

Empirical Bayes Test for Parameter of Inverse Exponential Distribution

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Abstract: The aim of this paper is to study the empirical Bayes test for the parameter of inverse exponential distribution. First, the Bayes test rule of one-sided test is derived in the case of independent and identically distributed random variables under weighted linear loss function. Then the empirical Bayes one-sided test rule is constructed by using the kernel-type density function and empirical distribution function. Finally, the asymptotically optimal property of the test function is obtained. It is shown that the convergence rates of the proposed empirical Bayes test rules can arbitrarily close to $O(n^{-1/2})$ under suitable conditions.

Keywords: Empirical Bayes Test, Asymptotic Optimality, Convergence Rates, Weighted Linear Loss Function, Inverse Exponential Distribution

1. Introduction

Since Robbins proposed empirical Bayes method, it has been extensively application many fields, such as reliability, lifetime prediction, medical research, ect. [1-6]. Empirical Bayes hypothesis test problem is the main content of empirical Bayes statistical inference, which attracts many scholars' attention. Qian and Wei [7] studied the two-sided empirical Bayes test of parameters for the scale exponential family under the weighed loss function. Chen et al.[8] discussed the two-sided empirical Bayes test problem for the parameter of continuous one-parameter exponential family with contaminated data (errors in variables) by a deconvolution kernel method. Wang et al.[9] studied the empirical Bayes test problem for two-parameter exponential distribution under Type-II censored samples. Lei and Qin [10] studied the empirical Bayes test problem in continuous one-parameter exponential families under dependent samples. More details about empirical Bayes test can be found in references [11-13].

In the life distribution, if the random variable X is distributed with exponential distribution, then X^{-1} is distributed with inverse exponential distribution. Assume random variable X distributed with the inverse Exponential distribution with the probability density function is

$$f(x|\theta) = \frac{\theta}{x^2} \exp\left(-\frac{\theta}{x}\right), x > 0 \quad (1)$$

Where $\theta > 0$ is the unknown shape parameter, $\Omega = \{x | x > 0\}$ is the sample space, and $\Theta = \left\{ \theta > 0 \mid \int_{\Omega} f(x|\theta) dx = 1 \right\}$ is the parameters space.

The inverse exponential distribution has many applications in life test and reliability study. Prakash [14] studied the properties of Bayes estimators of the parameter, reliability function and hazard rate of the inverse exponential model under symmetric and asymmetric loss functions. Rao [15] studied the reliability estimation of stress-strength model based on inverse exponential distribution. Singh et al. [16] proposed a two-stage group acceptance sampling plan for generalized inverse exponential distribution under truncated life test.

This paper will discuss the empirical Bayes test problem of the parameter of inverse exponential distribution under the weighted linear loss function. The organization of this paper is as follows: Section 2 will derive a Bayes decision rule for one-side hypothesis test problem. Then Section 3 will

construct a empirical Bayes decision rule based the kernel-type density function and empirical distribution function. Main results will given in Section 4 and Section 5.

2. Bayes Test

This section will discuss the hypothesis test problem:

$$H_0 : \theta \leq \theta_0 \leftrightarrow H_1 : \theta > \theta_0 \tag{2}$$

of inverse exponential distribution parameter under a weighted linear loss function. Here θ_0 is a given constant. For hypothesis test problem (2), let the weighted linear loss function have the following form:

$$L_0(\theta, d_0) = \frac{\theta - \theta_0}{\theta} I(\theta > \theta_0) \tag{3}$$

$$L_1(\theta, d_1) = \frac{\theta_0 - \theta}{\theta} I(\theta \leq \theta_0) \tag{4}$$

where $d = \{d_0, d_1\}$ is the action space, d_0 represents accepting H_0 , d_1 represents refusing H_0 . The weighted linear loss function has many advantages: it is invariant and make the expression of Bayes more concise and make the empirical Bayes test function is easy to construct.

Assume the prior distribution of parameters θ is $G(\theta)$, which is also unknown. Let the test function be

$$\delta(x) = P(\text{accept } H_0 | X = x) \tag{5}$$

Then, the risk function of $\delta(x)$ is

$$R(\delta(x), G(\theta)) = \int \int_{\Theta} [L_0(\theta, d_0) f(x|\theta) \delta(x) + L_1(\theta, d_1) f(x|\theta) (1 - \delta(x))] dx dG(\theta)$$

Then

$$R(\delta(x), G(\theta)) = \int_{\Omega} \beta(x) \delta(x) dx + C_G \tag{6}$$

Where $C_G = \int_{\Theta} L_1(\theta, d_1) dG(\theta)$, and

$$\beta(x) = \int_{\Theta} \frac{\theta - \theta_0}{\theta} f(x|\theta) dG(\theta) \tag{7}$$

Let the marginal probability density function of X is:

$$f_G(x) = \int_{\Theta} f(x|\theta) dG(\theta) \tag{8}$$

Therefore, by (5),

$$\begin{aligned} \beta(x) &= \int_{\Theta} f(x|\theta) dG(\theta) - \theta_0 \int_{\Theta} \frac{1}{\theta} f(x|\theta) dG(\theta) \\ &= f_G(x) - \theta_0 \int_{\Theta} \frac{1}{\theta} f(x|\theta) dG(\theta) \end{aligned}$$

Set

$$P_G(x) = \int_{\Theta} e^{-\frac{\theta}{x^2}} dG(\theta), \tag{9}$$

then

$$\beta(x) = f_G(x) - x^{-2} \theta_0 P_G(x) \tag{10}$$

By (4) we can know the Bayes test function is:

$$\delta_G(x) = \begin{cases} 1, & \beta(x) \leq 0 \\ 0, & \beta(x) > 0 \end{cases} \tag{11}$$

The Bayes risk function is:

$$R_G = \inf_{\delta} R(\delta(x), G(\theta)) = \int_{\Omega} \beta(x) \delta_G(x) dx + C_G \tag{12}$$

Knowing the fact that if the prior distribution $G(\theta)$ is known, $\delta(x)$ equals to $\delta_G(x)$, we can get R_G .

Remark 1. Unfortunately the $G(\theta)$ is unknown, then $\delta_G(x)$ is unknown too. Then the Bayes test function is no practical value, therefore, we need to introduce the empirical Bayes method, which requires construct the risk function can be arbitrary close to the empirical Bayes judgment function.

3. Empirical Bayes Test Function

This section we will construct the empirical Bayes test function according the following framework:

Suppose $X_1, X_2, \dots, X_n, X_{n+1}$ are the sequence of independent and identically distributed random variables, with the identical probability density function $f_G(x)$, X_1, X_2, \dots, X_n are the historical sample, X_{n+1} is the current sample. In this paper, we assume

(A1) $f_G(x) \in C_{s,\alpha}, x \in R^1$, where $C_{s,\alpha}$ represents probability density function of R^1 , with the existence of derivative s order, consecutive and absolute value does not exceed $\alpha, s \geq 2$ is a positive integer.

(A2) suppose $s \geq 2$ is an arbitrarily certain natural number, $K_r(x) (r = 0, 1, \dots, s-1)$ is a bounded function of Borel measurable function. When outside of the interval $(0, 1)$, their value are zero, and they satisfy the following conditions:

$$\frac{1}{t!} \int_0^1 y^t K_r(y) dy = \begin{cases} 1, & t = r \\ 0, & t \neq r, t = 0, 1, 2, \dots, s-1 \end{cases}$$

Define the Kernel density estimation of $f_G(x)$ as:

$$f_n(x) = \frac{1}{nb_n} \sum_{j=1}^n K_r\left(\frac{x - X_j}{b_n}\right) \tag{13}$$

Where $\{b_n\}$ is positive integer sequence, and $\lim_{n \rightarrow \infty} b_n = 0$,

By equations (1) and (9), we can get

$$\begin{aligned} E[I(X_i < x)] &= \int_{-\infty}^x \int_{\Theta} \frac{\theta}{x^2} e^{-\frac{\theta}{x}} dG(\theta) dx \\ &= \int_{\Theta} \left[\int_{-\infty}^x \frac{\theta}{x^2} e^{-\frac{\theta}{x}} dx \right] dG(\theta) \\ &= \int_{\Theta} \left[\int_0^x \frac{\theta}{x^2} e^{-\frac{\theta}{x}} dx \right] dG(\theta) \\ &= \int_{\Theta} [e^{-\frac{\theta}{x}}]_0^x dG(\theta) \\ &= P_G(x) \end{aligned}$$

Hence, the unbiased estimator of $P_G(x)$ can be constructed as

$$P_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i < x) \tag{14}$$

Therefore, the estimator of $\beta(x)$ is

$$\beta_n(x) = f_n(x) - x^{-2} \theta_0 P_n(x) \tag{15}$$

Finally, the empirical Bayes test function could be defined as follows:

$$\delta_n(x) = \begin{cases} 1, & \beta_n(x) \leq 0 \\ 0, & \beta_n(x) > 0 \end{cases} \tag{16}$$

Assume E_n represents the mean value of the joint distribution of X_1, X_2, \dots, X_n , hence the comprehensive risk of $\delta_n(x)$ is

$$R(\delta_n(x), G(\theta)) = \int_{\Omega} \beta(x) E_n[\delta_n(x)] dx + C_G \tag{17}$$

Definition 1. If $\lim_{n \rightarrow \infty} R(\delta_n(x), G(\theta)) = R_G$, then the random variable sequence $\{\delta_n(x)\}$ is called asymptotic optimality empirical Bayes test functions.

Definition 2. If $R(\delta_n(x), G(\theta)) - R_G = O(n^{-q}), q > 0$,

Then we say the convergence rate of the test function sequence $\{\delta_n(x)\}$ is $O(n^{-q})$.

By Definitions 1 and 2, we conclude that the optimal evaluation of the empirical Bayes test functions depends on the degree of its risk approximation Bayes risk.

Assuming c, c_1, c_2, \dots represent different constant, even in the same expression they also may take different values.

Lemma 1 [17] Suppose R_G and $R(\delta_n(x), G(\theta))$ definite in equation (11) and equation (16) separately, then

$$\begin{aligned} 0 &\leq R(\delta_n(x), G) - R_G \\ &\leq \int_{\Omega} |\beta(x)| P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|) dx \end{aligned} \tag{18}$$

Lemma 2 [18] Suppose $f_n(x)$ definite as (13), where

X_1, X_2, \dots are random variables sequence of independent and identically distributed, assume both (A1) and (A2) hold, for $\forall x \in \Omega$

(1) If $f_G(x)$ is continuous for x , when $\lim_{n \rightarrow \infty} b_n = 0$ and $\lim_{n \rightarrow \infty} n b_n = \infty$ hold, then

$$\lim_{n \rightarrow \infty} E_n |f_n(x) - f_G(x)|^2 = 0 \tag{19}$$

(2) If $f_G(x) \in C_{s,\alpha}$, let $b_n = n^{-\frac{1}{1+2s}}$, for $0 < \lambda \leq 1$, we have

$$E_n |f_n(x) - f_G(x)|^{2\lambda} \leq c \cdot n^{-\frac{\lambda s}{1+2s}} \tag{20}$$

Lemma 3 Suppose $P_G(x)$ and $P_n(x)$ definite in equation (9) and equation (14) separately, X_1, X_2, \dots are random variables sequence of independent and identically distributed, for $0 < \lambda \leq 1$, we have

$$E_n |P_n(x) - P_G(x)|^{2\lambda} \leq n^{-\lambda} \tag{21}$$

Proof. According to equation (14), we have

$$E_n [P_n(x) - P_G(x)]^2 = Var(P_n(x))$$

Where

$$\begin{aligned} Var(P_n(x)) &= E \left\{ \frac{1}{n} \sum_{i=1}^n [I(X_i < x) - P_G(x)] \right\}^2 \\ &= \frac{1}{n^2} \sum_{i=1}^n Var[I(X_i < x)] \\ &= \frac{1}{n} Var(I(X_i < x)) \\ &\leq \frac{1}{n} E(I(X_i < x))^2 \\ &\leq \frac{1}{n} \end{aligned}$$

Then

$$E_n [P_n(x) - P_G(x)]^2 \leq \frac{1}{n}$$

Hence, we have

$$\begin{aligned} E_n |P_n(x) - P_G(x)|^{2\lambda} &\leq \{E_n [P_n(x) - P_G(x)]^2\}^\lambda \\ &\leq n^{-\lambda} \end{aligned}$$

for any $0 < \lambda \leq 1$.

4. Asymptotic Optimality of Empirical Bayes Test

Theorem 1. Suppose $\delta_n(x)$ be the empirical Bayes test function defined in equation (16), X_1, X_2, \dots are random variables sequence of independent and identically distributed, when both (A1) and (A2) hold, if

(1) $\{b_n\}$ is a positive sequence, and satisfy $\lim_{n \rightarrow \infty} b_n = 0$, $\lim_{n \rightarrow \infty} nb_n = \infty$

(2) $\int_{\Theta} \frac{1}{\theta} dG(\theta) < \infty$

(3) $f_G(x)$ is continuous about x

Then $\lim_{n \rightarrow \infty} R(\delta_n(x), G(\theta)) = R_G$

Proof. Let $Q_n(x) = |\beta(x)| P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|)$,

Then $Q_n(x) \leq |\beta(x)|$.

By (5) and Fubini theorem, we have

$$\begin{aligned} \int_{\Omega} |\beta(x)| dx &\leq \int_{\Omega} f_G(x) dx + |\theta_0| \int \int_{\Theta} \frac{1}{\theta} f(x|\theta) dG(\theta) dx \\ &\leq 1 + |\theta_0| \int_{\Theta} \int_{\Omega} f(x|\theta) dx dG(\theta) \\ &= 1 + |\theta_0| \int_{\Theta} \frac{1}{\theta} dG(\theta) < \infty \end{aligned}$$

Then by Lemma 1 and Dominated convergence theorem, we have

$$0 \leq \lim_{n \rightarrow \infty} R(\delta_n(x), G(\theta)) - R_G \leq \int_{\Omega} \lim_{n \rightarrow \infty} Q_n(x) dx \quad (22)$$

According to formulas (8) and (13), then by Markov and Jensen inequality, we have

$$\begin{aligned} Q_n(x) &\leq E_n |\beta_n(x) - \beta(x)| \\ &\leq E_n |f_n(x) - f_G(x)| + |\theta_0 x^{-2}| E_n |P_n(x) - P_G(x)| \\ &\leq [E_n |f_n(x) - f_G(x)|^2]^{\frac{1}{2}} \\ &+ |\theta_0 x^{-2}| [E_n |P_n(x) - P_G(x)|^2]^{\frac{1}{2}} \end{aligned}$$

Then by lemma 2 and lemma 3, for $\forall x \in \Omega$, we have

$$\begin{aligned} 0 \leq \lim_{n \rightarrow \infty} Q_n(x) &\leq [\lim_{n \rightarrow \infty} E_n |f_n(x) - f_G(x)|^2]^{\frac{1}{2}} \\ &+ |\theta_0 c x^{-2}| [\lim_{n \rightarrow \infty} E_n |P_n(x) - P_G(x)|^2]^{\frac{1}{2}} = 0 \end{aligned}$$

Combining (19) with (20), the theorem is proved.

Theorem 2 Suppose $\delta_n(x)$ be the empirical Bayes test function defined in equation (16), X_1, X_2, \dots are random variables sequence of independent and identically distributed, both (A1) and (A2) hold, suppose $f_G(x) \in C_{s,\alpha}$, $0 < \lambda < 1$

and $s \geq 2$ is the positive integer, and it satisfies the following conditions:

$$(B1) \int_{\Omega} |\beta(x)|^{1-\lambda} dx < \infty$$

$$(B2) \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-2}|^{\lambda} dx < \infty$$

Then with the choice of $b_n = n^{-\frac{1}{2s+1}}$, we have

$$R(\delta_n, G) - R_G = O(n^{-\frac{\lambda s}{2s+1}})$$

Proof. First, we can easily get the following result:

$$\begin{aligned} &P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|) \\ &= \int_{|\beta_n(x) - \beta(x)| \geq |\beta(x)|} f(x|\theta) dG(\theta) \\ &\leq \int_{|\beta_n(x) - \beta(x)| \geq |\beta(x)|} \frac{|\beta_n(x) - \beta(x)|^{\lambda}}{|\beta(x)|^{\lambda}} f(x|\theta) dG(\theta) \\ &\leq \int_{\Omega} \frac{|\beta_n(x) - \beta(x)|^{\lambda}}{|\beta(x)|^{\lambda}} f(x|\theta) dG(\theta) \\ &= |\beta(x)|^{-\lambda} E |\beta_n(x) - \beta(x)|^{\lambda} \end{aligned}$$

Apply lemma 1 and Markov inequality, we conclude that

$$\begin{aligned} 0 \leq R(\delta_n, G) - R_G &\leq \int_{\Omega} |\beta(x)| P(|\beta_n(x) - \beta(x)| \geq |\beta(x)|) dx \\ &\leq \int_{\Omega} |\beta(x)|^{1-\lambda} E_n |\beta_n(x) - \beta_G(x)|^{\lambda} dx \\ &\leq c_1 \int_{\Omega} |\beta(x)|^{1-\lambda} E_n |f_n(x) - f_G(x)|^{\lambda} dx \\ &+ c_2 \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-3}|^{\lambda} E_n |P_n(x) - P_G(x)|^{\lambda} dx \\ &= A_n + B_n \end{aligned}$$

Where $A_n = c_1 \int_{\Omega} |\beta(x)|^{1-\lambda} E_n |f_n(x) - f_G(x)|^{\lambda} dx$,

$$B_n = c_2 \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-3}|^{\lambda} E_n |P_n(x) - P_G(x)|^{\lambda} dx$$

By Lemma 2 and Lemma 3 and condition (B1), we have

$$A_n \leq c_1 n^{-\frac{\lambda s}{2s+1}} \int_{\Omega} |\beta(x)|^{1-\lambda} |\theta_0|^{\lambda} dx \leq c_3 n^{-\frac{\lambda s}{2s+1}} \quad (23)$$

$$B_n \leq c_2 n^{-\frac{\lambda}{2}} \int_{\Omega} |\beta(x)|^{1-\lambda} |x^{-2}|^{\lambda} dx \leq c_4 n^{-\frac{\lambda}{2}} \quad (24)$$

Then $0 \leq R(\delta_n, G) - R_G \leq c_3 n^{-\frac{\lambda s}{2s+1}} + c_4 n^{-\frac{\lambda}{2}}$

Finally it shows that

$$R(\delta_n, G) - R_G = O(n^{-\frac{\lambda s}{2s+1}}).$$

Remark 2. $O(n^{-\frac{\lambda s}{2s+1}})$ can be arbitrary close to $O(n^{-\frac{1}{2}})$ for $\lambda \rightarrow 1, s \rightarrow \infty$.

5. Example

In this section, an example is given to show that the inverse exponential distribution and the prior distribution which satisfies theorems in this paper discussed. Let the probability density function of random variable X as defined in equation (1). The sample space is $\Omega = \{x | x > 0\}$, and the parameters space is $\Theta = \{\theta | \theta > 0\}$. Here we assume the prior distribution of the parameter θ is Gamma prior distribution with the following probability density function:

$$g(\theta) = \frac{1}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\theta}, \theta > 0, \alpha > 0 \tag{25}$$

Where $\alpha > 0$ is the given hyper parameter.

The marginal probability density function of X can be derived as follows:

$$\begin{aligned} f_G(x) &= \int_0^\infty \frac{\theta}{x^2} e^{-\frac{\theta}{x}} \frac{1}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\theta} d\theta \\ &= \frac{1}{x^2 \Gamma(\alpha)} \int_0^\infty \theta^{(\alpha+1)-1} e^{-(1+x^{-1})\theta} d\theta \\ &= \frac{1}{x^2 \Gamma(\alpha)} \frac{\Gamma(1+\alpha)}{(1+x^{-1})^{1+\alpha}} \\ &= \frac{\alpha}{x^2 (1+x^{-1})^{1+\alpha}} \end{aligned}$$

By directly calculating, it is easily to show that $f_G(x) \in C_{s,\alpha}$, and

$$\begin{aligned} \beta(x) &= f_G(x) - \theta_0 \int_\Theta f(x|\theta) dG(\theta) \\ &= \frac{\alpha}{x^2 (1+x^{-1})^{1+\alpha}} - \theta_0 \int_0^\infty \frac{1}{x^2} e^{-\frac{\theta}{x}} \frac{1}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-(1+x^{-1})\theta} d\theta \\ &= \frac{\alpha}{x^2 (1+x^{-1})^{1+\alpha}} - \theta_0 \frac{1}{x^2 (1+x^{-1})^\alpha} \\ &= \frac{1}{x^2 (1+x^{-1})^{1+\alpha}} [\alpha - \theta_0 (1+x^{-1})] \end{aligned}$$

Furthermore, we easily have

$$|\beta(x)| \leq \frac{M}{x^2 (1+x^{-1})^{1+\alpha}}$$

Where M is a positive number.

Then the conditions (B1) and (B2) hold true. Thus we have the conclusion:

Theorem 3 Let R_G and $R(\delta_n, G)$ be from (12) and (17), respectively, for the inverse exponential distribution (1), and assume the prior distribution of θ be defined by (25), with the choice of $n^{-\frac{\lambda s}{2s+1}}$, then we have

$$R(\delta_n, G) - R_G = O(n^{-\frac{\lambda s}{2s+1}}).$$

Remark 3. From Theorems 1 to 3, we know that the new established empirical Bayes test function is asymptotically optimal and its convergence rates can arbitrarily close to $O(n^{-1/2})$ under suitable conditions.

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