Science Research
2014; 2(1): 1-6
Published online January 30, 2014 (http://www.sciencepublishinggroup.com/j/sr)
doi: 10.11648/j.sr.20140201.11

New conceptions of transitivity and minimal mappings

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To cite this article:
Mohammed Nokhas Murad Kaki, New Conceptions of Transitivity and Minimal Mappings. *Science Research*.
Vol. 2, No.1, 2014, pp.1-6. doi: 10.11648/j.sr.20140201.11

Abstract: The concepts of topological δ- transitive maps, α-type transitive maps, δ-minimal and α-minimal mappings were introduced by M. Nokhas Murad Kaki. In this paper, the relationship between two different notions of transitive maps, namely topological δ-type transitive mapsandtopological α-type transitive maps has been studied and some of their properties in two topological spaces (X, τ^{δ})and (X, τ^{α}), τ^{δ} denotes the δ–topology (resp. τ^{α} denotes the α–topology) of a given topological space (X, τ) has been investigated.. Also, we have proved that there exists a dense orbit in X, where X is locally compact Hausdorff space and τ has a countable basis. The main results are the following propositions:Every topologically α-type transitive map is a topologically transitive map which implies topologically δ- transitive map, but the converse not necessarily true., and every α-minimal map is a minimal map which implies δ- minimal map in topological spaces, but the converse not necessarily true. Finally, we have proved that a map which is γr- conjugated to γ-transitive (resp. γ-minimal, γ-mixing) map is γ-transitive (resp. γ-minimal, γ-mixing).

Keywords: Topologically δ-Transitive, δ-Irresolute, δ-Type Transitive,δ-Dense, γ-Dense, γ Transitive

1. Introduction

Let A be a subset of a topological space (X, τ). The closure and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is said to be regular open [1] (resp. preopen [2]) if A = Int(Cl(A)) (resp. A ⊂Int(Cl(A))). A set A⊂ X is said to be δ-open [3] if it is the union of regular open sets of a space X. The complement of a regular open (resp. δ-open) set is called regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure [3] of A and is denoted by $Cl_{\delta}(A)$.. Recall that a set S is called regular closed if $S = Cl(Int(S))$. A point x ε X is called a δ-cluster point [3] of S if $S \cap U \neq \phi$ for each regular open set U containing x. The set of all δ-cluster points of S is called the δ-closure of S and is denoted by $Cl_{\delta}(S)$ .A subset S is called δ-closed if δCl(S) = S. The complement of a δ-closed set is called δ-open. The family of all δ-open sets of a space X is denoted by $\delta O(X,\tau)$..The δ-interior of S is denoted by $\mathrm{Int}_{\delta}(S)$ and it is defined as follows $\mathrm{Int}_{\delta}(S) = \{x \in X : x \in U \quad \subseteq \mathrm{Int}(Cl(U)) \subseteq S\}$ for some open set U of X.

The area of Dynamical Systems where one investigates dynamical properties that can be described in topological terms is called Topological Dynamics Let X be a compact topological space and let $f : X \to X$ be continuous. The pair (X, f) is so called topological system. The topological system (X, f) is called topologically δ-type transitive (or just δ-type transitive[4]) if for every pair of nonempty δ-open sets U and V in X there is a nonnegative integer n such that $f^n(U) \cap V \neq \phi$., If the space X has no isolated points, this is equivalent to the existence of a point x ε X whose orbit $O_f(x) = \{x, f(x), f^2(x), ...f^n(x),....\}$ is δ-dense in X. Consequently, a topologically δ-type transitive topological system cannot be decomposed into two disjoint sets with nonempty δ-interiors. For more information on topological δ-type transitivity see, e.g. [4] and references there.

In this paper, we will study some new class of topological transitive maps called topological δ-type transitive[4], also, we will study the relationship between two types of minimal mappings, namely, δ-minimal mapping and α-minimal mapping, and we will prove that the properties of δ-type transitive, δ-mixing and δ-minimal maps are preserved under δr-conjugacy and study some of its properties.

2. Preliminaries and Definitions

In this section, we recall some of the basic definitions. Let X be a space and $A \subset X$. The intersection (resp. closure) of A is denoted by Int(A) (resp. Cl(A).

Definition 2.1.Let (X,τ) be a space. A subset A of X is called dense in X if Cl(A)=X

Definition 2.2(i) A space X is said to be 2nd countable if it has a countable basis.

(ii) X is said to be of First Category if it is a countable union of nowhere dense subsets of X. It is of second Category if it is not of First Category.

Theorem 2.3Let X be a non-empty locally compact Hausdorff space. Then the intersection of a countable collection of open dense subsets of X is dense in X. Moreover, X is of second Category.

Definition 2.4Let (X,τ) be a topological space. X is second countable if and only if the topology of X has a countable basis.

Definition 2.5 Recall that a space X is said to be separable if X contains a countable dense subset.

Corollary 2.6 A subset A of a space (X,τ) is dense if and only if $A \cap U \neq \phi$ for all $U \in \tau$ other than $U = \phi$

Definition 2.7Let (X, τ) be a topological space , $f: X \to X$ be a continuous map then f is said to be topologically transitive if every pair of non-empty open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$..

The purpose of the following theorem is to prove that topological transitivity implies dense orbits in a space X where X is a non-empty locally compact Hausdorff topological space.

Theorem 2.8Let (X,f) be a topological system where X is a non-empty locally compact Hausdorff topological space *and* $f: X \to X$ *is a continuous map and that X is separable. Suppose that f is topologically transitive. Then there is* $x \in X$ *such that the orbit* $O_f(x) = \{x, f(x), f^2(x), ...f^n(x), ...\}$ *is dense in X.*

Proof: Let $B = \{U_i\}$ $i = 1, 2, 3, ...$ be a countable basis for the topology of *X*. For each *i*, let $O_i = \{x \in X : f^n(x) \in U_i$ for some $n \geq 0\}$

Then, clearly O_i is open and dense. It is open since *f is continuous, so,* $O_i = \bigcup_{i=1}^{\infty} f^{-1}(U_i)$ *is open and dense since f is topological transitive map. Further, for every open set V, there is a positive integer n such that* $f^n(V) \cap U_i \neq \phi$.

Now, apply theorem 2.3 to the countable dense sets $\{O_i\}$ to say that $\bigcap_{i=0}^{\infty} O_i$ is dense and so non-empty. Let $y \in \bigcap_{i=0}^{\infty} O_i$. This means that, for each i, there is a positive integer n such that $f^n(y) \in U_i$ for every i. Bycorollary 2.6 this implies that $O_f(x)$ is dense in X

Definition 2.9.If for $x \in X$ the set $\{f^n(x): n \in \mathbf{N}\}$ is dense in Xthenx is said to have a dense orbit. If there exists such an $x \in X$, then f is saidto have a dense orbit.

Definition2.10. A function $f: X \to X$ is called γr-homeomorphism if f is γ-irresolute bijective and $f^{-1}: X \to X$ is γ-irresolute.

Definition2.11 [19]Two topological systems $f: X \to X$, $x_{n+1} = f(x_n)$ and $g: Y \to Y$, $y_{n+1} = g(y_n)$ are said to be topologically γr-conjugate if there is γr-homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$ *(i.e. h(f(x)) = g(h(x)))*. We will call h a topological γr-conjugacy.

Remark 2.12[19] If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{ y_0 = h(x_0), y_1 = h(x_1), y_2 = h(x_2), .. \}$ yields an orbit of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n) = g(y_n)$. In particular, h maps periodic orbits of f onto periodic orbits of g .

In [19], we introduced and defined the new type of transitive called γ-type transitive in such a way that it is preserved under topologically γr- conjugation. It means; we have proved that a map which is γr- conjugated to γ-transitive (resp. γ-minimal, γ-mixing) map is γ-transitive (resp.γ-minimal, γ-mixing).

We proceed to prove the following important proposition:

Proposition 2.13 [19] Let (X,f) *and* (Y,g) be two topological systems, if $f: X \to X$ and $g: Y \to Y$ are topologically γr-conjugate. Then

(1) f is topologically γ- transitive if and only if g is topologically γ-transitive;

(2) f is γ-minimal if and only if g is γ-minimal;

(3) f is topologically γ-mixing if and only if g is topologically γ-mixing.

Proof (1)

Assume that $f: X \to X$ *and* $g: Y \to Y$ are topologically γr-conjugated by $h: X \to Y$. Suppose f is γ-type transitive. Let A, B be γ-open subsets of Y (*to show* $g^n(A) \cap B \neq \varphi$ *for some* $n > 0$).

$U = h^{-1}(A)$ *and* $V = h^{-1}(B)$ are γ–open subsets of X since h *is an* $\gamma - irresolute$

Then there exists some n>0 such *that* $f^n(U) \cap V \neq \varphi$ *since* f is γ–type transitive. Thus (as $f \circ h^{-1} = h^{-1} \circ g$ *implies* $f^n \circ h^{-1} = h^{-1} \circ g^n$),

$$\phi \neq f^n(h^{-1}(A)) \cap h^{-1}(B) = h^{-1}(g^n(A)) \cap h^{-1}(B)$$

Therefore, $h^{-1}(g^n(A) \cap B) \neq \phi$ implies $g^n(A) \cap B \neq \phi$ since h^{-1} *is invertible.*

Proof (2)

Assume that $f: X \to X$ *and* $g: Y \to Y$ are topological systems, which are topologically γr-conjugated by $h: Y \to X$. Thus, h is γr-homeomorphism (that is, h is bijective and thus invertible and both h and h^{-1} are γ-irresolute) and $h \circ g = f \circ h$, that is, the following diagram commutes:

$$\begin{array}{ccc} Y & \xrightarrow{g} & Y \\ h\downarrow & & \downarrow h \\ X & \xrightarrow[f]{} & X \end{array}$$

We show that ifgis γ-minimal, then f is γ-minimal. We want to show that for any $x \in X$, $O_f(x)$ *is* $\gamma - dense$. Since h is surjective, there exists x ϵX such that $y = h^{-1}(x)$. Since g isγ-minimal, $O_g(y)$ *is* $\gamma - dense$. For any non-empty γ- open subset U ofX, $h^{-1}(U)$ is an γ-open subset ofX since h^{-1} is γ-irresolute because the map h isγr-homeomorphism and it is non-empty since h is invertible map. By γ-density of $O_g(y)$ there exist k in N such that $g^k(y) \in h^{-1}(U) \Leftrightarrow h(g^k(y)) \in U$

Since h is γr-conjugacy; as $f \circ h = h \circ g$ implies $f^k \circ h = h \circ g^k$ *so* $f^k(h(y)) = h(g^k(y)) \in U$ thus $O_f(h(y))$ intersects U. This holds for any non-empty γ -open set U and thus shows that $O_f(x) = O_f(h(y))$ is γ-dense

Proof (3)

We only prove that if g is topologically γ-mixing then f is also topologically γ-mixing. Let U, V be two γ-open subsets of X. We have to show that there is N>0 such that for any n>N, $f^n(U) \cap V \neq \phi$.

$h^{-1}(U)$ and $h^{-1}(V)$ are two γ-open sets since the map h is γ-irresolute. If gis topologically γ-mixing then there is N >0 such that for any n>M, $g^n(h^{-1}(U)) \cap h^{-1}(V) \neq \phi$. Therefore thereexits $x \in g^n(h^{-1}(U)) \cap h^{-1}(V)$. That is, $x \in g^n(h^{-1}(U))$ *and* $x \in h^{-1}(V)$ if and only if $x = g^n(y)$ *for* $y \in h^{-1}(U)$ *and* $h(x) \in V$.

Thus, since $h \circ g^n = f^n \circ h$,sothat, $h(x) = h(g^n(y) = f^n(h(y)) \in f^n(U)$ and we have $h(x) \in V$ that is $f^n(U) \cap V \neq \phi$.

So, f is γ-mixing

3. Transitive and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system (X, f) [15] we mean a topological space X together with a continuous map $f: X \to X$. The space X is sometimes called the phase spaceof the system. A set $A \subseteq X$ is called $f - inveriant$ if $f(A) \subseteq A$.

A topological system (X, f) is called *minimal* if X does not contain any non-empty, proper, closed $f - inveriant$ subset. In such a case we also say that the map f itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them. Given a point x in X, $O_f(x) = \{x, f(x), f^2(x), ...\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its ω -limit set, i.e. the set of limit points of the sequence . $x, f(x), f^2(x), ...$ The following conditions are equivalent:

- (X, f) is α-minimal (resp. θ-minimal),
- every orbit is α-dense (resp. θ-dense) in X,
- $\omega_f(x) = X$ for every x ∈ X.

A minimal map f is necessarily surjective if X is assumed to be Hausdorff and compact.

Now, we will study the Existence of minimal sets. Given a dynamical system (X, f) ,a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed and invariant and if no proper subset of A has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f).

Let (X, f) be a topological system, and $f: X \to X$ αr-homeomorphism of X onto itself. For A and B subsets of X, we let $N(A, B) = \{n \in \mathbf{Z} : f^n(A) \cap B \neq \phi\}$

We write $N(A, B) = N(x, B)$ for a singleton $A = \{x\}$ thus $N(x, B) = \{n \in \mathbf{Z} : f^n(x) \in B\}$

For a point $x \in X$ we write $O_f(x) = \{f^n(x) : n \in \mathbf{Z}\}$ for the orbit of x and $Cl_\alpha(O_f(x))$ for the α-closure of $O_f(x)$. We say that the topological system (X, f) is α-type point transitive if there is a point $x \in X$ with $O_f(x)$ α-dense. Such a point is called α-type transitive. We say that the topological systems (X, f) is topologically α-type transitive (or just α-type transitive) if the set $N(U, V)$ is nonempty for every pair U and V of nonempty α-open subsets of X.

3.1. Topologically α-Transitive Maps

In [11], we introduced and defined a new class of transitive maps that are called topologically α-transitive maps on a topological space (X, τ), and we studied some of their properties and proved some results associated with these new definitions. We also defined and introduced a new class of α-minimal maps. In this paper we discuss the relationship between topologically α-transitive maps and θ-transitive maps. On the other hand, we discuss the relationship between α-minimal and θ-minimal in topological systems.

Definition 3.1.1 Let (X, τ) be a topological space. A subset A of X is called α-dense in X if $Cl_\alpha(A) = X$.

Note that, in general topology, for any subset A of the space X, $A \subset Cl_\alpha(A) \subset Cl(A)$, therefore if A is α-dense, in X, then A is dense in X.

Remark 3.1.2 Any α-dense subset in X intersects any α-open set in X.

Proof: Let A be an α-dense subset in X, then by definition, $Cl_\alpha(A) = X$, and let U be a non-empty α-open set in X. Suppose that A∩U=ϕ. Therefore $B = U^c$ is α-closed and $A \subset U^c$ =B. So $Cl_\alpha(A) \subset Cl_\alpha(B)$, i.e. $Cl_\alpha(A) \subset$B, but $Cl_\alpha(A) = X$, so X⊂B, this contradicts that U ≠ φ

Definition 3.1.3 [12] A map $f: X \to Y$ is called α-irresolute if for every α-open set H of Y, $f^{-1}(H)$ is α-open in X.

Example 3.1.4 [11] Let (X, τ) be a topological space such that X={a, b, c, d} and τ ={ϕ, X ,{a, b}, {b}}. We have the set of all α-open sets is α(X, τ)={ϕ, X, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}}and the set of all α-closed sets is αC(X, τ)={ϕ, X, {c, d, {a, c, d}, {a, d}, {}a, c}, {d}, {c}}. Then define the map f : X→X as follows f(a)= a, f(b)= b, f(c)= d, f(d)= c, we have f is α-irresolute because {b} is α-open and f^{1}({b})={b} is α-open; {a, b} is α-open and f^{1}({a, b})={a, b} is α-open; {b, c} is α-open and f^{1}({b, c})={b, d} is α-open; {a, b, c} is α-open and f^{1}({a, b, c})={a, b, d} is α-open; {a, b, d} is α-open and f^{1}({a, b, d})={a, b, c} is α-open so f is α-irresolute.

Definition 3.1.5 A subset A of a topological space (X,τ) is said to be nowhere α-dense, if its α-closure has an empty α-interior, that is, $\operatorname{int}_\alpha(Cl_\alpha(A)) = \phi$.

Definition 3.1.6 [11] Let (X, τ) be a topological space , $f: X \to X$ be α-irresolute map then f is said to be topological α-transitive if every pair of non-empty α-open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$. In the forgoing example 3.1.4: we have f is α-transitive because b belongs to any non-empty α-open set V and also belongs to f(U) for any α-open set it means that f(U)∩V≠ ϕ so f is . α - transitive.

Example 3.1.7[11] Let (X, τ) be a topological space such that X ={a, b, c}and τ={ϕ, {a}, X}.Then the set of all α-open sets is τ^α={ϕ, {a}, {a, b}, {a, c}, X}. Define f: X→X as follows f(a)=b, f(b)=b, f(c)=c. Clearly f is continuous because {a} is open and f({a})=ϕ is open. Note that f is transitive because f({a})={b} implies that f({a})∩{b}≠ϕ. But f is not α-transitive because for each n in N , f^n({a})∩{a, c}=ϕ; since f^n({a})={b} for every n ε N, and {b}∩{a, c}=ϕ. So we have f is not α-transitive, so we show that transitivity not implies α-transitivity.

Definition 3.1.8 Let (X, τ) be a topological space. A subset A of X is called θ-dense in X if $Cl_\theta(A) = X$.

Remark 3.1.9 Any θ-dense subset in X intersects any θ-open set in X.

Proof: Let A be a θ-dense subset in X, then by definition, $Cl_\theta(A) = X$, and let U be a non-empty θ-open set in X. Suppose that A∩U=ϕ. Therefore $B = U^c$ is θ-closed because B is the complement of θ-open and $A \subset U^c$=B. So $Cl_\theta(A) \subset Cl_\theta(B)$, i.e. $Cl_\theta(A) \subset$B, but $Cl_\theta(A) = X$, so X⊂B, this contradicts that U ≠ φ

Definition 3.1.10.[14] A function $f: X \to X$ is called θ − irresolute if the inverse image of each θ − open set is a θ − open set in X.

Definition 3.1.11 A subset A of a topological space (X,τ) is said to be nowhere θ-dense, if its θ-closure has an empty θ-interior, that is, $\operatorname{int}_\theta(Cl_\theta(A)) = \phi$.

Definition 3.1.12 [15] Let (X, τ) be a topological space, and $f: X \to X$ θ.-irresolute) map, then f is said to be topologically θ-type transitive map if for every pair of θ-open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$

Associated with this new definition we can prove the following new theorem.

Theorem 3.1.13 [11]: Let (X, τ) be a topological space and $f: X \to X$ be α -irresolute map. Then the following statements are equivalent:

(1) f is topological α-transitive map

(2) For every nonempty α-open set U in X, $\bigcup_{n=0}^{\infty} f^n(U)$ is α-dense in X

(3) For every nonempty α-open set U in X, $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is α-dense in X

(4) If $B \subset X$ is α-closed and B is f- invariant i.e. $f(B) \subset B$. then B=X or B is nowhere α-dense.

(5) If U is α-open and $f^{-1}(U) \subset U$ then U is either empty set or α-dense in X.

Theorem 3.1.14:[4] Let (X, τ) be a topological space and $f: X \to X$ be θ -irresolute map. Then the following statements are equivalent:

(1) f is θ-type transitive map

(2) $\bigcup_{n=0}^{\infty} f^n(D)$ is θ-dense in X, with D is θ-open set in X.

(3) $\bigcup_{n=0}^{\infty} f^{-n}(D)$ is θ-dense in X with D is θ-open set in X

(4) If $B \subset X$ is θ-closed and $f(B) \subset B$. then B=X or B is nowhere θ-dense

(5) If $f^{-1}(D) \subset D$ and D is θ-open in X then D=ϕ or D is θ-dense in X.

4. Minimal Functions

Weintroduced a new definition on α-minimal[11] (resp. δ-minimal[4]) maps and studied some new theorems associated with these definitions.

Given a topological space X, we ask whether there exists α-irresolute (resp. θ-irresolute) map on X such that the set $\{f^n(x): n \geq 0\}$, called the orbit of x and denoted by $O_f(x)$, is α-dense(resp. δ-dense) in X for each x ϵ X.. A partial answer will be given in this section. Let us begin with a new definition.

Definition 4.1 (α-minimal) Let X be a topological space and f be α-irresolute map on X with α-regular operator associated with the topology on X. Then the dynamical system (X, f) is called α-minimal system (or f is called α-minimal map on X) if one of the three equivalent conditions hold[11]:

1) The orbit of each point of X is α-dense in X.
2) $Cl_\alpha(O_f(x)) = X$ for each x ϵ X
3) Given x ϵ X and a nonempty α-open U in X, there exists n ϵ N such that $f^n(x) \in U$

A system *(X, f)*is called δ-*minimal* if X does not containany non-empty, proper, δ - closed f -invariant subset. In sucha case we also say that the map f itself is δ-minimal. Anotherdefinition of minimal function is that if the orbit of every pointx in X is dense in X then the map f is said to be minimal.

Theorem 4.2[4] For (X, f) the following statements are equivalent:

(1) f is an δ-minimal map.

(2) If E is an δ-closed subset of X with $f(E) \subset E$, we say E is invariant. Then E=ϕ or E=X.

(3) If U is a nonempty δ-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

5. Topological Systems and Conjugacy

Definition 5.1[4]Amap $h: Y \to X$ is said to be δr-homeomorphism if h is bijective and thus invertible and both h and h^{-1} are δr-irresolute

Definition5.2 Let (X, f) and (Y, g) be topological systems, then $f: X \to X$ and $g: Y \to Y$ are said to be topologically δr-conjugate if there is δr- homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$. We will call h a topological δr-conjugacy. Thus, the two topological systems with their respective function acting on them share the same dynamics

Associated with these definitions we have the following theorem:

Theorem 5.3 [4] Let (X, f) *and* (Y, g) be two systems, if $f: X \to X$ *and* $g: Y \to Y$ are topologically δr-conjugate. Then

(1) f is topologically δ-transitive if and only if g is topologically δ-transitive;

(2) f is δ-minimal if and only if g is δ-minimal;

(3) f is topologically δ-mixing if and only if g is topologically δ-mixing.

6. Conclusion

The main results are the following:

*Proposition 6.1*Every topologically α-type transitive map is a topologically transitive map which implies topologically δ- transitive map, but the converse not necessarily true..

*Proposition 6.2*Every α-minimal map is a minimal map which implies δ- minimal map in topological spaces, but the converse not necessarily true.

*Theorem 6.3*Let (X, f) be a topological system where X is a non-empty locally compact Hausdorff topological spaceand X is separable. Suppose that f is topologically transitive. Then there is $x \in X$ such that the orbit $O_f(x) = \{x, f(x), f^2(x), ...f^n(x),.....\}$ is dense in X.

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