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# Localization through compressive sensing: A survey

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**Abstract:** User mobile device or for wireless node detection localization is a primary concern not only in normal days but especially during emergency situations. There is variety of useful and necessary applications related to localization and it is an important technology playing critical role in wireless communication. The conceptual point of view is to sense the localization (coordinates of the user) from a specific region of interest (ROI). For reducing the complexity and increasing efficiency, the data samples for location sensing is limited in a term of taking sparsity of the detected signal in known transformed domain by taking fewer data samples. This whole phenomenon is called compressive sensing. This paper introduces this technology especially in location-sensing and discusses the present techniques.

**Keywords:** Cognitive Radio, Localization, Mobile Networks, Wireless Networks, Sparsity, Compressive Sensing, Signal Detection

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## 1. Introduction

In mobile and wireless network architecture location and mobility management have been an important factor for many good reasons such as giving on time rescue services, in case of GPS; in none availability of satellite plane of view directing users to their destination and many other useful applications. Location-sensing or localization is an automatic means of position determination for the user through signal detection from their devices. Efficient location-sensing require sampling of fewer data blocks from received signal and in many cases continuous signal is not received, fewer or interrupted signal has been detected. From these few samples based on sparsity technique location estimation is performed.

This paper discusses different present techniques for localization of user through compressed sensing. Localization has gained its popularity in many domains including mobile ad-hoc and vehicular networks, robotics and Public Protection and Disaster Relief (PPDR) communication system. There are surveys [1][37] purely based on location-sensing techniques through trilateration methods, none at the moment were related to compressive-sensing for localization of user nodes.

The outline of this survey is as follows: Section 2 discusses the main challenges and parameters for accurate localization of the user. Also what were the drawbacks of non-compressive techniques previously used for location-

sensing. Later in the section 3, compressive sensing is explained and the re-formulation of the location parameters in form of sparse values is explained. In section 4 focus will be on the effective algorithms for localization explaining the sparse techniques and recent developments to perform efficient localizations.

## 2. Localization Issues and Parameters

### 2.1. Linearizing Vs Non- Linear

For location estimation usually parameters are taken into 2 or 3D dimensional coordinates. In cellular network location parameters are taken within the network without the aid from external resources such as GPS. Mostly UE is known in normal days and if not known the parameters are detected from within the network. Not like GPS, cellular network localization parameters are detected from limited region of interest (ROI). While detecting signals the nodes may be moving generating time-stamped measurements. These parameters may be in non-linear coordinates. These parameters should be combined to form a trajectory leading to the user location.

For accurate and sparse calculations (discussed in next section 3) all parameters are converted to linear parameters. Geometric methods can locate an object distance and measurements. Before sparsity was not introduced multiple dimensional coordinates were used. Following kinds of non-

linear parameters exists

- 1) Lateration-Single Dimension
- 2) Trilateration-2D
- 3) Multi-lateration-3D

Every UE exhibits three or more parameters. For deploying compressive sensing, an efficient and less complex computation requires to convert all parameters in lateration to mere approximation values. The description is illustrated in the following figure 1.

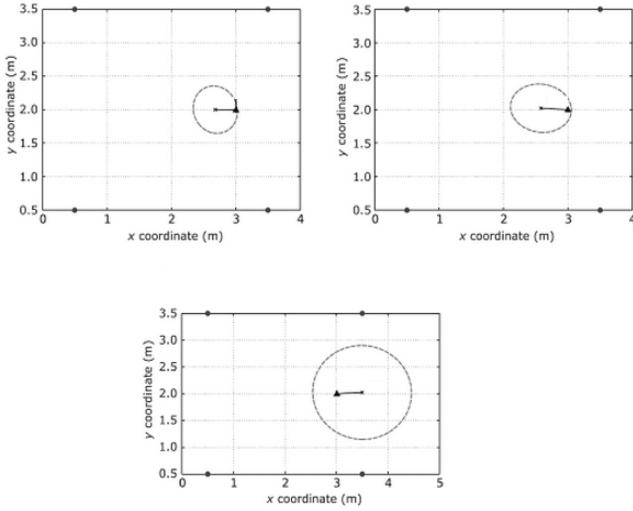


Figure 1. Location-sensing coordinates measurement

Location estimate in cellular/wireless networks are mostly processed through “RF finger prints” terminology. It is very similar to human finger printing. UE location-dependent signal parameters are extracted accordingly with their time-averages. There are two types of RF finger printing either reference or target [2]. The parameters obtained are unique set of geographic coordinates. A simple set of data matrix is shown below.

$$A = \begin{bmatrix} ID_1 & RSS_1 & RTD_1 \\ \dots & \dots & \dots \\ ID_{n-1} & RSS_{n-1} & RTD_{n-1} \end{bmatrix} \quad (2.1)$$

By using number of mathematical techniques such as Euclidean distance or Sum of Absolute Difference (SAD) these three dimensional values compute the distances with reference with the adjacent or known coordinate as shown in following equation

$$d_{i,j} = \sqrt{\sum_{m=1}^N \left( \left[ \frac{S_{i,j}(n_m, 2) - A(m, 2)}{\delta} \right]^2 \right)} \quad (2.2)$$

Here in above equation  $S_{i,j}$  is the N-dimensional RSS space. As discussed above for localization many non-linear parameters were considered and computed. As the paper concentrate on compressive sensing effectiveness for localization rather than using trilateral coordinate system, further in next section sparsity in compressive sensing

methodology is discussed in details.

### 3. Fundamentals of Compressive Sensing

#### 3.1. Sparse Representation

Taking the location parameters from region of interest (ROI) and re-formulating it in  $l$ -minimization matrix for compressive sensing on the data is termed as sparse representation. The reason to apply sparse transformation for location estimation is due to the in-efficiency of location-sensing technology those require computation on large amount of data that cost overhead to its management and require high budget for hardware and software. Re-formulation in compressive sensing provides fundamentally advance approach for cost-effective and time-consuming solutions.

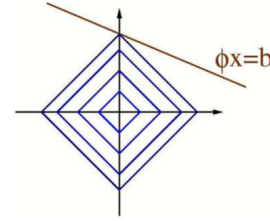


Figure 2. L-minimization [3]

By using fewer samples in linear domain compressive sensing implies sparsity. The explanation can be put forwarded as having a unknown signal vector  $\mathbf{x}^N$ , it is sampled using  $n$  functions for linearizing and later reconstruct it, where  $n < N$  when signal space is bigger than measurements. Nearly from mid-eighteen it has been researched that minimization on  $l$ -norm can recover sparse measurements as illustrated in above figure. Usually a sparse matrix derived from discrete-time domain signal is represented as follow

$$\Theta = \{\Omega_i\}_{i=1}^M \quad (3.1)$$

$$\Psi = \Theta e \quad (3.2)$$

The discrete signal is represented as  $S^M$  and  $e$  is  $M \times 1$  column vector of weighted product of co-efficient

$$e_i = \langle \Psi, \Omega_i \rangle = \Omega_i^T \Psi \quad (3.3)$$

The “T” symbol denotes transpose of the sparse vector. The above equation is the sparse representation of signal. Only the basis linear combination of  $k$  vectors are considered, meaning their values are the most significant such as

$$\begin{aligned} \text{if } e_i &\Rightarrow k \neq 0 \\ \text{then } M - k &= 0 \quad (k < M) \end{aligned} \quad (3.4)$$

$$M = (k \log N / k) \quad (3.5)$$

Where  $e_i$  is the linear projection of  $M$  signal, having  $N$  as intermediate acquiring samples. In a matrix if most of the

elements are non-zero then the matrix or vector is considered dense not sparse matrix. Transform coding is successfully processed on the data samples those are k-sparse signals through compressive sensing. This framework is considered incoherent and represented as sparse representation. There exists number of different techniques for sparsing the data such as wavelet transformation, Logan phenomena, Lasso, the matching pursuit and least absolute shrinkage. Using not all signal samples but only few intervals is actually sparsity of signal where the sample is most weighted one.

### 3.2. Compressive Sensing vs Data Compression

There are two types of compression lossless and lossy. Compression sensing and data compression are two very different technologies. Before discussing compressive sensing in detail, the difference between two techniques should be well cleared. Data compression is a methodology of discarding and reducing data for increasing bit storage. There are number of different models and coding techniques for performing data compression.

Compressive sensing (CS) is very similar to transform coding, involving large amount of data. Transforming code process input signals into dense form of high dimensional space. The signal is sampled into sparsity form in a known transform domain. By sparsity it is meant, the matrix having samples of most weighted coefficients of a received signal that through transformation becomes zero.

### 3.3. Spatial Sparsity in ROI

Incoherency and sparsity are the two main pillars on which CS relies. High-dimensional signals especially trilateral coordinates for localization can easily be presented using few small set of variables and co-efficient through sparsity as shown in the figure 3.

## 4. Present Techniques

This section discusses present algorithms and methodologies for localization of user nodes through compressive sensing. As discussed in above sections the significance of sparsity theory over certain old techniques like FFT and Nyquist sampling theorem. In the following algorithm [5], a pair-wise distance measured matrix is derived by using sparsity. The central node only transmit small noisy compressive signal and a pair-wise matrix is constructed from those samples. CS uses l-minimization matrix to find pair-wise matrix through sparsing. By applying l-minimization algorithm, a sparse pair-wise distance matrix is reconstructed for learning locations of nodes. Suppose  $S_k \in ROI^n$  is a sparse matrix  $S$  having pair-wise distance values. In the matrix each value is a two dimensional location vector as expressed in equation 4.1.

$$S_k = [S_{k1}, S_{k2}, \dots, S_{kn}] \quad (4.1)$$

$$\|S_{ki}\|_0 \leq k, i \in \{1, 2, \dots, n\}$$

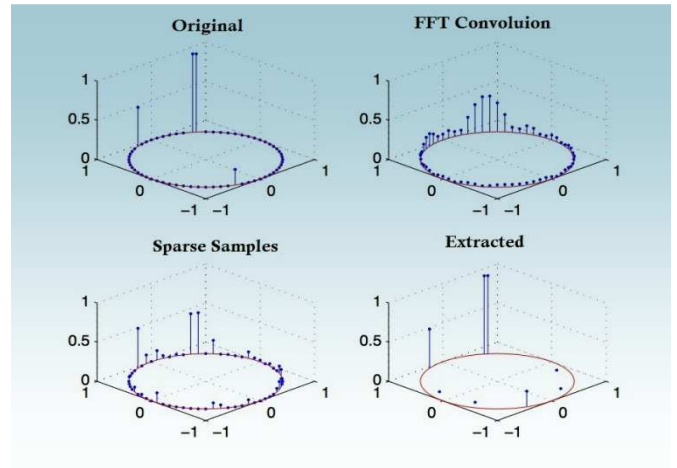


Figure 3. Sparse Data Samples

Further on three steps are performed on the matrix, in step 1, Floyd or Dijkstra path algorithm is applied on the values to recover sparse pair-wise values. In step 2, MDS algorithm is implied on the resultant matrix  $S'$ . The output from step 1 and 2 gives 3D relative coordinates of the nodes. Since this technique derive 3D coordinates for single node, the next techniques uses compressive sensing to derive the location of multiple points. The next popular technique [8] was evolved for missile launch system not for PPDR or public service schemes. The algorithm is very simple and straight forward by approaching the problem through Received Signal Strength (RSS) parameters. The RSS values are stored in a sparse matrix for pin-pointing the multiple location targets. The locations are then extracted from the sparse values through l-minimization matrix technique. Like previously discussed techniques that measured the k-sparse representations, instead RSS measurements in M-dimensional coordinates are measured accurately by convoluting with original received signal according to below equation

$$b = \Omega\Theta\xi + \epsilon \quad (4.2)$$

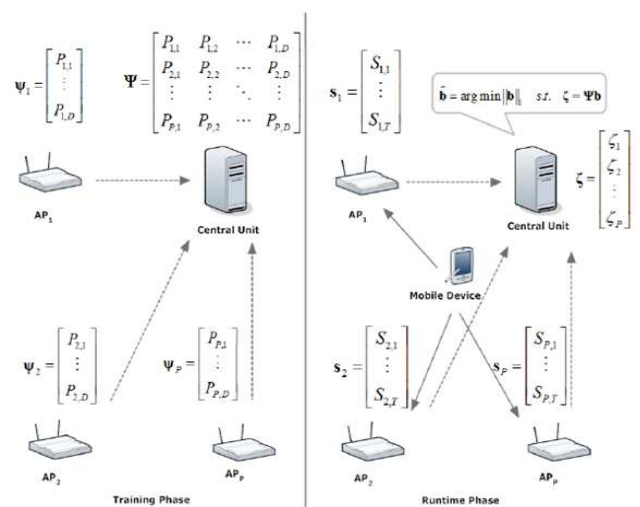


Figure 4. Coordinate Matrix [8]

Where  $\Omega$  is the sparse matrix having sparse coefficients  $\mathcal{E}$

A fixed power definition is specified through adopted channel model and according to the RSS matrix readings on the grid scale the location of the targets are estimated. The following algorithm [10] emphasize main concern on the distribution of the estimation for low-dimensional location coordinates. A projection matrix is specified that is incoherent with the sparse matrix. This algorithm is based on the spatial sparsity representation. If the received signal having of  $l$  length then  $l+2$  parameters would be required for the estimation of location points on the  $M$  target locations having  $k$  sparse samples containing amplitudes of source signals referred as "Localization via spatial sparsity". Algorithms [7][9], [10] is based on both previous techniques [4][5] discussed above. Extracting the received signal strength and plotting over  $k$  sparse spree, a simple illustration is shown in figure 4.

A new technique is proposed in algorithm [12] defined as Greedy Matching Pursuit "GMP". GMP is an algorithm similar to OMP and CoSAMP [39] algorithms that could offer much better performance in regard to the unknown target locations from a measured signal. By adopting target energy decay model [40], [41] the states of signal energy received at certain location for pointed target from another location  $j$  is approximated as:

$$C_{ij} = \frac{J_0 G_{ij}}{d_{ij}^\alpha} \quad (4.3)$$

Here  $J_0$  is the received signal intensity at  $i$ ,  $d_{ij}$  is the derived distance from Euclidean formula between the know target location  $i$  with the required / estimated target location  $j$ ,  $G_{ij}$  holds the Raleigh fading for the received target signal. Sparsity is implied on the resultant matrix, after getting sparse representation points on the grid, energy of the target signal will be highest where there are most of the targets resides.

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