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Adjusting Bookmaker's Odds to Allow for Overround

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Abstract: Several methods have been proposed to adjust bookmakers' implied probabilities, including an additive model, a normalization model, and an iterative method proposed by Shin. These approaches have one or more defects: the additive model can give negative adjusted probabilities, normalization does not account for favorite long-shot bias, and both the normalization and Shin approaches can produce bookmaker probabilities greater than 1 when applied in reverse. Moreover, it is shown that the Shin and additive methods are equivalent for races with two competitors. Vovk and Zhadanov (2009) and Clarke (2016) suggested a power method, where the implied probabilities are raised to a fixed power, which never produces bookmaker or fair probabilities outside the 0-1 range and allows for the favorite long-shot bias. This paper describes and applies the methods to three large bookmaker datasets, each in a different sport, and shows that the power method universally outperforms the multiplicative method and outperforms or is comparable to the Shin method.

Keywords: Adjusting Forecasts, Betting, Sports Forecasting, Probability Forecasting

1. Introduction

Bookmaker odds have a useful role for sports performance research and commercial applications. Bookmaker odds have been repeatedly shown to provide improved expectations about outcomes in sport [1-2], which can be used by practitioners to set more realistic expectations before and after competitive events. Efficiency in bookmaker odds is fundamental to the success of sports betting firms, an industry that continues to grow and have an influence on all professional sports [3].

Sports researchers and professionals can get the most use out of bookmaker odds if they have an accurate method to convert odds into event probabilities [4]. The probabilities π_i implied by bookmakers odds, or prices, invariably sum to more than 1. The total π of the implied probabilities is known as the booksum, and the excess π-1 the overround. The overround determines the expected return to punters, which is $1/\pi$ in the long run. Due to the overround, the implied probabilities from bookmaker odds require an adjustment to obtain the actual probability expectations of bookmakers.

While the need to remove the overround to estimate fair or true probabilities p_i is the most common situation in sport research, Clarke [5] gives an example of the reverse process. This previous study considered the commercial application of a major betting agency, in which a mathematical model produced fair probabilities for the number of runs in the next over of cricket. With only a small window while players changed ends to set odds and take bets, a mathematical formula was needed to convert the true probabilities to bookmaker odds with the required overround. In a second application the same process was used to set odds for the point score in the next game of tennis. With the expansion of sports betting, many bookmakers or exchanges now use mathematical models plus an adjustment for overround to determine initial prices. As the event nears, further adjustments are then made due to the weight of money on placed bets.

The present paper discusses and compares four methods for removing (or incorporating) overround. For simplicity racing terminology is used, but the analysis applies to any experiment (race, match, contest, etc.) with n outcomes on which betting takes place. Section 2 describes four adjustment methods of distributing the overround: additive, normalization, Shin and the power method. Section 3 compares the performance of each method on various data sets, and is followed by the conclusion.

2. Adjustment Methods

Four methods of adjustment for overround have been used in the literature. In the following subsections each method is described and its distinguishing features summarized.

2.1. The Additive Method

Better described as the additive method, additive uses an additive model where the overround is split evenly between the n outcomes. Thus, the true probability for the *ith* outcome, p_i, is

$$p_i = \pi_i - (\pi - 1)/n \text{ and } \pi_i = p_i + (\pi - 1)/n \quad (1)$$

Although used by Viney *et al.* and others [4], the additive method is rarely used in the literature, as the changes between the implied and adjusted probabilities for outsiders can be quite dramatic. Not infrequently, the additive method can produce negative probabilities for rank outsiders. In fact, this will occur whenever the ratio of the overround and implied probability is greater than the number of competitors, $(\pi - 1)/\pi_i > n$. The reverse process can also produce bookmaker probabilities greater than *1* for hot favorites.

2.2. The Multiplicative Method

The multiplicative or normalization method allocates the overround proportionally. So that,

$$p_i = \pi_i/\pi \text{ or } \pi_i = \pi p_i. \quad (2)$$

Because of its simplicity, this is the most commonly used method. While seemingly appropriate for totalisator data, an automated betting system that allocates the same proportion of the pool for all horses, it fails to account for the favorite longshot bias, where it is well known that long-shots tend to be overbet while favorites are underbet. Thus a greater proportion of overround needs to be removed/added to longshots than favorites. It also suffers from sometimes producing probabilities greater than 1 for favorites in the conversion from fair to bookmaker's probabilities.

2.3. The Shin Method

Shin [6-7] proposed a correction method based on an assumed fraction z of knowledgeable punters. As given in [8], this results in

$$\pi_i = \sqrt{zp_i + (1-z)p_i^2} \sum_{j=1}^{n} \sqrt{zp_j + (1-z)p_j^2} \quad (4)$$

or

$$p_i = \frac{\sqrt{z^2 + 4(1-z)^{\pi_j^2}/_{\pi} - z}}{2(1-z)} \quad (5)$$

where

$$z = \frac{\sum_{j=1}^{n} \sqrt{z^2 + 4(1-z)^{\pi_j^2}/_{\pi} - 2}}{n-2} \quad (6)$$

To create bookmakers odds from fair odds requires using (4) and (6) and iterating on z to produce the required overround. To adjust bookmaker's odds to produce fair odds requires using iteration on (5) and (6).

This method helps to protect against the favorite longshot bias, and has been shown to produce better predictive true probabilities than normalization [9-10]. However, it is shown in the Appendix that in the case of two outcomes the Shin method is equivalent to the simple additive method, and as such can adjust outsiders too much. While (5) implies it can never produce negative true probabilities, (4) can produce bookmaker's probabilities greater than *1* for hot favorites.

2.4. The Power Method

A natural extension of the additive method used in the additive approach (where probabilities are adjusted by a constant addition), and the multiplicative method used in normalization (where probabilities are adjusted by a constant multiplier), is to raise the probabilities to a constant power. Clarke [11] gives details of this method, used in a commercial application described in [5]. It was also described in Vovk and Zhdanov [12] and attributed to Victor Khutsishvili. The power approach proposed by these authors can be written as,

$$p_i = {\pi_i}^k \text{ or } \pi_i = {p_i}^{(\frac{1}{k})} \quad (7)$$

The logic behind this method stems from the idea that bookmaker probabilities derived from fair probabilities for joint events should satisfy the usual multiplicative law for independent events. In practical terms, this condition implies that the return to a punter from investing his winnings on subsequent events is the same as a single investment on the joint event. When the n competitors are all equally likely, the value for k is calculated as, $k = \frac{\log(n)}{\log(\frac{n}{\pi})}$. However, in most cases iteration on k is necessary to ensure $\sum p_i = 1$, or the required booksum.

A clear advantage of the power method is that it can never produce probabilities outside the [0, 1] range. Similarly, it can be applied directly to prices, as the fair and adjusted prices follow the same power law with the same k as the true and implied probabilities. The power method also ensures a greater change to outsider probabilities than favorites. However, when compared to Shin it adjusts favorites and longshots more but middle-of-the-range priced horses less.

3. Methods

The operational characteristics of each method are shown with several illustrative examples. Analyses are then presented on the actual predictive performance of each method on large-scale sports datasets for 3 different sports.

Historical bookmaker odds were gathered for 3 different sporting events: tennis, greyhound racing, and horse racing. These datasets were chosen to represent a range of competitor numbers and overround characteristics. The ATP dataset included nearly 15,000 men's singles matches from 2000 to the present. Bookmaker prices for this dataset were the average betting odds reported by Oddsportal. The greyhound data comprised tote data (from over 27,000 races) at 2,206 meetings in New Zealand between 1/8/2011 and 18/8/2016. The final dataset was gallop data that consisted of closing prices from the Victorian Tote on Australian thoroughbred races in the first half of 2008. Together, these datasets range from 2 to 12 competitor events and have average overrounds ranging between 6% and 27% (Table 1).

Table 1. *Description of Datasets used in Performance Evaluation.*

Dataset	Events	Average Number of Competitors	Average Overround (95% Interval)
ATP	14,925	2	6.1 (4.7 – 7.0)
Gallop	4,663	12	27.2 (9.0 – 78.2)
Greyhound	20,206	8	19.5 (11.1 – 22.2)

Three measures of performance were evaluated. The first was the distribution in the adjusted win probability assigned to the winning competitor. The higher the mean and the lower the variance in this probability, the better the predictive performance of the adjustment method. We also report the logloss, which is a loss measure that is closely connected to the Kelly betting criterion [13]. This measure is unique in that it penalizes inaccurate predictions that are made with higher confidence. For the non-binary events, a binary classifier was created that assigned one category to the winner and all other competitors to the losing category. Using the same binary classifier, we also evaluated the root-mean squared error, or Brier score, for each method. As with the logloss, a lower square-error indicates a superior prediction method.

4. Results

4.1. Operational Characteristics

Tables 2 and 3 show an example of transforming probabilities in both directions using the four adjustment methods. These tables clearly show the shortcomings of the additive method, and the varying degree to which the Shin and Power method adjust favorites and longshots. Later we compare the efficacy of the predictive power of the probabilities produced by these two methods.

Table 2. *Comparison of 4 Methods of Adjusting 1.25 Booksum to Produce Fair Probabilities.*

Prices and their Implied probs		Calculated True Probabilities				Calculated Fair Prices			
		Add.	Mult.	Shin	Power	Add.	Mult.	Shin	Power
$1.15	0.870	0.828	0.696	0.769	0.825	$1.21	$1.44	$1.30	$1.21
$5.00	0.200	0.158	0.160	0.148	0.110	$6.31	$6.25	$6.78	$9.12
$10.00	0.100	0.058	0.080	0.059	0.042	$17.12	$12.50	$16.94	$23.63
$20.00	0.050	0.008	0.040	0.020	0.016	$118.97	$24.99	$49.84	$61.23
$50.00	0.020	*-0.022*	0.016	0.004	0.005	*-$46.31*	$62.48	$264.78	$215.54
$100.00	0.010	*-0.032*	0.008	0.001	0.002	*-$31.65*	$124.96	$1,026.96	$558.47
Total	1.250	1.000	1.000	1.000	1.000				

Again Table 3 shows the possibility of both the multiplicative and Shin method producing probabilities greater than 1 for short priced favorites. Since probabilities in [0, 1] remain in [0, 1] when raised to any positive power, the power method always produces realistic transformations.

Table 3. *Comparison of 4 Methods of Adjusting true Probabilities to Produce a 1.25 Booksum.*

True Probs	Fair Prices	Adjusted Probabilities				Boookmaker Prices			
		Add.	Mult.	Shin	Power	Add.	Mult.	Shin	Power
0.01	$100.00	0.052	0.013	0.033	0.040	$19.35	$80.00	$30.74	$24.93
0.015	$66.67	0.057	0.019	0.041	0.053	$17.65	$53.33	$24.44	$18.78
0.02	$50.00	0.062	0.025	0.048	0.065	$16.22	$40.00	$20.64	$15.36
0.025	$40.00	0.067	0.031	0.055	0.076	$15.00	$32.00	$18.02	$13.15
0.03	$33.33	0.072	0.038	0.062	0.086	$13.95	$26.67	$16.08	$11.57
0.9	$1.11	0.942	1.125	1.010	0.929	$1.06	$0.89	*$0.99*	$1.08
1		1.250	1.250	1.250	1.250				

The findings indicate that the power method has some advantage over the other three methods in that it never produces improper probabilities. The Appendix shows that the additive and Shin method are equivalent for two-competitor races. The following subsection explores the predictive power of the probabilities produced by the various methods.

4.2. Predictive Performance

The results for the ATP data, with only two outcomes, confirmed in the Appendix, in that the additive and Shin methods always had the same result. In all three measures the Clarke power method achieved the best or equal best result, with the multiplicative method the worst (Table 4). For the gallop data, the additive model was superior, followed by Shin. The multiplicative model was the worst performing on all measures. Results for the greyhound data were more variable. The multiplicative model was again the worst performer on two measures, but second on the log loss measure, with Shin being the second worst on all measures. The additive method proved the best on the probability assigned to winner and RMSE, but only third on log loss. The power method was either first or second on each measure.

Table 4. *Performance Comparison of Alternative Methods of Removing Overround.*

Performance Measure	ATP	Gallop	Greyhound
Prob. Assigned to Winner, Mean (95% Interval)			
Power	62.8 (18.1 - 97.3)	19.2 (1.5 – 47.4)	24.8 (3.3 – 67.5)
Additive	62.4 (19.2 – 95.6)	20.4 (0.8 – 48.7)	25.6 (2.3 – 68.8)
Multiplicative	61.7 (21.1 – 93.4)	18.2 (2.4 – 44.6)	23.5 (4.0 – 59.4)
Shin	62.4 (19.2 – 95.6)	19.2 (1.6 – 46.7)	24.6 (3.1 – 64.4)
LogLoss			
Power	0.548	1.971	1.686
Additive	0.548	1.968	1.696
Multiplicative	0.550	1.994	1.692
Shin	0.548	1.968	1.696
RMSE			
Power	74.39	79.14	155.28
Additive	74.41	78.12	154.14
Multiplicative	74.50	79.75	156.90
Shin	74.41	79.03	155.42

Clearly the additive method has performed surprisingly well, but as pointed out earlier it does have some problems in producing probabilities outside the range [0, 1]. The multiplicative generally does very poorly. The power method outperforms the multiplicative method on all data sets on each measure. Similarly it universally outperforms or equals Shin, with the exception of the RMSE measure on the Greyhound data.

5. Conclusions

This is the first paper to give a complete description of the most popular methods for adjustment of bookmaker odds and provide the most comprehensive comparison of their performance with actual sporting data. While simple to apply, the additive method can produce negative probabilities, and the multiplicative or normalization method performed badly on all predictive performance measures. On the data sets analysed, the power method generally performed better than the Shin approach. It also performed better than all other methods on the ATP dataset, which is the only dataset obtained from bookmakers.

Given the comparability in performance between the Shin and power method, ease-of-implementation will be a critical consideration for practitioners and industry. Both the Shin and the power method require iteration. As with Shin, the power method has an underlying logical basis for its derivation. However, as a natural extension of the additive and multiplicative transformation the power method is conceptually simpler and generally easier to implement than Shin.

Past commercial applications also indicate an industry preference for the power method. Clarke [5] has used the power method successfully in a commercial application to incorporate overround into probabilities estimated from a mathematical model. To the authors' knowledge, at least two Australian companies currently use the power method to transform bookmakers' prices as a means to obtain an estimate of market knowledge about specific competitive events (personal communication).

There are multiple adjustment methods available to sports researchers and professionals for translating bookmaker odds into true event expectations. Considerations of performance, ease-of-implementation, and commercial record make the power adjustment method a strong competitor among approaches for correcting for overround.

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Appendix: Equivalence of Shin and the Additive Method for 2 Outcomes

Many events on which betting takes place have only two outcomes, usually win or loss. Betting on the line (whether a score will exceed or not exceed a given value), or laying (betting on an event not occurring) can reduce events with multiple outcomes to one with only two outcomes. Strumbelj [8] notes that in this special case, equation (5) above has a tractable solution. While this may be of interest in calculating the proportion of knowledgeable punters, it is not necessary to calculate z to find p_i, as we show here that for $n = 2$ the Shin probabilities are given by the additive method. Specifically,

$$p_1 = \pi_1 - \frac{(\pi-1)}{2}, \text{ and } p_2 = \pi_2 - \frac{(\pi-1)}{2}$$

Proof: For $n = 2$, we have π_1 and π_2 are bookmaker prices that sum to π and p_i are Shin adjusted prices that sum to *1*.

To simplify let

$$A_i = \sqrt{z^2 + 4(1-z)\frac{\pi_i^2}{\pi}} \quad \text{(A1)}$$

So from equation (6)

$$p_i = {(A_i - z)}\big/{2(1-z)} \quad \text{(A2)}$$

So $A_i = z + 2(1-z)p_i$

From (A2),

$$p_1^2\text{-}p_2^2 = \frac{(A_1^2 - 2A_1z + z^2)}{4(1-z)^2} - \frac{(A_2^2 - 2A_2z + z^2)}{4(1-z)^2}$$

So,

$$(p_1 - p_2)(p_1 + p_2) = \frac{(A_1^2 - A_2^2) - 2z(A_1 - A_2)}{4(1-z)^2}$$

$$= \{z^2 + 4(1-z)\frac{\pi_1^2}{\pi} - z^2 - 4(1-z)\frac{\pi_2^2}{\pi} - 2z(z + 2(1-z)p_1 - z - 2(1-z)p_2)\}/4(1-z)^2$$

$$= \{4(1-z)(\frac{\pi_1^2}{\pi} - \frac{\pi_2^2}{\pi}) - 4z(1-z)(p_1 - p_2)\}/4(1-z)^2$$

So,

$$p_1 - p_2 = \{(\pi_1 - \pi_2) - (z(p_1 - p_2)\}/(1-z)$$

Since $\pi = \pi_1 + \pi_2$ and $p_1 + p_2 = 1$.

Then, $(p_1 - p_2) = (\pi_1 - \pi_2)$

Solving using $p_1 + p_2 = 1$ gives $p_1 = {(1 + \pi_1 - \pi_2)}\big/{2}$, $p_2 = {(1 + \pi_2 - \pi_1)}\big/{2}$

or alternatively that $p_1 = \pi_1 - {(\pi_1 + \pi_2 - 1)}\big/{2}$, $p_2 = \pi_2 - {(\pi_1 + \pi_2 - 1)}\big/{2}$

as required

Alternatively, there is similar proof using (4)

Again for simplicity let $B_i = \sqrt{zp_i + (1-z)p_i^2}$

Then $\pi_1 - \pi_2 = B_1(B_1 + B_2) - B_2(B_1 + B_2)$

$= B_1^2 - B_2^2$

$= z(p_1 - p_2) + (1-z)(p_1^2 - p_2^2)$

$= (p_1 - p_2)(z + 1 - z)$ since $p_1 + p_2 = 1$

$= (p_1 - p_2) = (2p_1 - 1) = (1 - 2p_2)$

But $\pi_1 + \pi_2 = \pi$

Solving gives $\pi_1 = p_1 + {(\pi - 1)}\big/{2}$, $\pi_2 = p_2 - {(\pi - 1)}\big/{2}$ as required.

Note it is easily seen that this can result in π's greater than 1.

References

[1] C. Leitner, A. Zeileis, and K. Hornik, "Forecasting sports tournaments by ratings of (prob) abilities: A comparison for the EURO 2008." *International Journal of Forecasting* 26, no. 3, 2010, pp. 471-481.

[2] S. Kovalchik, "Searching for the GOAT of tennis win prediction." *Journal of Quantitative Analysis in Sports* 12, no. 3, 2016, pp. 127-138.

[3] L. Robinson, "The business of sport" in *Sport & Society: A Student Introduction*, Houlihan, B. Eds. London: SAGE, 2003, pp. 165-183.

[4] M. Viney A. Bedford, and E. Kondo, "Incorporating over-round into in-play Markov Chain models in tennis". 15th International Conference on Gambling & Risk-Taking, Las Vegas, USA, 2013.

[5] S. R. Clarke, "Successful applications of statistical modeling to betting markets". In IMA Sport 2007: First International conference on Mathematics in Sport. D. Percy, P Scarf & C Robinson, Eds., The Institute of Mathematics and its Applications: Salford, United Kingdom, 2007, pp. 35-43.

[6] H. S. Shin, "Prices of State Contingent Claims with Insider traders, and the Favorite-Longshot Bias". The Economic Journal, 1992, 102, pp. 426-435.

[7] H. S. Shin, "Measuring the Incidence of Insider Trading in a Market for State-Contingent Claims". The Economic Journal, 1993, 103(420), pp. 1141-1153.

[8] E. J. Strumbelj, "On Determining Probability Forecasts from Betting Odds." International Journal of Forecasting, 2014, 30(4), pp. 934-943.

[9] M. Cain, D. Law, and D. Peel, "The favorite-longshot bias, bookmaker margins and insider trading in a variety of betting markets". Bulletin of Economic Research, 2003, 55, pp. 263–273.

[10] M. A. Smith, D. Paton, and L. V. Williams, "Do bookmakers possess superior skills to bettors in predicting outcomes?" Journal of Economic Behavior & Organization, 2009, 71, 539 – 549.

[11] S. R. Clarke, "Adjusting true odds to allow for vigorish". In Proceedings of the 13th Australasian Conference on Mathematics and Computers in Sport. R. Stefani and A. Shembri, Eds., 2016: Melbourne, pp. 111-115.

[12] V. Vovk, and F. Zhdanov, "Prediction with Expert Advice for the Brier Game". Journal of Machine Learning Research, 2009, 10, pp. 2445-2471.

[13] L. H. Yuan, A. Liu, A., Yeh, A. *et al.* "A mixture-of-modelers approach to forecasting NCAA tournament outcomes". *Journal of Quantitative Analysis in Sports*, 2015, *11*(1), pp. 13-27.