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Analysis of Cognitive Radio Capacity in Fading Channels

Mohan Premkumar, Muthappa Perumal Chitra

Department of Electronics and Communication Engineering, Panimalar Institute of Technology, Chennai, India

Email address:
ermpremkumar@yahoo.co.in (M. Premkumar), chi_mp2003@yahoo.co.in (M. P. Chitra)

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Abstract: This research paper is intended to derive, simulate and analyze the capacity of cognitive radio (CR) system in fading channels. Capacity being an information theoretic perspective for a wireless system plays a role in the amount of information bits which can be transmitted. As wireless channel is subjected to multipath effects in a CR system caused by fading, capacity analysis needs to be done. Simulation carried out in terms of capacity, mean square error for channel estimation and bit error rate (BER) for the proposed system model of cognitive radio can give valuable information about performance of CR system in terms of data bits handling capacity. The amount of capacity in flat fading channels and frequency selective fading channels are simulated. The obtained results in terms of capacity can be used as reference for further analysis to be explored relating to design of cognitive radio systems for developing applications for 5G systems.

Keywords: Capacity, Cognitive Radio, Fading Channels, Channel Estimation, Detection

1. Introduction

As wireless technologies are marching towards 5G, necessity of enormous amounts of data rates for wireless applications emerging due to day to day requirements for human mankind, signal information transmission and reception in any frequency spectrum gains significance. Frequency spectrum allocation is done for all applications starting from low frequency to high frequency ranges but it is not used at all times. Hence, to utilize frequency spectrum effectively and efficiently a possible solution is the well known Mitola's Radio [1] and [2] popularly coined as cognitive radio (CR). Cognitive radio embedded with functionalities of sensing, adaptation, and intelligence provides efficient and effective information transfer between a transmitter in the form of a secondary user I and a receiver as secondary user II. Attributes of CR namely efficiency and effectiveness are convened by its ability to limit interference to primary users and sense the idleness of the frequency spectrum not used by primary users who have the license for operation. CR operations for wireless applications are mainly subjected to issues such as spectrum sensing, channel state information (CSI) and interference management [3]. However, spectrum sensing, channel state information, interference management have been investigated by many researchers in their research work, this paper enlists the capacity analysis of cognitive radio in wireless channels which can provide a front runner for analyzing spectral efficiency of cognitive radio systems.

Capacity analysis of a cognitive radio system provides an information theoretic limit for transmitting maximum amount of information irrespective of the wireless channel which it experiences. Wireless channels can be either flat fading or frequency selective fading and analysis of capacity of cognitive radio can give information relating to data detection thereby contributing to proper spectrum utilization. So, this research paper will be an add-on to the list of literatures relating to capacity of cognitive radio systems given by [4]-[10].

In the work of [4] capacity analysis of a MIMO CR is done by considering average received interference power constraints where minimum distance was considered along with primary users. Derivation of secondary user mean capacity of a cognitive radio system for primary and secondary link is presented with imperfect channel state information (CSI) in [5]. Capacity of a hybrid CR network is given at the level of the link and also at the level of the system [6]. Investigations on channel information for secondary user and primary user relating to outage capacity with average transmit power constraints is dealt in the research of considering Rayleigh channels [7]. Ergodic capacity and outage capacity for spectrum sharing relay network [8] is simulated with best relay selection in the primary link, where it infers that with increasing number of relays improvement in secondary network performance is obtained. Research paper [9] shows capacity of multiple access channel in a cognitive radio system under hyper fading channel, Gaussian fading and Nakagami fading channels along with number of secondary users. Acheivable rates in the form of spectral efficiency for cognitive radio systems are given where correlated fading channels are considered with imperfect channel sensing [10] and [11]. All of the above works [4]-[10] considered either instantaneous capacity, ergodic capacity or outage capacity in perfect CSI or imperfect CSI situtations.

However, a research paper considering imperfections due to channel estimation errors considering least squares (LS) [11], and linear minimum mean square error (LMMSE) algorithms for cognitive radio systems for capacity analysis can contribute significantly to the researchers working in cognitive radio domain. As aforementioned, derivation of capacity of a cognitive radio system in wireless channels is done. Wireless channels can be slow flat fading channels or slow frequency selective fading channels with perfect channel state information. Capacity analysis in channels also contribute to throughput analysis of networks [12].

Pertaining to imperfect channel estimates [13] LS and LMMSE algorithms are considered where least squares reduce the estimation error linearly and linear minimum mean square error reduces estimation error based on probability density function (PDF). Simulation results give information on the amount of achievable rates or capacity in bits/second or spectral efficiency in bits/sec/Hz for wireless channels. In another perspective throughput of networks can also be had based on capacity analysis [14] which could provide optimized performance for spectrum ulitilization through packets [15].

Outline of this paper is that section 1 gives the essential introduction about cognitive radio and requirement of capacity. Section 2 presents the system model for cognitive radio system in flat fading and frequency selective fading channels. Section 3 derives capacity of a cognitive radio (CR) channel in a wireless flat fading channel with methodology considering differential entropy and conditional entropy through mutual information statistics. Section 4 presents capacity of CR system with imperfect estimates in a wireless channel. Section 5 presents simulation results for capacity analysis of cognitive radio along with mean square error and bit error rate. Conclusion of the paper is given in section 6.

2. System Model

The system model for cognitive radio considers communication between primary transmitter and a primary receiver operating in the licensed frequency spectrum and the secondary transmitter and secondary receiver operate in unlicensed manner. Cognitive users namely secondary users operate by providing no interference to primary users by spectrum sensing concept. Single Input Single Output (SISO) model is considered which infers a single antenna is present in all the terminals relating to the cognitive system model.

A secondary transmitter after acquiring the frequency spectrum of primary users through sensing, if it intends to transmit a $l\times 1$ data vector d undergoing binary phase shift keying (BPSK), the $l\times 1$ received signal vector $\mathbf{r}_{cog}$ at the primary receiver takes the representation

$$\mathbf{r}_{cog} = h_{cog}\mathbf{d} + \mathbf{n}_{cog} \tag{1}$$

where h_{cog} is the scalar wireless channel experiencing flat fading undergoing Rayleigh distribution, $\mathbf{n}_{cog}$ is the $l\times 1$ noise vector at the primary receiver and it is a complex Gaussian noise vector $N(\mu,\sigma^2)$ having a mean μ and a variance of σ^2. Similarly a CR system in frequency selective fading channel is represented as

$$\mathbf{r}_{cog}[n] = h_{cog}(t;\tau)\mathbf{d}[n] + \mathbf{n}_{cog} \tag{2}$$

where $h_{cog}(t;\tau)$ represents the channel with time delay represented by using a multipath power delay profile.

3. Derivation of Capacity of Cognitive Radio System

As a starting point to derive the capacity of CR system the methodology for capacity can be defined as maximization of mutual information

$$C = \max_{f(d)} \left| I(\mathbf{d};\mathbf{r}_{cog}) \right| \tag{3}$$

where $I(\mathbf{d};\mathbf{r}_{cog})$ is mutual information and it is given as

$$I(\mathbf{d};\mathbf{r}_{cog}) = h(\mathbf{r}_{cog}) - h(\mathbf{r}_{cog}/\mathbf{d}) \tag{4}$$

where $h(\mathbf{r}_{cog})$ is differential entropy of the received signal vector; $h(\mathbf{r}_{cog}/\mathbf{d})$ is conditional entropy or noise entropy. The differential entropy is given as

$$h(\mathbf{r}_{cog}) = \frac{1}{2}\log_2 2\Pi e\sigma^2_{\mathbf{r}_{cog}} \tag{5}$$

${\sigma_{\mathbf{r}_{cog}}}^2$ is variance of received signal vector given by; $\sigma^2_{\mathbf{r}_{cog}} = E[\mathbf{r}_{cog}\mathbf{r}^H_{cog}]$; where *H*-refers to Hermitian which is complex conjugate and it is expanded as $\sigma^2_{\mathbf{r}_{cog}} = E[(h_{cog}\mathbf{d} + \mathbf{n}_{cog})(h_{cog}\mathbf{d} + \mathbf{n}_{cog})^H]$ and further it reaches to

$$\sigma^2_{\mathbf{r}_{cog}} = E[(h_{cog}\mathbf{d} + \mathbf{n}_{cog})(\mathbf{d}^H h_{cog} + \mathbf{n}^H_{cog})] \tag{6}$$

$${\sigma_{\mathbf{r}_{cog}}}^2 = E\left[h_{cog}\mathbf{d}\mathbf{d}^H h^H_{cog} + h_{cog}\mathbf{d}\mathbf{n}^H_{cog} + \mathbf{n}_{cog}\mathbf{d}^H\mathbf{n}^H_{cog} + \mathbf{n}_{cog}\mathbf{n}^H_{cog}\right] \tag{7}$$

$${\sigma_{\mathbf{r}\,cog}}^2 = E\left[h_{cog}\mathbf{d}\mathbf{d}^H h^H_{cog}\right] + E\left[h_{cog}\mathbf{d}\mathbf{n}^H_{cog}\right] + E\left[\mathbf{n}_{cog}\mathbf{d}^H\mathbf{n}^H_{cog}\right] + E\left[\mathbf{n}_{cog}\mathbf{n}^H_{cog}\right] \tag{8}$$

where second and third terms are zero as cross correlation is independent and hence it is zero. Hence, only the first and fourth terms are significant and it is

$$\sigma^2_{\mathbf{r}_{cog}} = \mathbf{d}h_{cog}h^H_{cog}\mathbf{d}^H + E[\mathbf{n}_{cog}\mathbf{n}^H_{cog}] \tag{9}$$

d-data; h_{cog} -channel which is flat fading channel or frequency selective channel. and $\mathbf{R}_{n_{cog}}$ is noise correlation matrix, which is random and follows Gaussian distribution

$$h(\mathbf{r}_{cog}) = \frac{1}{2}\log_2 2\Pi e[\mathbf{d}h_{cog}h^H_{cog}\mathbf{d}^H + \mathbf{R}_{n_{cog}}] \tag{10}$$

On the other hand conditional entropy is

$$h(\mathbf{r}_{cog}) = \frac{1}{2}\log_2 2\Pi e[\mathbf{R}_{\mathbf{n}_{cog}}] \tag{11}$$

Further mutual information is written as by substituting (10) and (11) in (4)

$$I(\mathbf{d} : \mathbf{r}_{cog}) = \frac{1}{2}\log_2 2\Pi e\left[\mathbf{d}h_{cog}h^H_{cog}\mathbf{d}^H + R_{n_{cog}}\right] - \frac{1}{2}\log_2 2\Pi e\left[\mathbf{R}_{n_{cog}}\right] \tag{12}$$

Continuing by using representation $\log(A/B) = \log A - \log B$

$$I(\mathbf{d};\mathbf{r}_{cog}) = \frac{1}{2}\log_2 \frac{2\pi e}{2\pi e}\left(\frac{\mathbf{d}h_{cog}h^H_{cog}\mathbf{d}^H + \mathbf{R}_{\mathbf{n}_{cog}}}{\mathbf{R}_{\mathbf{n}_{cog}}}\right) \tag{13}$$

$$I(\mathbf{d};\mathbf{r}_{cog}) = \frac{1}{2}\log_2 \frac{2\pi e}{2\pi e}\left(\frac{\mathbf{d}h_{cog}h^H_{og}\mathbf{d}^H +}{\mathbf{R}_{\mathbf{n}_{cog}}} + \frac{\mathbf{R}_{\mathbf{n}_{cog}}}{\mathbf{R}_{\mathbf{n}_{cog}}}\right) \tag{14}$$

The mutual information on simplification of (14) is

$$I(\mathbf{d} : \mathbf{r}_{cog}) = \frac{1}{2}\log_2\left(\mathbf{d}h_{cog}h^H_{cog}\mathbf{d}^H\mathbf{R}_{n_{cog}}{}^{-1} + 1\right) \tag{15}$$

The capacity of cognitive radio system in wireless fading channel is obtained by substitution (15) in (3) and it is given as

$$C = \max_{f_d(d)}\left|\frac{1}{2}\log_2(\mathbf{I} + \mathbf{d}h_{cog}h^H_{cog}\mathbf{d}^H\mathbf{R}_{n_{cog}}{}^{-1})\right| \tag{16}$$

4. Capacity of Cognitive Radio with Imperfect Channel Estimates

The $l\times 1$ received signal vector at the primary receiver with imperfect channel estimates can be formulated as

$$\mathbf{r}_{impcog} = \mathbf{d}\,(\hat{h}_{cog} + e_{rcog}) + \mathbf{n}_{cog} \tag{17}$$

where $\hat{h}_{cogLS}$ represents wireless channel obtained by LS channel estimation [11] algorithm e_{rcog} is the scalar error value which takes the representation

$$e_{rcog} = h_{cog} - \hat{h}_{cogLS} \tag{18}$$

where values of error are complex Gaussian having zero mean and variance $\sigma^2_{e_{rcog}}$. The capacity of CR system with imperfect channel estimates expressed as

$$C_{impcog} = \max_{f_{\mathbf{d}}(\mathbf{d})}[I(\mathbf{d};\mathbf{r}_{impcog})] \tag{19}$$

where $I(\mathbf{d};\mathbf{r}_{impcog})$ is the average mutual information between **d** and $\mathbf{r}_{impcog}$. The mutual information of CR system with imperfect channel estimates in terms of differential entropy is is defined as

$$I(\mathbf{d};\mathbf{r}_{impcog}) = h(\mathbf{r}_{impcog}) - h\left(\mathbf{r}_{impcog}/\mathbf{d}\right) \tag{20}$$

where $h(\mathbf{r}_{impcog})$ refers to differential entropy of the received signal vector with imperfect channel estimates and $h\left(\mathbf{r}_{impcog}/\mathbf{d}\right)$ refers to conditional differential entropy with with imperfect channel estimates. Mathematically, differential entropy $h(\mathbf{r}_{impcog})$ is

$$h(\mathbf{r}_{impcog}) = \frac{1}{2}\log_2 2\pi e\left|\mathbf{R}_{\mathbf{r}_{impcog}}\right| \tag{21}$$

where $\mathbf{R}_{\mathbf{r}_{impcog}}$ is the $L\times L$ correlation matrix of the $l\times 1$ received signal vector with imperfect channel estimates at the primary receiver with representation

$$\mathbf{R}_{r_{impcog}} = E[\mathbf{r}_{impcog}\mathbf{r}^H_{impcog}] \tag{22}$$

Substituting (17), (22) takes the form

$$E[\mathbf{r}_{impcog}\mathbf{r}_{impcog}{}^H] = E[(\mathbf{d}(\hat{h}_{cogLS} + e_{rcog}) + \mathbf{n}_{cog}) \\ (\mathbf{d}(\hat{h}_{cog} + e_{rcog}) + \mathbf{n}_{cog})^H] \tag{23}$$

As noise vector, wireless channel exhibit independency to each other (23) is simplified as

$$E[\mathbf{r}_{impcog}\mathbf{r}_{impcog}{}^H] = \hat{h}_{cogLS}\mathbf{d}\mathbf{d}^H\hat{h}^H_{cogLS} + e_{rcog}\mathbf{d}\mathbf{d}^H e^H_{rcog} + \mathbf{R}_{ncog} \tag{24}$$

where $\mathbf{R}_{n_{cog}} = E[\mathbf{n}_{cog}\mathbf{n}^H_{cog}]$ is $L\times L$ covariance matrix of the noise vector. Further substituting (24) in (21) the differential entropy of the $L\times 1$ received signal vector takes the formulation

$$h\left(\mathbf{r}_{impcog}\right) = \frac{1}{2}\log_2 2\pi e\left|\hat{h}_{cogLS}\mathbf{d}\mathbf{d}^H\hat{h}^H_{cogLS} + e_{rcog}\mathbf{d}\mathbf{d}^H e^H_{rcog} + \mathbf{R}_{ncog}\right| \tag{25}$$

Further, conditional differential entropy $h(\mathbf{r}_{impcog}/\mathbf{d})$ is $h(\mathbf{n}_{cog})$. Due to $\mathbf{n}_{cog}$ representing Gaussian distribution, noise entropy $h(\mathbf{r}_{impcog}/\mathbf{d})$ is

$$h(\mathbf{r}_{impcog}/d) = h(\mathbf{n}_{cog}) = \frac{1}{2}\log_2 2\pi e\left|e_{rcog}\mathbf{dd}^h e_{rcog}^H + \mathbf{R}_{n_d}\right| \tag{26}$$

Further, mutual information can be written as

$$\begin{aligned} I(\mathbf{d}:\mathbf{r}_{impcog}) = \frac{1}{2}\log_2 2\Pi e\left[\hat{h}_{cogLS}\mathbf{dd}^H\hat{h}_{cogLS}^H + e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}\right] - \\ \frac{1}{2}\log_2 2\Pi e\left[e_{rcog}\mathbf{dd}^h e_{rcog}^H + \mathbf{R}_{n_d}\right] \end{aligned} \tag{27}$$

Further, on solving

$$I(\mathbf{d};\mathbf{r}_{cog}) = \frac{1}{2}\log_2\frac{2\pi e}{2\pi e}\left(\frac{\hat{h}_{cogLS}\mathbf{dd}^H\hat{h}_{cogLS}^H + e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}}{e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}} + \frac{e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}}{e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}}\right) \tag{28}$$

The mutual information on simplification of (28) is

$$\begin{aligned} I(\mathbf{d}:\mathbf{r}_{impcog}) = \frac{1}{2}\log_2((\hat{h}_{cogLS}\mathbf{dd}^H\hat{h}_{cogLS}^H + e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}) \\ (e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog}) \quad +1) \end{aligned} \tag{29}$$

The capacity of cognitive radio system with imperfect channel estimates is obtained by substituting (29) in (19) and it is given as

$$C_{impcog} = \max_{f_d(d)}\left|\frac{1}{2}\log_2(\mathbf{I} + (\hat{h}_{cogLS}\mathbf{dd}^H\hat{h}_{cogLS}^H + e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog})(e_{rcog}\mathbf{dd}^H e_{rcog}^H + \mathbf{R}_{ncog})\right| \tag{30}$$

5. Simulation Results

In this section, simulation results for capacity against signal to noise ratio in decibels are plotted for flat fading channels following Rayleigh distribution and frequency selective fading channels. Capacity obtained is for the information transmitted between secondary transmitter the cognitive user and primary receiver.

In Figure 1, perfect CSI scenario of flat fading channel has a capacity of 0.25 bits/sec obtained for a signal to noise ratio (SNR) value of 10 dB. Whereas in frequency selective fading channel with multipath power delay profile the capacity value is 0.02 bits/sec at the same 10 dB. Flat fading channel has uniform attenuation in all frequencies but frequency selective fading channel has different attenuation (magnitude) in different frequencies based on the multipath power delay profile hence there is a reduction in capacity in frequency selective fading channel comparing to flat fading channel. Capacity refers to the instantaneous capacity of the cognitive radio system.

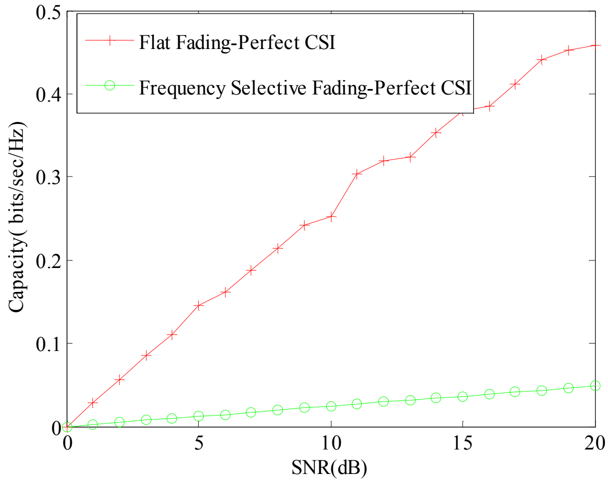

Figure 1. *Capacity vs SNR (dB) in wireless channels in Perfect CSI scenario.*

Table 1. *Capacity Vs SNR (dB) in wireless channels.*

SNR (dB)	Flat fading channel	Frequency Selective fading channel
5 dB	0.14 bits/sec	0.01 bits/sec
10 dB	0.25 bits/sec	0.025 bits/sec
15dB	0.38 bits/sec	0.038 bits/sec
20dB	0.45 bits/sec	0.049 bits/sec

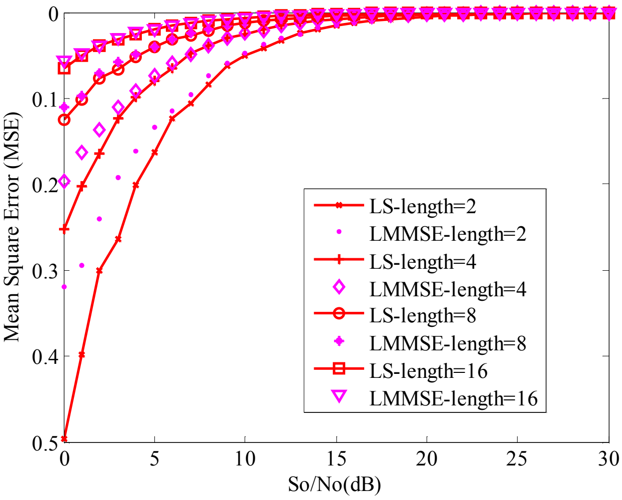

Figure 2. *Mean Square Error Vs Signal to Noise Ratio.*

Figure 2, shows mean square error against signal to noise ratio in dB for various training sequence lengths [11] for least squares and linear minimum mean square error (LMMSE) estimators. As the signal to noise ratio increases mean square error reduces and reaches to zero for both LS and LMMSE estimators. The LS and LMMSE estimators represent imperfect channel estimates for cognitive radio systems based on which simulation results of capacity and bit error rate could be obtained in flat fading and frequency selective fading channels.

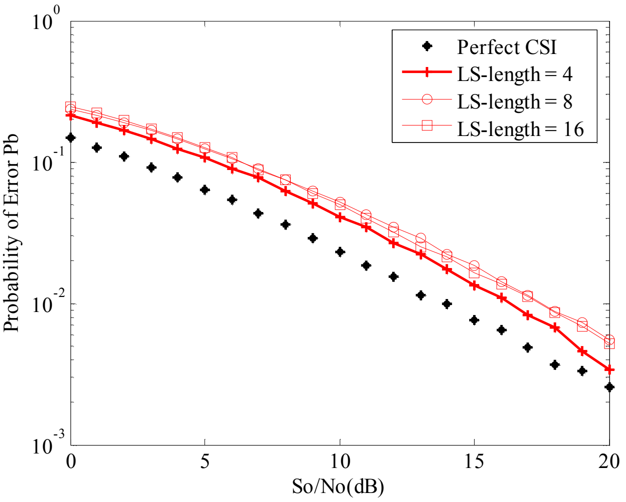

Figure 3. *Probability of Error Vs Signal to Noise Ratio.*

Figure 3, shows probability of error vs signal to noise ratio in dB for least squares algorithm for various sequence length values. When the signal to noise ratio value increases there lies a possibility of imperfect channel state estimates obtained through LS algorithm can reach perfect channel state information (CSI). This sort of a graph can give an information for design of cognitive radio systems in flat fading channels.

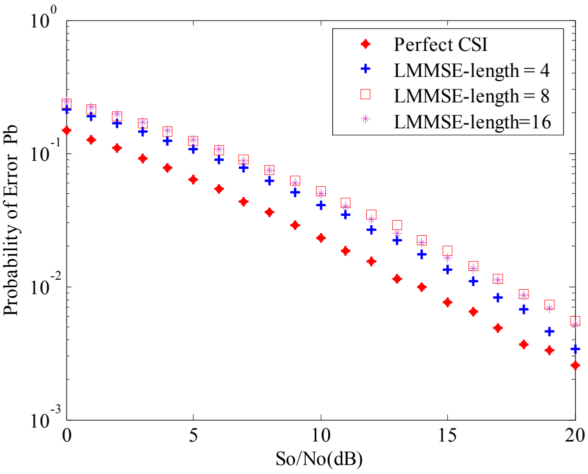

Figure 4. *Probability of Error Vs Signal to Noise Ratio.*

Figure 4, shows probability of error against signal to noise ratio in dB for various length of sequences with comparison against perfect channel state information. LMMSE algorithm represents the imperfect channel state estimates and it will tend to reach the perfect CSI state when signal to noise ratio increases for the considered cognitive radio system model. This bit error rate analysis in imperfect channel state estimates will help in data detection for designing and developing cognitive radio systems and its related applications. Detection performance such as probability of detection and probability of false alarm can also be considered as an extension to above research findings presented.

6. Conclusion

Concluding remarks for this paper suggests that analysis is done for cognitive radio in fading channels namely flat fading and frequency selective fading channels. Flat fading channel following Rayleigh distribution shows better performance in terms of capacity than frequency selective fading channel based on multipath power delay profile. CR system performance in fading channel with perfect and imperfect channel estimates obtained through LS and LMMSE algorithms gives an insight into how data is transmitted and received in imperfect situations. This paper on the whole can provide lending support to researchers working in cognitive radio domain to carry out design process and applications relating to cognitive radio systems which will be helpful to design 5G systems.

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