Effect of Chemical Reaction on Statistical Theory of Dusty Fluid MHD Turbulent Flow for Certain Variables at Three-Point Distribution Functions

M. Abul Kalam Azad, M. Abu Bkar Pk, Abdul Malek

Department of Applied Mathematics, University of Rajshahi, Rajshahi, Bangladesh

Email address: azad267@gmail.com (M. A. K. Azad), abubakarpk@yahoo.com (M. A. Bkar Pk), am.math.1970@gmail.com (A. Malek)

Abstract: In this paper, an attempt is made to study the three-point distribution functions in dusty fluid MHD turbulent flow for simultaneous velocity, magnetic temperature and concentration fields in a first order chemical reaction. It has been discussed the various properties of constructed distribution functions. From beginning to end out the study, the transport equation for three-point distribution functions in dusty fluid MHD turbulent flow undergoing a first order chemical reaction has been obtained. The obtained equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

Keywords: First Order Chemical Reaction, Dusty Fluid, MHD Turbulent Flow, Three-Point Distribution Functions, Magnetic, Temperature, Concentration

1. Introduction

Nowadays dusty viscous fluid motion has developed rapidly which plays an important role in turbulent flow. The phenomenon of this type occurs in the movement of dust-laden air, in problem of fluidization, in the use of dust in a gas cooling system and in the sedimentation problem of tidal rivers. The behavior of dust particle in a turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent fluid. In molecular kinetic theory in physics, a particle's distribution function is a function of several variables. Particle distribution functions are used in plasma physics to describe wave-particle interactions and velocity-space instabilities. Distribution functions are also used in fluid mechanics, statistical mechanics and nuclear physics. Also a first-order reaction is defined a reaction that proceeds at a rate that depends linearly only on one reactant concentration. In the past, several authors discussed the distribution functions in the statistical theory of turbulence such as Hopf (1952), Kraichnan (1956), Edward (1964), and Herring (1965). Further Lundgren (1967) derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-You (1966) in the kinetic theory of gasses. Bigler (1976) gave the hypothesis that in turbulent flames, the thermo chemical quantities can be related locally to few scalars and considered the probability density function of these scalars. Kishore (1978) studied the Distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope (1979) studied the statistical theory of turbulence flames. Pope (1981) derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kollman and Janicka (1982) derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient – flux model. Kishore and Singh (1984) derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh (1985) have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Dixit and Upadhayay (1989) considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Sarkar and Kishore (1991) discussed the distribution functions in the statistical theory of convective MHD


In this paper, the main purpose is to study the statistical theory of 3-point distribution function for simultaneous velocity, magnetic, temperature, concentration fields in MHD turbulence in presence of dust particles undergoing a first order reaction. Throughout the study, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The obtained three-point transport equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

2. Material and Methods

2.1. Basic Equations

The equations of motion and continuity for viscous incompressible dusty fluid MHD turbulent with constant reaction rate, the diffusion equations for the temperature and concentration are given by

\[
\frac{\partial u_x}{\partial t} + \frac{\partial}{\partial x_y} (u_x u_y - h_y h_x) = -\frac{1}{4\pi \tilde{\alpha}_x} \left[ \frac{\partial^2}{\partial x_y^2} - \frac{\partial^2}{\partial x_z^2} \right] \frac{\partial^2}{\partial x_z^2} + \nu \nabla^2 u_x
\]

\[
\frac{\partial h_y}{\partial t} + \frac{\partial}{\partial x_y} (h_y u_{y} - u_y h_y) = \lambda \nabla^2 h_y ,
\]

\[
\frac{\partial \theta}{\partial t} + u_x \frac{\partial \theta}{\partial x_y} = \chi \nabla^2 \theta ,
\]
\[ \frac{\partial c}{\partial t} + u_\alpha \frac{\partial c}{\partial x_\alpha} = D\nabla^2 c - Rc \]  

with

\[ \frac{\partial u_\alpha}{\partial x_\alpha} = \frac{\partial \rho}{\partial x_\alpha} = \frac{\partial h_\alpha}{\partial x_\alpha} = 0 \]

where \( u_\alpha(x,t) \), \( \alpha \) – component of turbulent velocity; \( h_\alpha(x,t) \), \( \alpha \) – component of magnetic field; \( \theta(x,t) \), temperature fluctuation; \( c \), concentration of contaminants; 
\( v_\alpha \), dust particle velocity; \( f = \frac{KN}{\rho} \), dimension of frequency; 
\( N \), constant number of density of the dust particle; 
\( \lambda \), Alfven velocity; \( \nu_\alpha \), magnetic diffusivity; \( \nu_c \), specific heat at constant pressure; \( k_\gamma \), thermal diffusivity; \( \sigma_\gamma \), electrical conductivity; \( \mu_\gamma \), magnetic permeability; \( D \), diffusion co-efficient for contaminants; \( R \), constant reaction rate.

The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. In the whole process \( u, h \) and \( x \) are the vector quantities.

### 2.2. Formulation of the Problem

It has considered that the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field and the reaction rate and the diffusivity are constant. It is also considered a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity \( u \), Alfven velocity \( h \), temperature \( \theta \) and concentration \( c \) are randomly distributed functions of position and time and satisfy their field. Different members of ensemble are subjected to different initial conditions and the aim is to find out a way by which we can determine the ensemble averages at the initial time.

Certain microscopic properties of conducting fluids such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct a joint distribution function for its evolution of three-point distribution functions

\[ \mathcal{F}^{(i)}(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}) = \{
\{ \delta(u^{(i)} - v^{(i)}) \} \delta(h^{(i)} - g^{(i)}) \delta(\theta^{(i)} - \phi^{(i)}) \delta(c^{(i)} - \psi^{(i)}) \}
\]

where \( \delta \) is the Dirac delta-function defined as

\[ \int \delta(\vec{u} - \vec{v}) d\vec{v} = \begin{cases} 1 & \text{at the point } \vec{u} = \vec{v} \\ 0 & \text{elsewhere} \end{cases} \]

in dusty fluid MHD turbulent flow undergoing first order reaction, study its properties and derive a transport equation for the joint distribution function of velocity, temperature and concentration in dusty fluid MHD turbulent flow undergoing a first order reaction.

### 2.3. Distribution Function in MHD Turbulence and Their Properties

In MHD turbulence, it is considered that the fluid velocity \( u \), Alfven velocity \( h \), temperature \( \theta \) and concentration \( c \) at each point of the flow field. Corresponding to each point of the flow field, there are four measurable characteristics represent by the four variables by \( v, g, \phi \) and \( \psi \) and denote the pairs of these variables at the points \( x^{(1)}, x^{(2)}, \cdots, x^{(n)} \) as \( \{ (v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}) \}, \{ (v^{(2)}, g^{(2)}, \phi^{(2)}, \psi^{(2)}) \}, \cdots, \{ (v^{(n)}, g^{(n)}, \phi^{(n)}, \psi^{(n)}) \} \) at a fixed instant of time.

It is possible that the same pair may be occurred more than once; therefore, it simplifies the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

\[ \{ (v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}) \}; \{ (v^{(2)}, g^{(2)}, \phi^{(2)}, \psi^{(2)}) \}; \cdots; \{ (v^{(n)}, g^{(n)}, \phi^{(n)}, \psi^{(n)}) \} \}

Instead of considering discrete points in the flow field, if it is considered the continuous distribution of the variables \( v, g, \phi \) and \( \psi \) over the entire flow field, statistically behavior of the fluid may be described by the distribution function

\[ F(v, g, \phi, \psi) \] which is normalized so that

\[ \int F(v, g, \phi, \psi) dvgdg d\phi d\psi = 1 \]

where the integration ranges over all the possible values of \( v, g, \phi \) and \( \psi \). We shall make use of the same normalization condition for the discrete distributions also.

The distribution functions of the above quantities can be defined in terms of Dirac delta function.

The one-point distribution function

\[ F^{(i)}(v^{(i)}, g^{(i)}, \phi^{(i)}, \psi^{(i)}) \], defined so that

\[ F^{(i)}(v^{(i)}, g^{(i)}, \phi^{(i)}, \psi^{(i)}) dv^{(i)} dg^{(i)} d\phi^{(i)} d\psi^{(i)} \]

is the probability that the fluid velocity, Alfven velocity, temperature and concentration at a time \( t \) are in the element \( dv^{(i)} \) about \( v^{(i)} \), \( dg^{(i)} \) about \( g^{(i)} \), \( d\phi^{(i)} \) about \( \phi^{(i)} \) and \( d\psi^{(i)} \) about \( \psi^{(i)} \) respectively and is given by

\[ \int \delta(u - v) dv = \begin{cases} 1 & \text{at the point } u = v \\ 0 & \text{elsewhere} \end{cases} \]
Two-point distribution function is given by

\[ F_{2}^{(1,2)} = \left\{ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \right\} \] (7)

and three point distribution function is given by

\[ F_{3}^{(1,2,3)} = \left\{ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \right\} \times \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \} \] (8)

Similarly, we can define an infinite numbers of multi-point distribution functions \( F_{4}^{(1,2,3,4)}, F_{5}^{(1,2,3,4,5)} \) and so on. The following properties of the constructed distribution functions can be deduced from the above definitions:

(A) Reduction Properties:
Integration with respect to pair of variables at one-point lowers the order of distribution function by one. For example,

\[
\int F_{1}^{(1)} dv^{(1)} = \left\{ \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\}, \\
\int F_{1}^{(1)} dg^{(1)} = \left\{ \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\}, \\
\int F_{1}^{(1)} d\varphi^{(1)} = \left\{ \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\},
\]

and so on.

(B) Separation Properties:
If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

\[
\lim_{x^{(2)} \rightarrow x^{(1)}} |x^{(2)} - x^{(1)}| \rightarrow \infty \quad F_{2}^{(1,2)}(x^{(1)}, x^{(2)}) = F_{1}^{(1)}(x^{(1)}) F_{1}^{(2)}(x^{(2)})
\]

and similarly,

\[
\lim_{r^{(3)} \rightarrow r^{(2)}} |r^{(3)} - r^{(2)}| \rightarrow \infty \quad F_{3}^{(1,2,3)}(r^{(1)}, r^{(2)}, r^{(3)}) = F_{2}^{(1,2)}(r^{(1)}, r^{(2)}) F_{1}^{(3)}(r^{(3)}) 	ext{ etc.}
\]

(C) Co-incidence Properties:
When two points coincide in the flow field, the components at these points should be obviously the same that is \( F_{2}^{(1,2)} \) must be zero.

Thus \( v^{(2)} = v^{(1)}, \quad g^{(2)} = g^{(1)}, \quad \varphi^{(2)} = \varphi^{(1)} \) and \( \psi^{(2)} = \psi^{(1)} \), but \( F_{2}^{(1,2)} \) must also have the property.

\[
\lim_{u^{(2)} \rightarrow u^{(1)}} |u^{(2)} - u^{(1)}| \rightarrow \infty \quad F_{2}^{(1,2)}(u^{(1)}, u^{(2)}) = F_{1}^{(1)}(u^{(1)})
\]

and hence it follows that

\[
\lim_{u^{(3)} \rightarrow u^{(1)}} |u^{(3)} - u^{(1)}| \rightarrow \infty \quad F_{3}^{(1,2,3)}(u^{(1)}, u^{(2)}, u^{(3)}) = F_{2}^{(1,2)}(u^{(1)}, u^{(2)}) F_{1}^{(3)}(u^{(3)}) 	ext{ etc.}
\]

(D) Symmetric Conditions:

\[
F_{n}^{(1,2,\ldots,k,\ldots,r,\ldots,n)} = F_{n}^{(1,2,\ldots,r,\ldots,k,\ldots,n)}.
\]

(E) Incompressibility Conditions:
2.4. Continuity Equation in Terms of Distribution Functions

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective MHD turbulent flow and are obtained directly by using \( \text{div } u = 0 \)

Taking ensemble average of equation (5), we get

\[
0 = \left( \frac{\partial u^{(i)}}{\partial x^{(i)}} \right) = \left\{ \frac{\partial}{\partial x^{(i)}} u^{(i)} \right\} \int F^{(i)}(x) di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

\[
= \frac{\partial}{\partial x^{(i)}} \int \left\{ u^{(i)} F^{(i)}(x) \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)} = \frac{\partial}{\partial x^{(i)}} \int v^{(i)} F^{(i)}(x) di^{(i)} d\varphi^{(i)} d\psi^{(i)} = \int \frac{\partial F^{(i)}}{\partial x^{(i)}} u^{(i)} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

and similarly,

\[
0 = \int \frac{\partial F^{(i)}}{\partial x^{(i)}} g^{(i)} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

Equation (15) and (16) are the first order continuity equations in which only one point distribution function is involved.

For second-order continuity equations, if we multiply the continuity equation by \( \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \delta (c^{(2)} - \psi^{(2)}) \) and if we take the ensemble average, we obtain

\[
o = \left\{ \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \delta (c^{(2)} - \psi^{(2)}) \right\} \frac{\partial u^{(i)}}{\partial x^{(i)}}
\]

\[
= \frac{\partial}{\partial x^{(i)}} \left\{ \int \left\{ u^{(i)} \delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \right\} \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

\[
\times \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \delta (c^{(2)} - \psi^{(2)}) \right\} \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

\[
= \frac{\partial}{\partial x^{(i)}} \int u^{(i)} \delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

and similarly,

\[
o = \frac{\partial}{\partial x^{(i)}} \int g^{(i)} \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

The Nth – order continuity equations are

\[
o = \frac{\partial}{\partial x^{(i)}} \int v^{(i)} \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

The continuity equations are symmetric in their arguments i.e.

\[
o = \frac{\partial}{\partial x^{(i)}} \int v^{(i)} \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \right\} di^{(i)} d\varphi^{(i)} d\psi^{(i)}
\]

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

\[
\frac{\partial}{\partial x^{(i)}} \int v^{(i)} F^{(i)}(x) di^{(i)} d\varphi^{(i)} d\psi^{(i)} \frac{\partial}{\partial x^{(i)}} \left\{ u^{(i)} \right\} = \left\{ \frac{\partial u^{(i)}}{\partial x^{(i)}} \right\} = \omega
\]

and all the properties of the distribution function obtained in section (4) can also be verified.

2.5. Equations for Three-Point Distribution Function \( F^{(1,2,3)} \)

It shall make use of equations (1) - (4) to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of \( u, h, \theta \) and \( c \) from the equations (1) - (4).

Differentiating equation (8) with respect to time and using equation (1) - (4), we get,
\[
\frac{\partial F^{(1,2,3)}}{\partial t} = \frac{\partial}{\partial t} \left\{ \delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \right. \\
\left. + \delta (c^{(2)} - \psi^{(2)}) \delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \right. \\
\left. + \delta (c^{(3)} - \psi^{(3)}) \delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta (h^{(1)} - g^{(1)}) \right. \\
\left. + \delta (c^{(2)} - \psi^{(2)}) \delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta (h^{(2)} - g^{(2)}) \right. \\
\left. + \delta (c^{(3)} - \psi^{(3)}) \delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \frac{\partial}{\partial t} \delta (h^{(1)} - g^{(1)}) \right. \\
\left. \right\} 
\]
Using equations (1) to (4), we get from the above equation

\begin{align*}
&+\left\{ -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})
\right.
\left.\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial h^{(3)}}{\partial t}\frac{\partial}{\partial c^{(3)}}\delta(\theta^{(3)}-\phi^{(3)})\right\}
\end{align*}

\begin{align*}
&+\left\{ -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})
\right.
\left.\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\frac{\partial}{\partial \psi^{(3)}}\delta(\theta^{(3)}-\phi^{(3)})\right\}
\end{align*}

\begin{align*}
&+\left\{ -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})
\right.
\left.\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\frac{\partial}{\partial \psi^{(3)}}\delta(\theta^{(3)}-\phi^{(3)})\right\}
\end{align*}

Using equations (1) to (4), we get from the above equation

\begin{align*}
\frac{\partial F^{(1,2,3)}}{\partial t} &= -\left\{ -\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})
\right.
\left.\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\right\}
\end{align*}
\[
\{ - \frac{\partial}{\partial x_1^2} (h_1 u_1' u_2'' - u_1'' h_2') + \lambda \nabla^2 h_2' \} \times \frac{\partial}{\partial y} \delta (k_1 - g_2') \\
+ \langle - \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (c_1'' - y_1'') \delta (u_2'' - v_2'') \delta (h_2' - g_2') \delta (\theta_1'' - \phi_1'') \\
\delta (c_2'' - y_2'') \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_2' - \phi_2'') \delta (c_2'' - y_2'') \delta (\theta_2' - \phi_2'') \delta (c_2'' - y_2'') \delta (\theta_2' - \phi_2'') \\
\delta (\theta_2' - \phi_2'') \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \\
\times \{ - u_1'' \frac{\partial \theta_1''}{\partial y} + \nabla^2 \theta_1'' \} \times \frac{\partial}{\partial y} \delta (\theta_1'' - \phi_1'') \\
+ \langle - \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_1'' - \phi_1'') \delta (c_1'' - y_1'') \delta (u_2'' - v_2'') \delta (h_2' - g_2') \delta (\theta_2' - \phi_2'') \\
\delta (\theta_2' - \phi_2'') \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \\
\{ - u_2'' \frac{\partial \theta_2''}{\partial y} + \nabla^2 \theta_2'' \} \times \frac{\partial}{\partial y} \delta (\theta_2'' - \phi_2'') \\
+ \langle - \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_1'' - \phi_1'') \delta (c_1'' - y_1'') \delta (u_2'' - v_2'') \delta (h_2' - g_2') \delta (\theta_2' - \phi_2'') \\
\delta (\theta_2' - \phi_2'') \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \\
\times \{ - u_3'' \frac{\partial \theta_3''}{\partial y} + \nabla^2 \theta_3'' \} \times \frac{\partial}{\partial y} \delta (\theta_3'' - \phi_3'') \\
+ \langle - \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_1'' - \phi_1'') \delta (c_1'' - y_1'') \delta (u_2'' - v_2'') \delta (h_2' - g_2') \delta (\theta_2' - \phi_2'') \\
\delta (\theta_2' - \phi_2'') \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \\
\times \{ - u_3'' \frac{\partial \theta_3''}{\partial y} + \nabla^2 \theta_3'' \} \times \frac{\partial}{\partial y} \delta (\theta_3'' - \phi_3'') \\
+ \langle - \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_1'' - \phi_1'') \delta (c_1'' - y_1'') \delta (u_2'' - v_2'') \delta (h_2' - g_2') \delta (\theta_2' - \phi_2'') \\
\delta (\theta_2' - \phi_2'') \delta (u_1'' - v_1'') \delta (h_1 - g_2') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \delta (c_2'' - y_2'') \delta (\theta_3' - \phi_3') \\
\times \{ - u_3'' \frac{\partial \theta_3''}{\partial y} + \nabla^2 \theta_3'' \} \times \frac{\partial}{\partial y} \delta (\theta_3'' - \phi_3'') \rangle.
\]
\[
\left\{ -\frac{\partial}{\partial x_1^3}(h_1^3 u_1^3 - u_1^3 h_1^3) + A \nabla^2 h_1^3 \right\} \frac{\partial}{\partial x_2^3} \delta(h_1^3 - g_1^3) \\
+ \left\{ -\frac{\partial}{\partial x_3^3}(u_1^3 - v_1^3) \delta(h_1^3 - g_1^3) \delta(\theta_1^3 - \phi_1^3) \delta(c_1^3 - \psi_1^3) \delta(u_2^3 - v_2^3) \delta(h_2^3 - g_2^3) \delta(\theta_2^3 - \phi_2^3) \delta(c_2^3 - \psi_2^3) \\
\frac{\partial}{\partial x_2^3} \delta(h_3^3 - g_3^3) \right\} \times
\left\{ -\frac{\partial \theta_1^3}{\partial x_3^3} + \frac{\partial \phi_1^3}{\partial x_3^3} \delta(\theta_1^3 - \phi_1^3) \right\} \\
+ \left\{ -\frac{\partial}{\partial x_3^3}(u_1^3 - v_1^3) \delta(h_1^3 - g_1^3) \delta(\theta_1^3 - \phi_1^3) \delta(c_1^3 - \psi_1^3) \delta(u_2^3 - v_2^3) \delta(h_2^3 - g_2^3) \delta(\theta_2^3 - \phi_2^3) \delta(c_2^3 - \psi_2^3) \\
\frac{\partial}{\partial x_2^3} \delta(h_3^3 - g_3^3) \right\} \times
\left\{ -\frac{\partial \theta_2^3}{\partial x_3^3} + \frac{\partial \phi_2^3}{\partial x_3^3} \delta(\theta_2^3 - \phi_2^3) \right\} \\
+ \left\{ -\frac{\partial}{\partial x_3^3}(u_1^3 - v_1^3) \delta(h_1^3 - g_1^3) \delta(\theta_1^3 - \phi_1^3) \delta(c_1^3 - \psi_1^3) \delta(u_2^3 - v_2^3) \delta(h_2^3 - g_2^3) \delta(\theta_2^3 - \phi_2^3) \delta(c_2^3 - \psi_2^3) \\
\frac{\partial}{\partial x_2^3} \delta(h_3^3 - g_3^3) \right\} \times
\left\{ -\frac{\partial \theta_3^3}{\partial x_3^3} + \frac{\partial \phi_3^3}{\partial x_3^3} \delta(\theta_3^3 - \phi_3^3) \right\} \\
\right\}
\]
\begin{align*}
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times\partial_{\varphi^{(1)}}\delta(\theta^{(1)}-\varphi^{(1)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times u^{(1)}\frac{\partial}{\partial u^{(1)}}\delta(u^{(1)}-\nu^{(1)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times DU^{(1)}\frac{\partial}{\partial u^{(1)}}\delta(c^{(1)}-\nu^{(1)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\nu^{(1)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\nu^{(1)})\times RC^{(1)}\frac{\partial}{\partial u^{(1)}}\delta(c^{(1)}-\nu^{(1)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times \frac{\partial u^{(2)}u^{(2)}}{\partial u^{(2)}}\frac{\partial}{\partial u^{(2)}}\delta(u^{(2)}-\nu^{(2)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times \frac{\partial h^{(2)}h^{(2)}}{\partial u^{(2)}}\frac{\partial h^{(2)}}{\partial u^{(2)}}\delta(u^{(2)}-\nu^{(2)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times \frac{\partial}{\partial u^{(1)}}\left\{ \frac{\partial u^{(2)}\partial u^{(2)}\partial h^{(2)}\partial h^{(2)}}{\partial u^{(2)}\partial u^{(2)}\partial u^{(2)}\partial u^{(2)}} \right\} \\
&\times \frac{\partial h^{(2)}}{\partial u^{(2)}}\delta(u^{(2)}-\nu^{(2)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times f(u^{(2)}-\nu^{(2)})\delta(u^{(2)}-\nu^{(2)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times \frac{\partial u^{(2)}u^{(2)}}{\partial h^{(1)}}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times \frac{\partial u^{(2)}h^{(2)}}{\partial h^{(1)}}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\} \\
&+\{(\delta u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-\nu^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times 2\partial h^{(2)}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\} \\
&\times \delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times 2\partial h^{(2)}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\} \\
&\times \delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times 2\partial h^{(2)}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\} \\
&\times \delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times 2\partial h^{(2)}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\} \\
&\times \delta(u^{(1)}-\nu^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\times 2\partial h^{(2)}\frac{\partial}{\partial h^{(1)}}\delta(h^{(1)}-g^{(1)})\}
\[
+ \langle \delta (u_i - v_i) \delta (h_i - g_i) \delta (\theta_i - \phi_i) \delta (c_i - \psi_i) \delta (u_j - v_j) \rangle \delta (h_j - g_j) \delta (c_j - \psi_j) \delta (u_k - v_k) \delta (h_k - g_k) \delta (c_k - \psi_k) \\
\delta (u_l - v_l) \delta (h_l - g_l) \delta (\theta_l - \phi_l) \delta (c_l - \psi_l) \delta (u_m - v_m) \delta (h_m - g_m) \delta (c_m - \psi_m) \times u_i^2 \frac{\partial \theta_i}{\partial x_{\beta}} \frac{\partial}{\partial \theta_i} \delta (\theta_i - \phi_i) \rangle \}
\]

\[
+ \langle -\delta (u_i - v_i) \delta (h_i - g_i) \delta (\theta_i - \phi_i) \delta (c_i - \psi_i) \rangle \delta (u_j - v_j) \delta (h_j - g_j) \delta (c_j - \psi_j) \delta (u_k - v_k) \delta (h_k - g_k) \delta (c_k - \psi_k) \delta (u_l - v_l) \delta (h_l - g_l) \delta (\theta_l - \phi_l) \rangle \}
\]

\[
+ \langle \delta (u_i - v_i) \delta (h_i - g_i) \delta (\theta_i - \phi_i) \delta (c_i - \psi_i) \delta (u_j - v_j) \delta (h_j - g_j) \delta (c_j - \psi_j) \times u_i^2 \frac{\partial \theta_i}{\partial x_{\beta}} \frac{\partial}{\partial \theta_i} \delta (\theta_i - \phi_i) \rangle \}
\]
Various terms in the above equation can be simplified as that they may be expressed in terms of one-, two-, three- and four-point distribution functions.

The 1st term in the above equation is simplified as follows:

\[
\begin{align*}
\delta(h^{(i)} - g^{(i)}) &\delta(\theta^{(i)} - \phi^{(i)}) \delta(c^{(i)} - \psi^{(i)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
\delta(u^{(3)} - v^{(3)}) &\delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u^{(3)}_\beta}{\partial x^{(3)}_\beta} \frac{\partial \psi^{(3)}}{\partial \psi^{(3)}} \delta(\psi^{(3)} - \phi^{(3)}) \\
\end{align*}
\]

Similarly, sixth, ninth and eleventh terms of right hand-side of equation (70) can be simplified as follows:

\[
\begin{align*}
\delta(h^{(i)} - g^{(i)}) &\delta(\theta^{(i)} - \phi^{(i)}) \delta(c^{(i)} - \psi^{(i)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
\delta(u^{(3)} - v^{(3)}) &\delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times \frac{\partial u^{(3)}_\beta}{\partial x^{(3)}_\beta} \frac{\partial \psi^{(3)}}{\partial \psi^{(3)}} \delta(\psi^{(3)} - \phi^{(3)}) \\
\end{align*}
\]

nineth term,
\[
\left\{ \begin{align*}
&\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\delta(u^{(2)}-u^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times u^{(3)}_{\beta}\frac{\partial}{\partial x^{(3)}_{\beta}}\delta(\theta^{(3)}-\varphi^{(3)})
\end{align*}\}
\]

Adding these equations from (71) to (74), we get

\[
\left\{ \begin{align*}
&\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times u^{(3)}_{\beta}\frac{\partial}{\partial x^{(3)}_{\beta}}\delta(\theta^{(3)}-\varphi^{(3)})
\end{align*}\}
\]

and eleventh term

\[
\left\{ \begin{align*}
&\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times u^{(3)}_{\beta}\frac{\partial}{\partial x^{(3)}_{\beta}}\delta(\theta^{(3)}-\varphi^{(3)})
\end{align*}\}
\]

Adding these equations from (71) to (74), we get

\[
\left\{ \begin{align*}
&\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times u^{(3)}_{\beta}\frac{\partial}{\partial x^{(3)}_{\beta}}\delta(\theta^{(3)}-\varphi^{(3)})
\end{align*}\}
\]

Similarly, 14th, 19th, 22nd and 24th terms of right hand-side of equation (17) can be simplified as follows;

\[
\left\{ \begin{align*}
&\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\varphi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\varphi^{(2)})\delta(c^{(2)}-\psi^{(2)})
&\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\varphi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times u^{(3)}_{\beta}\frac{\partial}{\partial x^{(3)}_{\beta}}\delta(\theta^{(3)}-\varphi^{(3)})
\end{align*}\}
\]
\[ \delta(x^{(2)} - \psi^{(2)}) \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(u^{(2)} - \nu^{(2)}) \] (23)

19th term,

\[ \{ \delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(h^{(2)} - g^{(2)}) \} \]

\[ = \{ -\delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(h^{(2)} - g^{(2)}) \} \] (24)

22nd term,

\[ \{ \delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(h^{(2)} - g^{(2)}) \} \]

\[ = \{ -\delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(h^{(2)} - g^{(2)}) \} \] (25)

And 24th term,

\[ \{ \delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(c^{(2)} - \psi^{(2)}) \} \]

\[ = \{ -\delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(c^{(2)} - \psi^{(2)}) \} \] (26)

Adding equations (76) to (79), we get

\[ - \frac{\partial}{\partial x^{(2)}_{\beta}} \{ u^{(2)}_{\beta} \} \{ -\delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \} \]

\[ \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(h^{(2)} - g^{(2)}) \} \]

\[ = -v^{(2)}_{\beta} \frac{\partial F^{(1,2,3)}}{\partial x^{(2)}_{\beta}} \] (27)

Similarly, 27th, 32nd, 35th and 37th terms of right hand-side of equation (70) can be simplified as follows;

\[ \{ -\delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - \nu^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \} \]

\[ \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(2)}_{\beta} \frac{\partial}{\partial x^{(2)}_{\beta}} \delta(h^{(2)} - g^{(2)}) \} \]

\[ = \{ -\delta(u^{(1)} - \nu^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(3)} - \nu^{(3)}) \delta(h^{(2)} - g^{(2)}) \} \]
\[ \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_{j} \frac{\partial}{\partial x_{j}} \delta(u^{(3)} - v^{(3)}) \ \} \]  

(28)

32nd term,

\[ \{ \delta(u^{(1)} - v^{(1)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \]

\[ \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_j \frac{\partial}{\partial x_{j}} \delta(h^{(3)} - g^{(3)}) \ \} \]

(29)

35th term,

\[ \{ \delta(u^{(1)} - v^{(1)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \]

\[ \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_j \frac{\partial}{\partial x_{j}} \delta(h^{(3)} - g^{(3)}) \ \} \]

(30)

and 37th term,

\[ \{ \delta(u^{(1)} - v^{(1)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \]

\[ \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_j \frac{\partial}{\partial x_{j}} \delta(h^{(3)} - g^{(3)}) \ \} \]

(31)

Adding equations (81) to (84), we get

\[ - \frac{\partial}{\partial x_{p}} \left\{ u^{(3)}_j - \delta(u^{(1)} - v^{(1)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_j \frac{\partial}{\partial x_{j}} \delta(u^{(3)} - v^{(3)}) \right\} = - \delta^{(3)} \frac{\partial F_{1}^{(1,2,3)}}{\partial x_{p}} \]

(32)

Similarly, 2nd, 7th, 15th, 20th, 28th and 33rd terms of right hand-side of equation (70) can be simplified as follows;

\[ \{ - \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \]

\[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_j \frac{\partial}{\partial x_{j}} \delta(u^{(1)} - v^{(1)}) \right\} = - \delta^{(3)} \frac{\partial g_{a}^{(1)}}{\partial v_{a}^{(1)}} \frac{\partial F_{3}^{(1,2,3)}}{\partial x_{p}} \]

(33)

7th term,

\[ \{ - \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \]

\[ \delta(u^{(1)} - v^{(1)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \times u^{(3)}_j \frac{\partial}{\partial x_{j}} \delta(h^{(1)} - g^{(1)}) \right\} = - \delta^{(3)} \frac{\partial g_{a}^{(1)}}{\partial v_{a}^{(1)}} \frac{\partial F_{3}^{(1,2,3)}}{\partial x_{p}} \]

(34)
\begin{align*}
\text{15}^{\text{th}} \text{ term,} & \\
& \left\{- \delta \left( u^{(1)} - v^{(1)} \right) \delta \left( h^{(1)} - g^{(1)} \right) \delta \left( \theta^{(1)} - \phi^{(1)} \right) \delta \left( c^{(1)} - \psi^{(1)} \right) \delta \left( h^{(2)} - g^{(2)} \right) \delta \left( \theta^{(2)} - \phi^{(2)} \right) \delta \left( c^{(2)} - \psi^{(2)} \right) \right\} \times \frac{\partial (u_{\alpha}^{(3)} h_{\beta}^{(3)})}{\partial x_{\gamma}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta \left( u^{(2)} - v^{(2)} \right) = - \alpha \beta \frac{\partial u_{\alpha}^{(3)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_{1}^{(3,2,3)}}{\partial x_{\beta}^{(2)}} \tag{35} \\
\text{20}^{\text{th}} \text{ term,} & \\
& \left\{- \delta \left( u^{(1)} - v^{(1)} \right) \delta \left( h^{(1)} - g^{(1)} \right) \delta \left( \theta^{(1)} - \phi^{(1)} \right) \delta \left( c^{(1)} - \psi^{(1)} \right) \delta \left( u^{(2)} - v^{(2)} \right) \delta \left( \theta^{(2)} - \phi^{(2)} \right) \delta \left( c^{(2)} - \psi^{(2)} \right) \right\} \times \frac{\partial (u_{\alpha}^{(3)} h_{\beta}^{(3)})}{\partial x_{\gamma}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta \left( h^{(2)} - g^{(2)} \right) = - \alpha \beta \frac{\partial u_{\alpha}^{(3)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_{1}^{(3,2,3)}}{\partial x_{\beta}^{(2)}} \tag{36} \\
\text{28}^{\text{th}} \text{ term,} & \\
& \left\{- \delta \left( u^{(1)} - v^{(1)} \right) \delta \left( h^{(1)} - g^{(1)} \right) \delta \left( \theta^{(1)} - \phi^{(1)} \right) \delta \left( c^{(1)} - \psi^{(1)} \right) \delta \left( u^{(2)} - v^{(2)} \right) \delta \left( h^{(2)} - g^{(2)} \right) \delta \left( \theta^{(2)} - \phi^{(2)} \right) \right\} \times \frac{\partial (u_{\alpha}^{(3)} h_{\beta}^{(3)})}{\partial x_{\gamma}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta \left( u^{(3)} - g^{(3)} \right) = - \alpha \beta \frac{\partial u_{\alpha}^{(3)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_{1}^{(3,2,3)}}{\partial x_{\beta}^{(2)}} \tag{37} \\
\text{and 33}^{\text{rd}} \text{ term,} & \\
& \left\{- \delta \left( u^{(1)} - v^{(1)} \right) \delta \left( h^{(1)} - g^{(1)} \right) \delta \left( \theta^{(1)} - \phi^{(1)} \right) \delta \left( c^{(1)} - \psi^{(1)} \right) \delta \left( u^{(2)} - v^{(2)} \right) \delta \left( h^{(2)} - g^{(2)} \right) \delta \left( \theta^{(2)} - \phi^{(2)} \right) \right\} \times \frac{\partial (u_{\alpha}^{(3)} h_{\beta}^{(3)})}{\partial x_{\gamma}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta \left( u^{(3)} - g^{(3)} \right) = - \alpha \beta \frac{\partial u_{\alpha}^{(3)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_{1}^{(3,2,3)}}{\partial x_{\beta}^{(2)}} \tag{38} \\
\text{Fourth term can be reduced as} & \\
& \left\{- \delta \left( h^{(1)} - g^{(1)} \right) \delta \left( \theta^{(1)} - \phi^{(1)} \right) \delta \left( c^{(1)} - \psi^{(1)} \right) \delta \left( u^{(2)} - v^{(2)} \right) \delta \left( h^{(2)} - g^{(2)} \right) \delta \left( \theta^{(2)} - \phi^{(2)} \right) \delta \left( c^{(2)} - \psi^{(2)} \right) \right\} \times \frac{\partial (u_{\alpha}^{(3)} h_{\beta}^{(3)})}{\partial x_{\gamma}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta \left( u^{(3)} - v^{(3)} \right) = - \alpha \beta \frac{\partial u_{\alpha}^{(3)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_{1}^{(3,2,3)}}{\partial x_{\beta}^{(2)}} \tag{39}
\end{align*}
Similarly, $8^{th}, 10^{th}, 12^{th}, 17^{th}, 21^{st}, 23^{rd}, 25^{th}, 30^{th}, 34^{th}, 36^{th}$ and $38^{th}$ terms of right hand-side of equation (70) can be simplified as follows;

\begin{align*}
&\left\{ -\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)}) \\
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times \lambda \nabla^2 h^{(0)}_{\alpha} \frac{\partial}{\partial g^{(0)}_{\alpha}} \delta(h^{(4)}-g^{(4)}) \right\} \\
&= -\lambda \frac{\partial}{\partial g^{(0)}_{\alpha}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int g^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)} \\
&=-\lambda \frac{\partial}{\partial g^{(0)}_{\alpha}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int g^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)} \\
&=-\lambda \frac{\partial}{\partial g^{(0)}_{\alpha}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int g^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)}
\end{align*}

(40)

10\textsuperscript{th} term,

\begin{align*}
&\left\{ -\delta(u^{(1)}-v^{(1)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)}) \\
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times \lambda \nabla^2 h^{(0)}_{\alpha} \frac{\partial}{\partial g^{(0)}_{\alpha}} \delta(h^{(4)}-g^{(4)}) \right\} \\
&= -\gamma \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)} \\
&=-\gamma \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)}
\end{align*}

(41)

12\textsuperscript{th} term,

\begin{align*}
&\left\{ -\delta(u^{(1)}-v^{(1)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)}) \\
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times \lambda \nabla^2 h^{(0)}_{\alpha} \frac{\partial}{\partial g^{(0)}_{\alpha}} \delta(h^{(4)}-g^{(4)}) \right\} \\
&= -D \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)} \\
&=-D \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)}
\end{align*}

(42)

17\textsuperscript{th} term,

\begin{align*}
&\left\{ -\delta(u^{(1)}-v^{(1)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)}) \\
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times \lambda \nabla^2 h^{(0)}_{\alpha} \frac{\partial}{\partial g^{(0)}_{\alpha}} \delta(h^{(4)}-g^{(4)}) \right\} \\
&= -\nu \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)} \\
&=-\nu \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)}
\end{align*}

(43)

21\textsuperscript{st} term,

\begin{align*}
&\left\{ -\delta(u^{(1)}-v^{(1)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)}) \\
&\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times \lambda \nabla^2 h^{(0)}_{\alpha} \frac{\partial}{\partial g^{(0)}_{\alpha}} \delta(h^{(4)}-g^{(4)}) \right\} \\
&= -\Lambda \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)} \\
&=-\Lambda \frac{\partial}{\partial \psi^{(0)}} \lim_{x(4) \to x(1)} \frac{\partial^2}{\partial x^{(4)}_\alpha \partial x^{(4)}_\beta} \int \psi^{(4)}_{\alpha} F^{(1,2,3,4)}_{4} dv^{(4)} dg^{(4)} d\psi^{(4)} d\psi^{(4)}
\end{align*}

(44)

23\textsuperscript{rd} term,
\begin{align*}
\langle -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) 
&\delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times \mathcal{V}^2 \theta^{(3)} \frac{\partial}{\partial \theta^{(3)}} \delta (\theta^{(2)} - \phi^{(2)}) \rangle \\
&= -\gamma \frac{\partial}{\partial \phi^{(2)}} \frac{\partial^2}{\partial \chi^2} \left( \frac{1}{\chi(4)} - \frac{1}{\chi(2)} \right) \frac{\partial^2}{\partial \chi^2} \int \psi^{(4)} F_4^{(1,2,3,4)} d\psi^{(4)} d\theta^{(4)} d\phi^{(4)} d\psi^{(4)}
\end{align*}
\begin{equation}
(45)
\end{equation}

25th term,
\begin{align*}
\langle -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) 
&\delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times \mathcal{V}^2 \theta^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta (\psi^{(2)} - \psi^{(2)}) \rangle \\
&= -D \frac{\partial}{\partial \psi^{(2)}} \frac{\partial^2}{\partial \chi^2} \left( \frac{1}{\chi(4)} - \frac{1}{\chi(2)} \right) \frac{\partial^2}{\partial \chi^2} \int \psi^{(4)} F_4^{(1,2,3,4)} d\psi^{(4)} d\theta^{(4)} d\phi^{(4)} d\psi^{(4)}
\end{align*}
\begin{equation}
(46)
\end{equation}

30th term,
\begin{align*}
\langle -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) 
&\delta (c^{(2)} - \psi^{(2)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times \mathcal{V}^2 u^{(3)} \frac{\partial}{\partial u^{(3)}} \delta (u^{(3)} - \nu^{(3)}) \rangle \\
&= -\nu \frac{\partial}{\partial \psi^{(3)}} \frac{\partial^2}{\partial \chi^2} \left( \frac{1}{\chi(4)} - \frac{1}{\chi(3)} \right) \frac{\partial^2}{\partial \chi^2} \int \psi^{(4)} F_4^{(1,2,3,4)} d\psi^{(4)} d\theta^{(4)} d\phi^{(4)} d\psi^{(4)}
\end{align*}
\begin{equation}
(47)
\end{equation}

34th term,
\begin{align*}
\langle -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) 
&\delta (c^{(2)} - \psi^{(2)}) \delta (u^{(3)} - v^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times \mathcal{V}^2 h^{(3)} \frac{\partial}{\partial h^{(3)}} \delta (h^{(3)} - \gamma^{(3)}) \rangle \\
&= -\lambda \frac{\partial}{\partial \psi^{(3)}} \frac{\partial^2}{\partial \chi^2} \left( \frac{1}{\chi(4)} - \frac{1}{\chi(3)} \right) \frac{\partial^2}{\partial \chi^2} \int \psi^{(4)} F_4^{(1,2,3,4)} d\psi^{(4)} d\theta^{(4)} d\phi^{(4)} d\psi^{(4)}
\end{align*}
\begin{equation}
(48)
\end{equation}

36th term,
\begin{align*}
\langle -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) 
&\delta (c^{(2)} - \psi^{(2)}) \delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times \mathcal{V}^2 \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta (\psi^{(3)} - \theta^{(3)}) \rangle \\
&= -\gamma \frac{\partial}{\partial \phi^{(3)}} \frac{\partial^2}{\partial \chi^2} \left( \frac{1}{\chi(4)} - \frac{1}{\chi(3)} \right) \frac{\partial^2}{\partial \chi^2} \int \psi^{(4)} F_4^{(1,2,3,4)} d\psi^{(4)} d\theta^{(4)} d\phi^{(4)} d\psi^{(4)}
\end{align*}
\begin{equation}
(49)
\end{equation}

38th term,
\begin{align*}
\langle -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) 
&\delta (c^{(2)} - \psi^{(2)}) \delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times \mathcal{V}^2 \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta (\psi^{(3)} - \theta^{(3)}) \rangle
\end{align*}
\[ D \frac{\partial}{\partial \psi^{(4)}} \lim_{x^{(4)} \to x^{(3)}} \frac{\partial^2}{\partial x^{(4)} \partial \psi^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\varphi^{(4)} d\psi^{(4)} \]  

(50)

We reduce the third term of right hand side of equation (17),

\[ + \left\{ \delta (\theta^{(3)} - \varphi^{(3)}) \delta (\theta^{(3)} - \varphi^{(3)}) \delta (\theta^{(3)} - \varphi^{(3)}) \delta (\theta^{(3)} - \varphi^{(3)}) \delta (\theta^{(3)} - \varphi^{(3)}) \delta (\theta^{(3)} - \varphi^{(3)}) \right\} \]

Similarly, 16th term,

\[ = \frac{\partial}{\partial v^{(2)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x^{(2)}} \left( \frac{1}{\sqrt{v(x^{(1)} - x^{(2)})}} \right) \left( \frac{\partial v^{(4)}}{\partial x^{(4)}} \frac{\partial v^{(4)}}{\partial x^{(4)}} \frac{\partial v^{(4)}}{\partial x^{(4)}} \frac{\partial v^{(4)}}{\partial x^{(4)}} \right) \int \frac{dx^{(2)}}{\sqrt{v(x^{(1)} - x^{(2)})}} \delta (u^{(2)} - v^{(2)}) \right] \]

(51)

Similarly, 29th term,

\[ \int \frac{\partial}{\partial x^{(2)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x^{(2)}} \left( \frac{1}{\sqrt{v(x^{(1)} - x^{(2)})}} \right) \left( \frac{\partial v^{(4)}}{\partial x^{(4)}} \frac{\partial v^{(4)}}{\partial x^{(4)}} \frac{\partial v^{(4)}}{\partial x^{(4)}} \frac{\partial v^{(4)}}{\partial x^{(4)}} \right) \int \frac{dx^{(2)}}{\sqrt{v(x^{(1)} - x^{(2)})}} \delta (u^{(2)} - v^{(2)}) \right] \]

(52)

Similarly, 5th term of right hand side of equation (17),

\[ \left\{ -\delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (h^{(3)} - g^{(3)}) \right\} \]

Similarly, 18th, 33rd and 34th terms of right hand side of equation (17),

(53)

(54)
18th term,
\[
\begin{align*}
&\left( -\delta (u^{(1)} - v^{(1)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \right) \\
&\delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times f \left( u^{(3)}_a - v^{(3)}_a \right) \frac{\partial}{\partial \psi^{(3)}_a} \delta (u^{(3)} - v^{(3)}) \\
&= -f \left( u^{(3)}_a - v^{(3)}_a \right) \frac{\partial}{\partial \psi^{(3)}_a} F_{3}^{(1,2,3)} \quad (55)
\end{align*}
\]

31st term,
\[
\begin{align*}
&\left( -\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \right) \\
&\delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \delta (c^{(2)} - \psi^{(2)}) \\
&\delta (c^{(1)} - \psi^{(1)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \times f \left( u^{(3)}_a - v^{(3)}_a \right) \frac{\partial}{\partial \psi^{(3)}_a} \delta (u^{(3)} - v^{(3)}) \\
&= -f \left( u^{(3)}_a - v^{(3)}_a \right) \frac{\partial}{\partial \psi^{(3)}_a} F_{3}^{(1,2,3)} \quad (56)
\end{align*}
\]

13th term of Equation (17)
\[
\begin{align*}
&\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \\
&\delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \delta (c^{(2)} - \psi^{(2)}) \\
&\delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \delta (c^{(3)} - \psi^{(3)}) \\
&\times R e^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta (u^{(3)} - v^{(3)}) \\
&= R \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}} F_{3}^{(1,2,3)} \quad (57)
\end{align*}
\]

26th term of Equation (17)
\[
\begin{align*}
&\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \\
&\delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \\
&\delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \\
&\delta (c^{(3)} - \psi^{(3)}) \\
&\times R e^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta (u^{(3)} - v^{(3)}) \\
&= R \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}} F_{3}^{(1,2,3)} \quad (58)
\end{align*}
\]

39th term of Equation (17)
\[
\begin{align*}
&\delta (u^{(1)} - v^{(1)}) \delta (h^{(1)} - g^{(1)}) \delta (\theta^{(1)} - \phi^{(1)}) \delta (c^{(1)} - \psi^{(1)}) \\
&\delta (u^{(2)} - v^{(2)}) \delta (h^{(2)} - g^{(2)}) \delta (\theta^{(2)} - \phi^{(2)}) \\
&\delta (u^{(3)} - v^{(3)}) \delta (h^{(3)} - g^{(3)}) \delta (\theta^{(3)} - \phi^{(3)}) \\
&\delta (c^{(3)} - \psi^{(3)}) \\
&\times R e^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta (u^{(3)} - v^{(3)}) \\
&= R \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}} F_{3}^{(1,2,3)} \quad (59)
\end{align*}
\]

3. Results

Substituting the results (18) – (59) in equation (20) we get the transport equation for three-point distribution function $F_{3}^{(1,2,3)}(v, g, \phi, \psi)$ in MHD turbulent flow for velocity, magnetic temperature and concentration in presence of dust particles undergoing a first order reaction as
Continuing this way, we can derive the equations for evolution of \( F_n \) (\( n \) is an integer) but the system of equations so obtained is not closed. Certain approximations will be required thus obtained.

4. Discussions

If \( R=0 \), i.e, the reaction rate is absent, the transport equation for three-point distribution function in MHD turbulent flow (60) becomes

\[
\frac{\partial F_{3}^{(1,2,3)}}{\partial t} + \left( v_{x}^{(1)} \frac{\partial}{\partial x_{x}^{(1)}} + v_{x}^{(2)} \frac{\partial}{\partial x_{x}^{(2)}} + v_{x}^{(3)} \frac{\partial}{\partial x_{x}^{(3)}} \right) F_{3}^{(1,2,3)} + \left[ g_{\beta}^{(1)} \left( \frac{\partial g_{\alpha}^{(1)}}{\partial x_{x}^{(1)}} + \frac{\partial g_{\alpha}^{(1)}}{\partial x_{x}^{(2)}} \right) \frac{\partial}{\partial x_{x}^{(1)}} \right] F_{3}^{(1,2,3)} + \left[ g_{\beta}^{(2)} \left( \frac{\partial g_{\alpha}^{(2)}}{\partial x_{x}^{(2)}} + \frac{\partial g_{\alpha}^{(2)}}{\partial x_{x}^{(3)}} \right) \frac{\partial}{\partial x_{x}^{(2)}} \right] F_{3}^{(1,2,3)} + \left[ g_{\beta}^{(3)} \left( \frac{\partial g_{\alpha}^{(3)}}{\partial x_{x}^{(3)}} + \frac{\partial g_{\alpha}^{(3)}}{\partial x_{x}^{(1)}} \right) \frac{\partial}{\partial x_{x}^{(3)}} \right] F_{3}^{(1,2,3)} + \left[ f \left( u_{x}^{(1)} - v_{a}^{(1)} \right) \frac{\partial}{\partial v_{x}^{(1)}} + f \left( u_{x}^{(2)} - v_{a}^{(2)} \right) \frac{\partial}{\partial v_{x}^{(2)}} + f \left( u_{x}^{(3)} - v_{a}^{(3)} \right) \frac{\partial}{\partial v_{x}^{(3)}} \right] F_{3}^{(1,2,3)} = 0
\]

which was obtained earlier by M. N. Islam et al (2014).

If the fluid is clean then \( f=0 \), the transport equation for three-point distribution function in MHD turbulent flow (60) becomes

\[
\frac{\partial F_{3}^{(1,2,3)}}{\partial t} + \left( v_{x}^{(1)} \frac{\partial}{\partial x_{x}^{(1)}} + v_{x}^{(2)} \frac{\partial}{\partial x_{x}^{(2)}} + v_{x}^{(3)} \frac{\partial}{\partial x_{x}^{(3)}} \right) F_{3}^{(1,2,3)} + \left[ g_{\beta}^{(1)} \left( \frac{\partial g_{\alpha}^{(1)}}{\partial x_{x}^{(1)}} + \frac{\partial g_{\alpha}^{(1)}}{\partial x_{x}^{(2)}} \right) \frac{\partial}{\partial x_{x}^{(1)}} \right] F_{3}^{(1,2,3)} + \left[ g_{\beta}^{(2)} \left( \frac{\partial g_{\alpha}^{(2)}}{\partial x_{x}^{(2)}} + \frac{\partial g_{\alpha}^{(2)}}{\partial x_{x}^{(3)}} \right) \frac{\partial}{\partial x_{x}^{(2)}} \right] F_{3}^{(1,2,3)} + \left[ g_{\beta}^{(3)} \left( \frac{\partial g_{\alpha}^{(3)}}{\partial x_{x}^{(3)}} + \frac{\partial g_{\alpha}^{(3)}}{\partial x_{x}^{(1)}} \right) \frac{\partial}{\partial x_{x}^{(3)}} \right] F_{3}^{(1,2,3)} + \left[ f \left( u_{x}^{(1)} - v_{a}^{(1)} \right) \frac{\partial}{\partial v_{x}^{(1)}} + f \left( u_{x}^{(2)} - v_{a}^{(2)} \right) \frac{\partial}{\partial v_{x}^{(2)}} + f \left( u_{x}^{(3)} - v_{a}^{(3)} \right) \frac{\partial}{\partial v_{x}^{(3)}} \right] F_{3}^{(1,2,3)} = 0.
\]
It was obtained earlier by Azad et al (2014a) if we drop the viscous, magnetic and thermal diffusive and concentration terms from the three point evolution equation (60), we have

\[
\begin{align*}
\frac{\partial F_3^{(1,2,3)}}{\partial t} + & \left( \psi_1^{(1)} \frac{\partial}{\partial \psi_1^{(1)}} + \psi_2^{(1)} \frac{\partial}{\partial \psi_2^{(1)}} + \psi_3^{(1)} \frac{\partial}{\partial \psi_3^{(1)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \frac{\partial}{\partial \psi_1^{(3)}} + g_\beta^{(2)} \frac{\partial}{\partial \psi_2^{(3)}} + g_\beta^{(3)} \frac{\partial}{\partial \psi_3^{(3)}} \right] F_3^{(1,2,3)} \\
+ & \left( \psi_1^{(2)} \frac{\partial}{\partial \psi_1^{(2)}} + \psi_2^{(2)} \frac{\partial}{\partial \psi_2^{(2)}} + \psi_3^{(2)} \frac{\partial}{\partial \psi_3^{(2)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \frac{\partial}{\partial \psi_1^{(3)}} + g_\beta^{(2)} \frac{\partial}{\partial \psi_2^{(3)}} + g_\beta^{(3)} \frac{\partial}{\partial \psi_3^{(3)}} \right] F_3^{(1,2,3)} \\
+ & \left( \psi_1^{(3)} \frac{\partial}{\partial \psi_1^{(3)}} + \psi_2^{(3)} \frac{\partial}{\partial \psi_2^{(3)}} + \psi_3^{(3)} \frac{\partial}{\partial \psi_3^{(3)}} \right) F_3^{(1,2,3)} + \left[ g_\beta^{(1)} \frac{\partial}{\partial \psi_1^{(3)}} + g_\beta^{(2)} \frac{\partial}{\partial \psi_2^{(3)}} + g_\beta^{(3)} \frac{\partial}{\partial \psi_3^{(3)}} \right] F_3^{(1,2,3)} \\
= & 0
\end{align*}
\]

The existence of the terms
\[
(\frac{\partial g_\alpha^{(1)}}{\partial \psi_1^{(3)}} + \frac{\partial g_\alpha^{(3)}}{\partial \psi_1^{(3)}}), (\frac{\partial g_\alpha^{(2)}}{\partial \psi_2^{(3)}} + \frac{\partial g_\alpha^{(3)}}{\partial \psi_2^{(3)}}) \text{ and } (\frac{\partial g_\alpha^{(3)}}{\partial \psi_3^{(3)}} + \frac{\partial g_\alpha^{(3)}}{\partial \psi_3^{(3)}})
\]

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at point \(x^{(1)}\), \(x^{(2)}\) and \(x^{(3)}\).

We can exhibit an analogy of this equation with the 1st equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given by Lundgren (1969) as

\[
\frac{\partial \psi_1^{(1)}}{\partial t} + \frac{1}{m} \frac{\partial \psi_1^{(1)}}{\partial \psi_1^{(1)}} F_1^{(1)} = n \iint \frac{\partial \psi_{1,2}}{\partial x_{1,2}} \frac{\partial F_2^{(1,2)}}{\partial x_{1,2}} - \delta x_{1,2} \delta v_{1,2} \quad (64)
\]

where \(\psi_{1,2} = \psi_{1,2}^{(2)} - \psi_{1,2}^{(1)}\) is the intermolecular potential.

Acknowledgement

Authors are grateful to the Department of Applied Mathematics, University of Rajshahi, Bangladesh for giving all facilities and support to carry out this work.

References


Effect of Chemical Reaction on Statistical Theory of Dusty Fluid MHD Turbulent Flow for Certain Variables at Three-Point Distribution Functions


