A Method Based on Combination Operation of Detecting Harmonic in Unbalanced Power System

Junli Wan, Hao Liu, Linjuan Wu, Shirun Cheng

College of Electrical Engineering & New Energy, China Three Gorges University, Yi Chang, China

Email address: wjlem@163.com (Junli Wan)
*Corresponding author

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Abstract: Aiming at defects of traditional harmonic detection in three-phase unbalanced condition, an improved \( i_p - i_q \) detecting method is proposed based on instantaneous reactive power theory. When power grid voltage unbalanced or voltage distortion, through coordinate transformation, trigonometric transforming and combination operation of the three-phase voltage, the fundamental positive sequence voltage is extracted, and the average filter string into the low pass filter (LPF) to reduce the ripples in fundamental positive sequence current. The improved method is more accurate and quick than traditional \( i_p - i_q \) detecting method when extracting positive sequence fundamental voltage, detecting the negative sequence current of power grid and harmonic reactive current, significantly to reduce THD of fundamental positive sequence current and the detection error caused by unbalance and distortion voltage. Simulative results verify the correctness and validity of theoretical analysis.

Keywords: Instantaneous Reactive Power, Unbalanced Power Grid, Fundamental, Positive Sequence Voltage Extracting, Average Filter, Harmonic Detection

1. Introduction

With power electronic equipment as well as other non-linear loads widely being used in power systems, harmonic pollution problem arises making more serious impacting on power quality. Using active power filter (APF) to suppress the harmonic of power grid has become an important means of harmonic governance [1], meanwhile, harmonic detection of active power filter is the key to achieve accurate, real-time compensating current, also important for power system security, stability and economic operation. The mainly method of current harmonic detection including Fourier analysis and detection [2], Neural network adaptive testing [3], the instantaneous reactive power theory testing [4], etc. Based on Fast Fourier Transform (FFT) harmonic detection algorithm simply programming, easily implementing embedded system, however, it needs a certain period of time discrete sampling values when detecting. Harmonic detection algorithm based on nerve network features of speed, high accuracy, good real-time performance, capacity of study, real-time and anti-interference, nevertheless, there are still many problems in the practical application of nerve networks such as no standard method of neural network construction, no specific method for determining sample sizes and need a lot of training samples [5]. Harmonic detection algorithm based on nerve network is not mature, and further research needs to be optimized. Harmonic current detection based on instantaneous reactive power theory is widely used in harmonic detection since it has a small amount of calculation, simple model structure, and the advantages of good real-time performance [6-7]. The mainly method of detecting harmonic current based on instantaneous reactive power theory includes \( p-q \) method and \( i_p - i_q \) method. The \( p-q \) method can only be used to detect the harmonic and reactive current of the three-phase voltage symmetrical system. Despite the \( i_p - i_q \) method could detect harmonic when the system of three-phase voltage distortion and asymmetry, there is a large phase difference between the fundamental positive sequence voltage and the power grid voltage when power grid voltage distortion and imbalance, effecting the precision of phase locking and generating detection error [8], which cannot meet the requirements of APF precise compensation. In [9-10], the linear equations were constructed through the differential
operation of three-phase voltage to separate fundamental positive sequence component of distortion power grid voltage, and then using phase locked loop (PLL) technique to compensate phase jump of power grid voltage to realize fast tracking of phase of fundamental positive sequence voltage component, but, when distorting voltage contain a lot of harmonic, could lead to dimension too many of the equations as well as complexity calculation, affecting dynamic response speed of the system. To slave the situation of slow response dynamic in single $d - q$ transforming, double synchronous transform phase locked loop and a decoupling network is adopted to extract positive sequence voltage component in imbalanced system [11-12], nevertheless, when harmonic contained in voltage, equipment cannot isolate fundamental component. The digital PLL of vary period sampling phase shift control loop is proposed by [13], the phase shifting controller is used to eliminate disturbance of the frequency doubling of the power grid, but it is unable to filter out other harmonic component. In [14], the improvement of the soft phase lock loop is improved by transfer filter function and reducing forward integral element, the dynamic phase tracking performance and unbalanced voltage detection performance are significantly improved. In [15], the basic principle of the non phase locked loop is analyzed and the average value theory is introduced. The method of the non phase locked loop harmonic detection based on the average value theory is proposed, but it needs to be transformed repeatedly and the computation is large. For harmonic detection error caused by traditional $i_p - i_q$ method in three-phase imbalanced, this paper, based on $i_p - i_q$ method, using virtual sine and cosine function fundamental positive sequence component of a phase voltage, existing in three-phase unbalance and the voltage distortion, are extracted by synchronizing the coordinate and trigonometric transformation; to eliminate the errors disturbing PLL detection by asymmetric and voltage distortion. Adding an average value filter to the Butterworth filter replace the traditional LPF obtaining current direct current component, to reduce the ripple current in the fundamental positive sequence.

2. Traditional Harmonic Detection Algorithm Based on $i_p - i_q$ Theory and Reactive Power

Based on instantaneous reactive power theory the traditional $i_p - i_q$ algorithm as in Fig. 1 [16-17], this method use the voltage of phase-A passing by the PLL getting sine signal in-phase of the voltage of phase-A and the corresponding cosine signal, and three-phase load current $i_a, i_b, i_c$ firstly obtain two-phase orthogonal current $i_{p}, i_{q}$ by $\alpha-\beta$ transforming of the matrix $C_{12}$, as in (1), then obtaining active and reactive current $i_{pa}, i_{qa}$ by transformation of the matrix $C_{12}$, as in (2). And then getting the direct current component by LPF, finally getting the three-phase fundamental wave current $i_{pa}, i_{qa}, i_{qb}$ by reverse transformation of the matrix $C_{12}$, as in (3), and getting three-phase harmonic current $i_{pa}, i_{qa}, i_{qb}$ by subtracting fundamental of three-phase current respectively.

![Fig. 1. The principle of harmonic detection based on $i_p - i_q$ method.](image)

$$
\begin{bmatrix}
  i_{pa} \\
  i_{qa}
\end{bmatrix}
= \begin{bmatrix}
  \frac{1}{2} & \frac{\sqrt{3}}{2} \\
  -\frac{\sqrt{3}}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
  i_p \\
  i_q
\end{bmatrix}
= C_{32}
\begin{bmatrix}
  i_p \\
  i_q
\end{bmatrix}
$$

(1)

$$
\begin{bmatrix}
  i_p \\
  i_q
\end{bmatrix}
= \begin{bmatrix}
  \sin \omega t & -\cos \omega t \\
  -\cos \omega t & \sin \omega t
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b
\end{bmatrix}
= C \begin{bmatrix}
  i_a \\
  i_b
\end{bmatrix}
$$

(2)

$$
\begin{bmatrix}
  i_{pa} \\
  i_{qa}
\end{bmatrix}
= \frac{1}{\sqrt{3}}
\begin{bmatrix}
  \frac{1}{2} & 0 \\
  0 & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
  \sin \omega t & -\cos \omega t \\
  -\cos \omega t & \sin \omega t
\end{bmatrix}
\begin{bmatrix}
  i_p \\
  i_q
\end{bmatrix}
= C_{12} \begin{bmatrix}
  \frac{1}{2} \\
  \frac{1}{2}
\end{bmatrix}
$$

(3)

3. The Modified Harmonic Detection Method

When the asymmetry and distorted of three-phase voltage, the voltage of phase-A contains fundamental positive sequence voltage, negative sequence voltage, zero sequence voltage and harmonic voltage, making the phase difference between the voltage of phase-A and the positive sequence voltage of phase-A component. In the other hand, harmonic voltage will make great interference to the PLL, causing phase error of PLL tracking, ultimately affecting the harmonic and reactive current detection [18]; moreover, LPF of traditional $i_p - i_q$ detection method makes large fundamental existing ripple
because of the low order [19], which is more obvious when voltage imbalance and load unbalanced.

The principle of harmonic detection based on improved \( i_p - i_q \) detection method as in Fig. 2, aiming at the characteristics of three-phase voltage imbalance and distortion, the synchronous rotating reference coordinate transformation and LPF being introduced before PLL to obtain the fundamental positive sequence voltage, and the LPF obtain direct current component has been improved at the same.

![Fig. 2. The principle of improved \( i_p - i_q \) detection method.](image)

### 3.1. The Extraction of the Fundamental Positive Sequence Voltage of Phase-A

The principle of extracting fundamental positive sequence voltage in phase-A as in Fig. 3, using virtual sine and cosine function, makes synchronous reference frame coordinate transforming, triangle function transforming and combination operation to eliminate direct current component of fundamental negative sequence voltage in phase-A. Furthermore, the fundamental positive sequence component is obtained by low pass filter. Ultimately, the fundamental positive sequence voltage in phase-A is constructed based on trigonometric.

![Fig. 3. The principle of extracting on fundamental positive sequence in a phase voltage.](image)

If the three-phase grid voltage asymmetric containing harmonics as well, it can be expressed as:

\[
\begin{bmatrix}
  u_a \\
  u_b \\
  u_c 
\end{bmatrix} = \begin{bmatrix}
  \sum_{n=1}^{\infty} U_{1n} \sin(n\alpha + \phi_{1n}) + U_{2n} \sin(n\beta + \phi_{2n}) \\
  \sum_{n=1}^{\infty} U_{1n} \sin(n\alpha + \phi_{1n} - \frac{2\pi}{3}) + U_{2n} \sin(n\beta + \phi_{2n} - \frac{2\pi}{3}) \\
  \sum_{n=1}^{\infty} U_{1n} \sin(n\alpha + \phi_{1n} + \frac{2\pi}{3}) + U_{2n} \sin(n\beta + \phi_{2n} + \frac{2\pi}{3}) 
\end{bmatrix}
\]

\[\text{(4)}\]

in (4), \( U_{1n}, U_{2n} \) are respective positive and negative sequence component of valid values; \( \phi_{1n}, \phi_{2n} \) are respective the initial phase of positive and negative sequence component; \( \omega \) is the rotational angular frequency of grid voltage; \( n \) is the order of harmonics. Transforming three-phase voltage into \( \alpha - \beta \) coordinates it can be got that:

\[
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 
\end{bmatrix} = C_{32} \begin{bmatrix}
  u_a \\
  u_b \\
  u_c 
\end{bmatrix} = \begin{bmatrix}
  \sqrt{\sum_{n=1}^{\infty} U_{1n} \sin(n\alpha + \phi_{1n}) + U_{2n} \sin(n\alpha + \phi_{2n})} \\
  \sqrt{\sum_{n=1}^{\infty} U_{1n} \sin(n\alpha + \phi_{1n} - \frac{2\pi}{3}) + U_{2n} \sin(n\alpha + \phi_{2n} - \frac{2\pi}{3})} \\
  \sqrt{\sum_{n=1}^{\infty} U_{1n} \sin(n\alpha + \phi_{1n} + \frac{2\pi}{3}) + U_{2n} \sin(n\alpha + \phi_{2n} + \frac{2\pi}{3})} 
\end{bmatrix} \begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 
\end{bmatrix}
\]

\[\text{(5)}\]

Using (5) multiply unit sine and cosine matrix which are constructed it can be got that:

\[
\begin{bmatrix}
  u_{\alpha} \\
  u_{\beta} \\
  u_{\alpha} \\
\end{bmatrix} = \begin{bmatrix}
  \sin(\alpha + \alpha) \\
  \cos(\alpha + \alpha) \\
  \sin(\alpha + \alpha) \\
\end{bmatrix} = \begin{bmatrix}
  \sin(\alpha + \alpha) \\
  \cos(\alpha + \alpha) \\
  \sin(\alpha + \alpha) \\
\end{bmatrix} \begin{bmatrix}
  u_{\alpha} \\
  u_{\beta} \\
  u_{\alpha} \\
\end{bmatrix}
\]

\[\text{(6)}\]

\[\text{(7)}\]

And properties of trigonometric function can be used to make combination operation among \( u_{\alpha}, u_{\beta}, u_{\alpha}, u_{\beta} \), as in (8), fundamental negative sequence components are eliminated...
after being expanded. AC component is filtered by LPF, containing only fundamental positive sequence component value can be got as in (9).

\[
\begin{bmatrix}
U_{ux} - U_{fy} \\
U_{ux} + U_{fy}
\end{bmatrix} = \begin{bmatrix}
\sqrt{3} \sum_{n=1}^{\infty} \left[ U_{u_n} \cos((n-1)\alpha + \varphi_{u_n} - \alpha) - U_{u_n} \cos((n+1)\alpha + \varphi_{u_n} + \alpha) \right] \\
\sqrt{3} \sum_{n=1}^{\infty} \left[ U_{u_n} \sin((n-1)\alpha + \varphi_{u_n} - \alpha) - U_{u_n} \sin((n+1)\alpha + \varphi_{u_n} + \alpha) \right]
\end{bmatrix}
\]

(8)

\[
\begin{bmatrix}
U_{ux} - U_{fy} \\
U_{ux} + U_{fy}
\end{bmatrix} = \begin{bmatrix}
\sqrt{3} U_{11} \cos(\varphi_{11} - \alpha) \\
\sqrt{3} U_{11} \sin(\varphi_{11} - \alpha)
\end{bmatrix}
\]

(9)

Where, \( U_{11} \) is the valid value of positive sequence component; \( \varphi_{11} \) is the initial phase of positive sequence. By using change product into sum or difference of trigonometric function, the constructed fundamental positive sequence in a phase voltage \( u_{\alpha-ref} \) can be got [20], as in (10).

\[
u_{\alpha-ref} = \frac{1}{2} \left[ (u_{\alpha} - u_{\beta}) \sin(\alpha + \alpha) + (u_{\alpha} + u_{\beta}) \cos(\alpha + \alpha) \right] = \sqrt{2} U_{11} \sin(\alpha + \alpha) + \sqrt{2} u_{11} \cos(\alpha + \alpha) \sin(\alpha + \alpha)
\]

(10)

The extractive fundamental positive sequence voltage of a phase are being harmonic detected by phase-locked loop to eliminate the phase difference of a phase voltage lock when voltage asymmetry and distortion, and avoid harmonic voltage signal interference on PLL.

### 3.2. The Design of Improved Low Pass Filter

In traditional \( i_{\alpha}-i_{\beta} \) detection method low pass filter (LPF) is used to obtain \( i_{\alpha} \) and \( i_{\beta} \) direct current signal. When the higher the order, filter filtering effect is the better, but slower dynamic response. On the contrary, the lower the order of filter, the better of dynamic response but poor filtering effects nonetheless. When the order is same, the higher of cutoff frequency, the faster of dynamic response with low accuracy; the lower of cutoff frequency, the higher filtering precision with slower dynamic response. In a word, delay and filtering effect of the low pass filter is contradictory [21]. The advantage of filter based on Butterworth is balanced in three aspects: attenuation slope, linear phase and load characteristics [22]. Considered both the effect of filter and sensitive of response, the second-order Butterworth filter is selected and cut-off frequency regarded as 30Hz. This filter will bring a large ripple into direct component, especially when three-phase voltage unbalanced and load imbalance, which leads the fundamental positive sequence current containing higher harmonic, causing the harmonic detection resulting error.

The averaging filtering method uses module which calculates the averaging of \( i_{\alpha} \) and \( i_{\beta} \) instead of LPF in \( i_{\alpha}-i_{\beta} \) algorithm. The mean current is a straight flow corresponded to current frequency component [23]. The principle of mean filtering as in Fig. 4:

![Fig. 4. Current mean filtering method.](image)

Replace LPF with integral, time delay and gain link, algorithm implemented easily, and harmonic current detection delay is reduced to T/6. The averaging filtering method has faster dynamic response speed. When voltage stability and load symmetric, it has better real-time performance. But when the voltage distorted or load asymmetry, detection performance is relatively poor [24].

Considering the advantages and disadvantages of the above two kinds of filter algorithm, in this paper, use Butterworth filter and averaging filter which in series as a harmonic detection, in the form of modified LPF. The model diagram as in Fig. 5:

![Fig. 5. Improved LPF based on harmonic detection method.](image)

### 4. Simulation Results and Analysis

In order to verify the validity of the improved harmonic test, based on the above theoretical analysis and derivation, model and simulate of improved harmonic detection by matlab/simulink.
power system simulation software. Suppose three-phase power
grids unbalanced, voltage of the simulation as in (11):

\[
\omega = \frac{2 \pi}{f}, f = 50 \text{ Hz}
\]

where, \(\omega = \frac{2 \pi}{f}\). Three-phase voltage contains the
fundamental positive sequence voltage and negative sequence
voltage, 3rd and 5th harmonic voltage. Harmonic source is
asymmetric load: three-phase controlled rectifier bridge and A,
B two-phase resistive load, DC side of the rectifier circuit is
resistive inductive load (R= 5\(^\Omega\), L=10 mH). The resistance of
A and B is respective 5\(^\Omega\)and 10\(^\Omega\).

Unbalanced three-phase power grid voltage simulation
waveform is shown as in Fig. 6, and three-phase voltage
unbalance and distortion; a phase grid current and frequency
spectrum analysis is shown as in Fig. 7. As can be seen that it
contains a lot of harmonic current which is mainly 3rd, 5rd,
7rd, 9rd harmonics, and total harmonic distortion (THD) is
35.42%.

\[\text{Fig. 6. Three-phase power grid voltage.}\]

\[\text{Fig. 7. Grid current of a phase.}\]

Fig. 8 is simulation waveform got by traditional \(i_p - i_q\)
method. Fig. 9 is simulation waveform got by improved \(i_p - i_q\)
method.
Comparing Fig. 8 (c) with Fig. 9 (c), it can be got that the fundamental positive sequence active current which will lead to big error when detecting reactive current. Improved $i_{p1q}$ method, whereas, can accurately obtains the fundamental positive sequence active current when three-phase voltage asymmetry and distorted. Comparing Fig. 8 (c) with Fig. 9 (c), it can be got that the fundamental positive current of a phase by improved $i_{p1q}$ method has large of ripple. Comparing of the frequency analysis results of fundamental positive sequence current of a phase by this two methods, it can be got that THD of fundamental positive sequence current after improving is 3.14%, while THD of fundamental positive sequence current of traditional method is 3.14%, while THD of fundamental positive sequence current got by traditional $i_{p1q}$ method has large of ripple. According to the results, it can be known that improved $i_{p1q}$ method keeps better dynamic response speed when three-phase imbalance, and it can accurately detects the sum of negative sequence current, reactive current and harmonic current.

5. Conclusion

This paper based on the theory of instantaneous reactive power, analyses the harmonic detection results got by traditional $i_{p1q}$ method when three-phase imbalance, and an improved $i_{p1q}$ method is put forward. The improved $i_{p1q}$ method has simple circuits and computation, and more accurately and quickly to extract the positive sequence fundamental voltage, detecting the negative sequence current of power grid, harmonic and reactive current when three-phase imbalance compared with traditional method. It can eliminate the error caused by imbalance and distorted voltage, and improved LPF in the link of current filter is used to reduce ripple in the positive sequence, the detection accuracy improved at the same time. Simulation results show the effectiveness of harmonic detection by improved $i_{p1q}$ method when three-phase system of power grid unbalanced.

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References


**Biography**

**Junli Wan** was born in 1957. Now he is a professor and Master supervisor in the College of Electrical Engineering & New Energy, China Three Gorges University. His present research interests include Smart Grid Technologies, intelligent information processing technology and its application in power systems.

**Hao Liu** was born in 1990. Now he is a student in the College of Electrical Engineering & New Energy, China Three Gorges University. His present research interests are about the smart grid and power quality.

**Shirun Cheng** was born in 1989. Now he is a student in the College of Electrical Engineering & New Energy, China Three Gorges University. He present research interests are about the smart grid and intelligent control.

**Linjuan Wu** was born in 1994. Now she is a student in the College of Electrical Engineering & New Energy, China Three Gorges University. Her research interests are about the smart grid and power quality.