, β , and m_1, m_2 the estimator t_3 will be better than usual estimator under measurement errors.

Also,

$$MSE(t_4)_{\min} \leq V(\bar{y}_m) \quad (4.2)$$

5. Empirical Study

Data statistics: The data used for empirical study has been taken from Gujrati and Sangeetha (2007).

Where, Y_i = True consumption expenditure,

X_i = True income,

y_i = Measured consumption expenditure,

x_i = Measured income.

From the data given, we get the following parameter values

n	μ_y	μ_x	σ_y^2	σ_x^2	ρ	σ_u^2	σ_v^2
10	127	170	1278	3300	0.964	36.00	36.00

Table 5.1. Showing the Percent Relative Efficiencies' PRE's of estimators with respect to $\bar{y}_m$.

Estimators	Values of α and β	PRE
$\bar{y}_m$	-	100.00
t_1	-	437.59
t_{regm}	$\alpha = 1, \beta_m = \frac{V_{yxm}}{\sqrt{V_{xm}V_{ym}}}$	946.54
t_{2min}	$\alpha = 0, \beta = 0$	944.94
t_{3min}	$\alpha = 1, \beta = 1$	123.23
	$\alpha = 1, \beta = 0$	603.01
	$\alpha = 0, \beta = 1$	437.27
	$\alpha = 1, \beta = -1$	437.27
	$\alpha = 1, \beta = 0$	1012.77
t_{4min}	$\alpha = 0, \beta = 1$	1031.13
	$\alpha = 1, \beta = 1$	948.35
	$\alpha = 1, \beta = -1$	1031.11

Table 5.2. Showing the MSE's of the estimators with and without measurement errors.

Estimators	MSE without measurement error	Contribution of measurement error in MSE	MSE with measurement error
$\bar{y}_m$	127.800	3.600	131.400
t_1	25.925	4.102	30.028
t_{reg}	9.000	4.896	13.882
t_{2min}	$(\alpha = 0, \beta = 0)$	4.910	13.905
	$(\alpha = 1, \beta = 0)$	4.587	21.790
	$(\alpha = 0, \beta = 1)$	4.252	30.050
	$(\alpha = 1, \beta = 1)$	4.747	106.621
	$(\alpha = 1, \beta = -1)$	4.278	30.050
t_{3min}	$(\alpha = 1, \beta = 0)$	4.577	12.974
	$(\alpha = 0, \beta = 1)$	4.207	12.743
	$(\alpha = 1, \beta = 1)$	4.865	13.855
	$(\alpha = 1, \beta = -1)$	4.874	12.742
	$(\alpha = 1, \beta = -1)$	7.868	

6. Conclusion

We observe that our proposed estimator t_4 is the most appropriate estimator given the set of optimality conditions depicted in Table 5.1. That is, the MSE of our proposed estimator is lower than the MSE of estimators that have been studied in this paper. Furthermore, it shall be noted that the class of usual estimator is the least that is impacted by the measurement error, and unequivocally it has maintained its topological stability. Our result that is illustrated in Table 5.2. confirms that it is imperative to consider observational errors in order to obtain true variances, and to minimize the topological overshooting and undershooting of measurement errors. Our Future research could potentially include the fuzzy efficiency of our estimator t_4 with a single Fuzzy Logic Controller. This de novo investigation perhaps enables us to test the sensitivity and specificity of various data structures more decisively under fuzzy measurement errors.

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