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Preferences Involved Comprehensive Evaluation in Educational Management and Decision Making

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Abstract: Evaluation functions are very crucial in educational comprehensive evaluation. This study summarizes some classical evaluation functions and shows their usage in Pedagogic evaluation applications. The study also presents and illustrates some hybrid evaluation function with some types of preferences of decision makers involved. Some illustrations with examples show that different types of preferences can embody both educators' teaching experience and their optimism/pessimism decision attitudes. Therefore, the analyses in this study can help first line teachers select suitable, flexible and reasonable models for their practical educational comprehensive evaluations.

Keywords: Aggregation Functions, Choquet Integrals, Decision Making, Pedagogic Management, Educational Evaluation, OWA Operators

1. Introduction

Pedagogic administration and decision making studies are very important in the both applications and theoretical studies of management sciences [2], [7], [10]. Schools and educational organizations have difference from business corporations in many aspects, while the difference in the decision making methods and models between these two types of organizations is slight [2]. Performance evaluation is a sine qua non of educational management and decision making. School as a basic decision making structure, needs strict, reasonable and normative models to help the managements make decisions. Judgments of value are generally complex in school management. For example, teachers may favor one student over the other; and headmaster may similarly prefer one teacher to another. The phenomenon is pervasive because frequently general evaluations can not fully and accurately reflect students' or teachers' true performance and behavior in school. The preferences of different managements not only reflect their intuition in decision, but also represent some important experiences of their own. Therefore, preferences involved decision and evaluation models are more important and effective than those based only on superficial facts and datum. In this study, different from traditional evaluation models such as Weighted Average mean, we present some preferences involved adjustable and adaptive evaluation models which are more reasonable and effective.

Educational evaluations are very important for the future works of every school, educational organization or system, which include many numerical evaluation models such as multi-attribute decision-making. For example, when we wish to evaluate the comprehensive performance of a certain student in one semester, we can assign weights with different magnitudes to different subjects, and then we can use the Weighted Average Mean to evaluate the average performance of that student in that semester. For example, let $C=\{C_1,C_2,\ldots,C_n\}$ be the courses collection including n courses/subjects with different importance; let $\mathbf{x}=(x_1,x_2,\ldots,x_n)$ be the score vector corresponding to n courses in C, i.e., x_i is the score one student obtains for ith course; and finally let $\mathbf{w}=(w_1,w_2,\ldots,w_n)$ ($\sum_{i=1}^{n} w_i=1$) be the weighting vector representing the different importance for different courses respectively; the a comprehensive score S can be obtained by the simple inner produce of vector x and w, i.e., $S = \mathbf{x} \cdot \mathbf{w} = \mathbf{x}(\mathbf{w})^T = \sum_{i=1}^{n} x_i w_i$. This type of evaluation, though maybe simpler or more complex using different merging functions [1], [3], [6], [9], is important and needs to be reasonable and fair, because the evaluation not only expresses the study status of that student (to compare with others'), but also can remind the student to judge if or not an adaptive adjustment for the current course learning will be necessary.

In this study, we will summarize some important and useful evaluation functions [1], [8], [9], [11], [12] to show they can be flexibly and widely used in educational decision making especially when educators' preferences and their practical experiences will be involved. We will also introduce some merging functions and hybrid preferences involved evaluation models, as they can be shown better suitable for some certain situations.

The remainder of this study is organized as follows: Section 2 reviews Weighted Average and OWA operators and shows their application in educational evaluation. Section 3 discusses two hybrid methods melting Weighted Average and OWA operators which can be used in the evaluation with dual-preference involved. Section 4 presents the usage of Choquet Integral in educational decision making. Section 5 summarizes the main results and conclusions.

2. The Usage of Weighted Average and OWA Operators in Educational Evaluation

The comprehensive judgments of students' performances in several different subjects or courses are very crucial in the educational evaluation. The fair and reasonable evaluation not only effectively helps to determine the completeness of teaching objectives of teachers, but also helps to compare between different students. Without loss of generality, suppose that we have n subjects S_i , $i = 1, 2, ..., n$, needing to give performances for every student, and that each performance of *n* subjects is within interval [0, 1]. Because different subjects have different importance, the general method to give a comprehensive score for all the subjects is to use Weighted Average. That is, assume that one student has n scores, represented by a vector $\mathbf{s} = (s_1, s_2, ..., s_n)$, corresponding to n subjects respectively; and that the normalized weights for different subjects is given also by one weighting vector $\mathbf{w} = (w_1, w_2, ..., w_n)$. Thus, we have the weighted average, seen also as the simple inner product, $s = \sum_{i=1}^{n} w_i s_i = \mathbf{w}(\mathbf{s})^T = \mathbf{w} \cdot \mathbf{s}$, where s is the comprehensive score we need. Different weights (also called weighting vectors) represent the preferences derived by educators' long time teaching experiences. For example, if $n = 4$, $\mathbf{s} = (1, 0.5, 0.2, 0.8)$ and $\mathbf{w} = (0.3, 0.4, 0.1, 0.2)$, we will obtain

$$s = 0.3(1) + 0.4(0.5) + 0.1(0.2) + 0.2(0.8) = 0.86 .$$

Another type of preferences of first-line educators can be expressed by their optimistic or pessimistic attitudes. The Ordered Weighted Averaging (OWA) operators [11], [12] perfectly embody these two opposite attitudes. For example, suppose we have four scores $s_1 = 1$, $s_2 = 0.5$, $s_3 = 0.2$ and $s_4 = 0.8$ needing to be aggregated into one comprehensive score s. If one decision maker is with the most optimistic attitude, the final result should be $s = Max(s_1, s_2, s_3, s_4) = s_1 = 1$; conversely, if (s)he has the extreme pessimistic attitude, then we will select the Min function to get $s = Min(s_1, s_2, s_3, s_4) = s_3 = 0.2$. We specially note that Max also corresponds to logic "Or", while Min corresponds to logic "And". However, the huge continuum between these two extreme attitudes is neglected, unless we use some special function like OWA operators. An OWA operator of dimension n is a mapping $F : [0,1]^n \to [0,1]$, which has an associated weighting vector $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ satisfying the following properties

$$\sum_{j=1}^{n} w_j = 1; \qquad 0 \le w_j \le 1; \qquad j = 1, 2, \ldots, n$$

and such that

$$F_{\mathbf{w}}(\mathbf{s}) = F_{\mathbf{w}}(s_1, s_2, \ldots, s_n) = \sum_{j=1}^{n} w_j \cdot s_{\sigma(j)} = \mathbf{w}(\mathbf{s}_\sigma)^T \qquad (1)$$

where $\sigma : \{1, ..., n\} \to \{1, ..., n\}$ is a permutation satisfying $s_{\sigma(i)} \ge s_{\sigma(j)}$ whenever $i < j$; and $\mathbf{s}_\sigma = (s_{\sigma(1)}, s_{\sigma(2)}, ..., s_{\sigma(n)})$.

Simply speaking, let $\mathbf{s} = (s_1, s_2, ..., s_n)$ and w be the OWA weighting vector; then we do not perform the direct inner product $\mathbf{w} \cdot \mathbf{s}$, but must firstly rearrange the elements of s according to their magnitudes to obtain the ordered input vector $\mathbf{s}_\sigma$, and then perform the inner product $\mathbf{w} \cdot \mathbf{s}_\sigma = \mathbf{w}(\mathbf{s}_\sigma)^T$. For example, let $\mathbf{s} = (0.5, 0, 0.2, 1)$ and $\mathbf{w} = (0.4, 0.3, 0.2, 0.1)$, we firstly rearrange s into $\mathbf{s}_\sigma = (1, 0.5, 0.2, 0)$, then we can get the final OWA aggregation result as

$$F_{\mathbf{w}}(\mathbf{s}) = \mathbf{w}(\mathbf{s}_\sigma)^T = 0.4(1) + 0.3(0.5) + 0.2(0.2) + 0.1(0) = 0.59 .$$

Clearly, the optimism/pessimism involved OWA weighting vector w and the permutation σ play crucial rules in this aggregating process. If the decision maker is with the extreme optimism, then the corresponding OWA weighting vector should be $\mathbf{w}^* = (1, 0, \ldots, 0)$, and the aggregation result should be $F_{\mathbf{w}^*}(\mathbf{s}) = \max(s_i)$; conversely, if the decision maker is with the extreme pessimism, then the weights correspondingly should be $\mathbf{w}_* = (0, \ldots, 0, 1)$, and the aggregation result should be $F_{\mathbf{w}_*}(\mathbf{s}) = \min(s_i)$. Due to its flexible nature, OWA aggregation process can reflect the continuum between these two extreme attitudes. For example, when we have a moderate attitude, with neither optimism nor pessimism, we may generally select OWA weighting vector

$\mathbf{w}_A = (1/n, 1/n, \ldots, 1/n)$. Note that $F_{\mathbf{w}_A}(\mathbf{s}) = (\sum_{i=1}^{n} s_i)/n$, which is also known to be Average Mean, the most often used way in comprehensive evaluation.

The most important measure for OWA operators is the orness degree [11], which is actually a function mapping the collection of all OWA operators into real interval [0, 1]. In decision making, the larger orness of an OWA weighting vector, the more optimism it expresses, and vice versa. The degree of "*orness*" associated with OWA operator is defined as [11]

$$\text{orness}(\mathbf{w}) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i = \sum_{i=1}^{n}\frac{n-i}{n-1}w_i. \quad (2)$$

The measure of "*andness*" associated with an OWA operator is the complement of its "*orness*", and is defined as

$$\text{andness}(\mathbf{w}) = 1 - \text{orness}(\mathbf{w})$$

or

$$\text{andness}(\mathbf{w}) = \frac{1}{n-1}\sum_{i=1}^{n}(i-1)w_i = \sum_{i=1}^{n}\frac{i-1}{n-1}w_i.$$

Two basic properties about orness are reviewed as follows.

Proposition 1 [11] The max, min and average operator correspond to $\mathbf{w}^* = (1, 0, \ldots, 0)$, $\mathbf{w}_* = (0, \ldots, 0, 1)$ and $\mathbf{w}_A = (1/n, 1/n, \ldots, 1/n)$, respectively, and $\text{orness}(\mathbf{w}^*) = 1$, $\text{orness}(\mathbf{w}_*) = 0$ and $\text{orness}(\mathbf{w}_A) = 1/2$.

Proposition 2 [11] (Reverse Property) For any OWA weighting vector $\mathbf{w} = (w_1, w_2, \ldots, w_n)$, $\text{orness}(\mathbf{w}) = \alpha$, then for the reverse of w: $\mathbf{w}' = (w_1', w_2', \ldots, w_n') = (w_n, w_{n-1}, \ldots, w_1)$, $\text{orness}(\mathbf{w}') = 1 - \alpha$.

Some important recently studied OWA weighting vectors include: Stancu OWA operators [8] which generalize several classes of OWA weights, and Recursive OWA operators [4] which can be very suitable in long time educational evaluation.

We summarize this section by comparing the two aggregation models discussed in this section in order to illustrate their difference in usage. Recall we have shown that if $\mathbf{s} = (1, 0.5, 0.2, 0.8)$ and $\mathbf{w} = (0.3, 0.4, 0.1, 0.2)$, using Weighted Average we have

$$s = 0.3(1) + 0.4(0.5) + 0.1(0.2) + 0.2(0.8) = 0.86.$$

Now suppose here **w** is an OWA weighting vector, then the corresponding evaluation result should be obtained from the following two steps:

Step 1: Rearrange s to obtain $\mathbf{s}_\sigma = (1, 0.8, 0.5, 0.2)$;

Step 2: Compute $\mathbf{w}(\mathbf{s}_\sigma)^T$ to obtain

$$F_{\mathbf{w}}(\mathbf{s}) = \mathbf{w}(\mathbf{s}_\sigma)^T = 0.3(1) + 0.4(0.8) + 0.1(0.5) + 0.2(0.2) = 0.71.$$

And note that in this case $orness(\mathbf{w}) = 0.6$, representing a slightly optimism of one decision maker.

3. Hybrid Evaluation Methods Melting Weighted Average and OWA Operators

In Section 2 we have shown that both Weighted Average and OWA operators are very useful in evaluation problems, and they can be selected to use respectively in different decision contexts. However, when one decision maker has both optimism/pessimism preference and the inclination to use the traditional Weighted Average, we may face the dilemma about how choose all of them. As a simpler solution to find out, we can "multiply" both weighting vectors by multiply all of their respective entries. That is, if we denote by $\mathbf{w} = (w_1, \ldots, w_n)$ the OWA weighting vector and by $\mathbf{u} = (u_1, \ldots, u_n)$ the weights for Weighted Average, then we can create a hybrid weighting vector $\mathbf{r} = (r_1, \ldots, r_n)$ such that:

$$r_i = w_i u_i / \sum_{j=1}^{n} w_j u_j \text{ for all } i = 1, 2, \ldots, n. \quad (3)$$

(Note that in formula (3) we posit the general condition $w_k > 0$ for all $k = 1, 2, \ldots, n$, which can be easily satisfied due to the diversity of OWA weighting vectors.)

Another hybrid method can be fulfilled by the adjustment process. Suppose the weighting vector for Weighted Average is still $\mathbf{u} = (u_1, \ldots, u_n)$, and the preference of decision maker is an OWA operator $\mathbf{w} = (w_1, \ldots, w_n)$. We take u as original weights which can be adjusted by operator w using some special techniques. Here we firstly review the Decreasing Class $C_{\mathbf{w}} = \{\mathbf{w}^{(1)}, \mathbf{w}^{(2)}, \ldots, \mathbf{w}^{(n)}\}$ of n-dimensional OWA operator $\mathbf{w} = (w_1, \ldots, w_n)$ which can be created by the following way [5]:

$\mathbf{w}^{(i)} = (w_{i1}, w_{i2}, \ldots, w_{ii})$, i = 1, 2, …, n, is the i-dimensional OWA operator satisfying

$$w_{i-1,j} = \frac{i-j}{i-1}w_{ij} + \frac{j}{i-1}w_{i,j+1} \quad (j = 1, \ldots, i-1;\ i = 2, \ldots, n) \quad (4)$$

and $\mathbf{w}^{(n)} = (w_{n1}, w_{n2}, \ldots, w_{nn}) \triangleq \mathbf{w}$.

And n is called the degree of that Decreasing Class. In addition, since it is actually generated by w from (4), we say that w is its Generator [5], or that the Decreasing Class is generated from w.

For example, we can use one 4-dimensioonal OWA operator $\mathbf{w} = (w_1, w_2, w_3, w_4) = (0.3, 0, 0.3, 0.4)$ to generate its Decreasing Class:

$$\mathbf{w}^{(4)} = (w_{41}, w_{42}, w_{43}, w_{44}) = (0.3, 0, 0.3, 0.4);$$

$$\mathbf{w}^{(3)} = (w_{31}, w_{32}, w_{33}) = (0.3, 0.2, 0.5);$$

$$\mathbf{w}^{(2)} = (w_{21}, w_{22}) = (0.4, 0.6);$$

$$\mathbf{w}^{(1)} = (w_{11}) = (1).$$

Thus, we can generate an optimism/pessimism Preference

Matrix W from one decreasing class. For example, as for the above mentioned decreasing class generated from **w**, we can build W by:

$$W=\begin{bmatrix}\mathbf{w}^{(1)},0,0,0\\ \mathbf{w}^{(2)},0,0\\ \mathbf{w}^{(3)},0\\ \mathbf{w}^{(4)}\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0.4&0.6&0&0\\0.3&0.2&0.5&0\\0.3&0&0.3&0.4\end{bmatrix}.$$

We find that the orness degrees for all the weighting vectors (apart from $\mathbf{w}^{(1)}$ as the degenerated case) are the same value 0.4.

Therefore, W now serves as a linear transformation, and instead of w, W can be as a multiplier for the original weighting vector u. We show some details as follows.

We firstly rearrange the ordering of u by the corresponding scores. Suppose the student's score of different subjects is a vector $\mathbf{s}=(s_1,s_2,...,s_n)$ such that s_i is the score of subject S_i which is assigned the weight u_i , $i=1,2,...,n$. We can always rearrange s in a decreasing way; that is, by using one permutation $\tau(\cdot)$. And we define

$$\mathbf{s}'=(s_1',s_2',...,s_n')=(s_{\tau(1)},s_{\tau(2)},...,s_{\tau(n)})$$

being the rearranged one such that $s_{\tau(i)}\geq s_{\tau(j)}$ whenever $i<j$. Then the corresponding rearranged weighting vector u should be

$\mathbf{u}'=(u_1',u_2',...,u_n')=(u_{\tau(1)},u_{\tau(2)},...,u_{\tau(n)})$ such that $s_{\tau(i)}\geq s_{\tau(j)}$ whenever $i<j$.

For example, if originally $\mathbf{s}=(0.5,0,0.8,0.2)$, and $\mathbf{u}=(0.1,0.2,0.3,0.4)$, we have

$\mathbf{s}'=(0.8,0.5,0.2,0)$ and $\mathbf{u}'=(0.3,0.1,0.4,0.2)$.

In the next step, we use preference matrix M as a transformation to adjust $\mathbf{u}'$ to obtain

$$\mathbf{u}''=\mathbf{u}'W .$$

Specially note that here since $\mathbf{s}'$ and $\mathbf{u}'$ are the rearranged ones with decreasing order, then W represents an optimism attitude sine it will enlarge the weights of $\mathbf{u}'$ at the top positions, and the larger orness of generator w, the more optimism one decision maker has.

As the adjusted weighting vector, we still need to use $\mathbf{u}''$ and rearranged score $\mathbf{s}'$ to get the final comprehensive score with preference involved:

$$Score=\mathbf{u}''(\mathbf{s}')^T .$$

For example, just as the above mentioned case, we have

$$\mathbf{u}''=\mathbf{u}'W=(0.3,0.1,0.4,0.2)\begin{bmatrix}1&0&0&0\\0.4&0.6&0&0\\0.3&0.2&0.5&0\\0.3&0&0.3&0.4\end{bmatrix}=(0.52,0.14,0.26,0.08),$$

with

$$Score=\mathbf{u}''(\mathbf{s}')^T=0.52(0.8)+0.14(0.5)+0.26(0.2)+0.08(0)=0.538 .$$

To compare with the result of general Weighted Average using original weighting vector **u**, we compute and obtain:

$$S=\mathbf{s}(\mathbf{u})^T=0.1(0.5)+0.2(0)+0.3(0.8)+0.4(0.2)=0.37 .$$

The new Score is better than the original one; this is because the decision maker is optimistic with extent 0.4, equaling to the orness of OWA operator **w**.

4. Choquet Integral in Educational Comprehensive Evaluation

The weights, no matter they are for Weighted Average or OWA operators, express the variance of importance in different subjects or educational evaluation objects. Both type of weights add up to unity, and more strictly, they are additive. In most of decision making environments, however, the importance and related "weights" are not additive. For example, suppose we have four subjects needing to associate weights: 1) Mathematics; 2) English; 3) Computer sciences; and 4) Physics. The teachers put the absolute necessity and importance on the learning of English. That is to say, no matter how well the other three subjects one student learns, if (s)he can not have good English learning, then his/her comprehensive score will be poor and can not be accepted by school. To model this situation, fuzzy measure and related Choquet integral [1] (also Sugeno integral [9]) are some very important and useful tools. Simply speaking, we can assign different weights to different combination of those four subjects. For example, we can assign 0.5 to {English}, and assign 0 to {Mathematics}, but assign 0.6 to {English, Mathematics} and finally assign the largest 1 to all of them {Mathematics, English, Computer sciences, Physics}.

Let $N=\{1,2,...,n\}$ be the sets whose elements represent all of the evaluation objects, e.g., all concerned subjects; and let $\mathbf{x}=(x_1,x_2,...,x_n)$, $x_i\in[0,1]$, be the vector whose entries represent the single valuations for all objects, respectively. We then review some necessary preparations for Choquet integral in educational comprehensive evaluation.

Definition 1 [1] A fuzzy measure on N is a mapping $\mu:2^N\to[0,1]$ satisfying

a) $\mu(\varnothing)=0$, $\mu(N)=1$

b) $A\subseteq B$ implies $\mu(A)\leq\mu(B)$.

Definition 2 [1] A fuzzy measure μ on N is symmetric if for all $A, B \in 2^N$ such that |A| = |B|, we have $\mu(A) = \mu(B)$.

Definition 3 [1] Let μ be a fuzzy measure on N and $f: N \to [0, \infty)$. The Choquet integral of f with respect to μ is defined by

$$(C)\int f \, d\mu = \int_0^\infty \mu(\{\omega \in N \mid f(\omega) > \alpha\}) \, d\alpha .$$

Note that this original definition is applicable to continuous spaces. For discrete space and corresponding nonnegative vectors, the usual integral notation $\int$ could be abandoned.

Definition 4 [1] Let μ be a fuzzy measure on N and $\mathbf{x} \in [0,1]^n$. The Choquet integral of x w.r.t. μ is defined by

$$C_\mu(\mathbf{x}) = \sum_{i=1}^{n} (x_{\sigma(i)} - x_{\sigma(i-1)}) \mu(A_{\sigma(i)}) \quad (5)$$

with σ being a permutation on N such that $x_{\sigma(i)} \le x_{\sigma(j)}$ whenever $i < j$, with the convention $x_{\sigma(0)} = 0$ and $A_{\sigma(i)} = \{\sigma(i), \cdots, \sigma(n)\}$.

The following proposition presents the interesting relation between OWA operators and Choquet integral.

Proposition 3 [1] Let μ be a fuzzy measure on N and $\mathbf{w} = (w_1, w_2, \cdots, w_n)$ be an OWA operator. For any $\mathbf{a} = (a_1, a_2, \cdots, a_n)$, $C_\mu(\mathbf{a}) = F_{\mathbf{w}}(\mathbf{a})$ if and only if μ is symmetric, with $w_i = \mu(A_i) - \mu(A_{i-1})$, i = 1, 2, …, n, where A_i is any subset of N with $|A_i| = i$ and $A_0 = \varnothing$.

Suppose we have four subjects needing to evaluate, and we still use the above mentioned four, i.e., $N = \{1, 2, 3, 4\}$ representing {1 Mathematics, 2 English, 3 Computer sciences, 4 Physics}. Assume we have the follow fuzzy measure μ :

$\mu(\{1\}) = \mu(\{3\}) = \mu(\{4\}) = 0$ (showing Mathematics, Computer sciences and Physics independently only have 0 importance);

$\mu(\{2\}) = 0.5$ (showing English independently can have 0.5 importance);

$\mu(\{1,2\}) = 0.6$ (showing English and Mathematics combined can have 0.6 importance, clearly more important than anyone of other single subjects);

$\mu(\{2,3\}) = 0.8$ (showing English and Computer sciences combined are further more important);

$\mu(\{2,4\}) = 0.5$ (showing English and Physics have on combination advantages over English itself);

$\mu(\{1,3\}) = \mu(\{1,4\}) = \mu(\{3,4\}) = \mu(\{1,3,4\}) = 0$ (showing that without English any other combination of subjects is with no importance, which stresses the leading importance of English);

$\mu(\{1,2,3\}) = \mu(\{1,2,4\}) = \mu(\{2,3,4\}) = 0.9$ (showing that every three subjects combined, if involving English, is very important);

$\mu(\{1,2,3,4\}) = 1$ (showing the perfect case when all subjects are combined).

Now suppose one student's scores for all subjects are represented by a score vector $\mathbf{x} = (1, 0.6, 0.8, 0.5)$, then using formula (5) for discrete Choquet integral, we can obtain the comprehensive score for that student:

$$\begin{aligned} C_\mu(\mathbf{x}) &= \sum_{i=1}^{n} (x_{\sigma(i)} - x_{\sigma(i-1)}) \mu(A_{\sigma(i)}) \\ &= (1-0.8)\mu(\{1\}) + (0.8-0.6)\mu(\{1,3\}) \\ &\quad + (0.6-0.5)\ \mu(\{1,2,3\}) + (0.5-0)\mu(\{1,2,3,4\}) \\ &= (1-0.8)0 + (0.8-0.6)0 + (0.6-0.5)0.9 + (0.5-0)1 \\ &= 0.59 . \end{aligned}$$

Due to the properties of given fuzzy measure μ, obtaining this score is majorly because the student's English score is not high.

By the above analysis and illustration, we can find that Choquet integral with respect to fuzzy measure is a much flexible method which can assign importance (or weights) using a large variety of preferences of decision makers. Therefore, it is also an effective and applicable method in educational decision making and evaluation problems.

5. Conclusions

This study reviewed and summarized some types of aggregation functions such as Weighted Average and OWA operators which can be suitably used in educational applications of comprehensive evaluation problems. The study also discussed two hybrid methods melting Weighted Average and OWA operators which can be used in the evaluation with dual-preference involved. The usage of Choquet Integral in educational comprehensive evaluation was presented, which can have some special flexibility and advantages. Some illustrative examples of those discussed models were shown in this study. The study can help educators select suitable, flexible and reasonable models for their practical educational comprehensive evaluations.

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