The Research on Middle School Mathematics Teaching Based on BOPPPS Model -- Taking the Teaching Design of "Determination of Congruent Triangles" as an Example

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Abstract: The Mathematics Curriculum Standards for Compulsory Education (2022 Edition) places higher demands on junior high school mathematics teaching, emphasizing students' active participation in classroom activities and the development of core literacies, mobilizing students' interest in learning as much as possible, fostering students' initiative and enthusiasm for learning, and making students the masters of the classroom, with the teacher merely a "guide" of knowledge, returning the classroom to the students. The BOPPPS model is a teaching model in which students participate in all aspects as well as play an important role in teaching objectives. Applying this model in middle school mathematics teaching can help teachers improve teaching efficiency and also increase the proportion of students' participation in the classroom. In this study, this model is applied to the design of middle school mathematics teaching, and the first lesson of "Determination of congruent triangles" is used as a case study for analysis. Through the introduction of real-life scenarios, specific and testable classroom objectives, review of old knowledge as a pre-assessment, active and participatory learning of students, post-test of the content in the classroom, and summary and self-evaluation of students, students can experience the process of knowledge occurrence and development, establish a systematic knowledge system of the content, develop students' core literacy, improve the teaching efficiency of the classroom, and provide a new direction for teachers to think about their teaching.

Keywords: BOPPPS Model, Middle School Mathematics, Teaching Research

1. Introduction

The National Medium- and Long-term Education Reform and Development Plan (2010-2020) calls for schools at all levels to deepen educational reform and explore teaching models such as inspirational, inquiry-based, discussion-based, and participatory models to help students learn to learn [1]. The Curriculum Standards for Compulsory Education in Mathematics (2022 Edition) issued by the Ministry of Education has made many improvements to the implementation part of the curriculum, requiring that teaching objectives should point to the cultivation of students' core literacy, further emphasizing that students are the main subjects of learning, and that teaching activities should focus on stimulating students' interest in learning and provoking their active thinking, while reflecting the diversity of evaluation subjects [2]. In addition, through compulsory education, students need to be able to see and observe the world through the eyes of mathematics, to think about the real world using mathematical thinking, and finally to express the real world in mathematical language, which we refer to as the "three skills".

Middle school is a critical period for developing students' core literacies such as abstraction ability, geometric intuition and reasoning ability, and it is important to insist that students acquire basic knowledge, basic skills, basic ideas and basic activity experiences in mathematics. However, the current junior high school classroom suffers from a lack of vitality and fails to stimulate students' interest well [3]. The traditional teaching model is not accurate enough in grasping
the focus of the classroom, does not pay much attention to conveying specific teaching objectives to students, and students' self-assessment is relatively lacking, which cannot fully satisfy students' overall development [4]. The core aspects of the BOPPPPS model, such as "participatory learning", communicating concrete and testable objectives to students, and student self-assessment, are conducive to solving these problems.

In this paper, the BOPPPPS model is applied to the teaching of middle school mathematics, taking the "Determination of congruent triangles" as an example, using a lively and interesting introduction, requiring the class to be guided by clear objectives, i.e., what students need to learn, how to learn it and to what extent. At the end of the class, a summary and a self-assessment are conducted, in which students are asked to clarify their mastery of the triangle and are assigned corresponding homework assignments, both individual and group, to achieve a systematic grasp of knowledge. At the same time, core literacies such as geometric intuition and reasoning skills are infused in the classroom to enhance the nurturing value of mathematics.

2. Meaning and Value of BOPPPPS Model

The BOPPPPS model was first used in teachers' skills training [5], but in recent years it has been focused on by educators and applied to actual teaching, with the distinctive feature of "effective teaching" and the concept of standardized and refined teaching, which consists of six components, namely bridge-in, classroom objectives, classroom pre-assessment, participatory learning, classroom post-test and classroom summary [6], as shown in Figure 1 below:

![Figure 1. BOPPPPS model link.](image)

In this paper, the BOPPPPS model is integrated into the classroom teaching and learning design, taking Chapter 12, Section 2 "Determination of congruent triangles, Lesson 1" in the first book of Grade 8 of the Compulsory Education Textbook "Mathematics" of the Human Education Edition as an example.

3. The Design of Middle School Classroom Instruction Based on the BOPPPPS Model

3.1. Classroom Introduction (Bridge-in)

Question 1: The new movie "The First Slam Dunk" was released on April 20th. Did you go to the cinema to see it? Many people fell in love with the sport of basketball because of this manga, and it is the initiation anime for many people who love playing basketball because it conveys to us the enterprising spirit of never giving up and striving hard. We should also always keep this spirit of thought in mind in our daily lives and study hard to serve our country in the future. Did students sign up for the basketball event in the recent field day? Zhang San actively participated in this competition, but yesterday, because of training to play basketball, he broke the triangular glass window of the equipment room, which was two identical glass windows now only one is left, how can we help him solve this problem?

Students speak freely and help Zhang San come up with solutions. The teacher captures students' interest in inquiry and leads to the learning objectives of the lesson.

Design Intent: Creating problem situations with current hot topics stimulates students' interest in learning, generates desire for knowledge, shifts students' excitement to the classroom, focuses their attention on teaching, and shifts from passive acceptance of knowledge to active inquiry and understanding of knowledge, thus improving teaching efficiency. A wonderful classroom introduction can lay a good foundation for the whole lesson. Teachers should be good at grasping students' preferences to achieve an appropriate introduction, avoiding a rigid and far-fetched introduction of the situation.
3.2. Classroom Teaching Objective (Objective)

1) What to learn: The content of the "side by side" theorem and the use of the ruler to make an angle equal to a known angle.

2) How to learn: In the process of investigating the determination of congruence of triangles, the teacher is only a guide, and each member of the group does his or her own job to ensure that the students' enthusiasm and initiative are brought into play.

3) What have you learned: Be familiar with the content of the "side by side" theorem, be able to use the theorem to determine the congruence of two triangles, and use the ruler tool to make an angle equal to a known angle.

4) Emotional and Attitudinal Values: The students will experience the process of determining the congruence of triangles and the process of drawing mathematical conclusions by manipulation and categorical discussion, which will penetrate the core literacy of abstraction ability and geometric intuition. Experience the sense of accomplishment of working together to reach a conclusion, so that each student's initiative can be given full play, thus stimulating the interest in learning mathematics.

Design Intent: In traditional classroom lectures, teachers are mostly concerned with the completeness, fluency, and proficiency of the content and less concerned with whether the classroom objectives are clear and clear, appropriately sized, and used with effective means to help students reach the objectives. The BOPPPS model, however, is different in that its instructional objectives are designed according to what the lesson is to be learned, by what means, to what extent, and the development of emotional and attitudinal values and the objectives are assessable and easy for students to understand. The other five links will all revolve around the teaching objectives, which will be finally implemented through a lesson to see if the teaching objectives are achieved, so it is both a starting point and a landing point [9].

3.3. Pre-Assessment in the Classroom (Pre-Assessment)

Question 1: First review the previous lesson and try to see if you can help Zhang San solve this problem. What is a congruent triangle? What are the properties of congruent triangles?

Student: Two triangles that can overlap exactly are called congruent triangles; the property of congruent triangles is that the corresponding sides are equal and the corresponding angles are equal.

Question 2: Given the conditions \( \triangle ABC \cong \triangle DEF \), As shown in Figure 2: find the sides and angles that are equal.

\[ \begin{align*}
\angle A &= \angle D, \\
\angle B &= \angle E, \\
\angle C &= \angle F, \\
AB &= DE, \\
AC &= DF, \\
BC &= EF
\end{align*} \]

Figure 2. Question 2.

Question 3: If we measure the values of the three sides and three angles of the triangular glass, we can make an identical piece of glass, but it is a little inconvenient to measure a little too many quantities, and mathematics is about the beauty of simplicity, so it is better to solve mathematical problems in a simple way. If we measure only a few of the quantities, can we guarantee that the triangular glass will be identical to the original one? For example, measure only two corners of the glass, or only three sides of the glass.

Students are confused and begin to have different answers, some think they can and some think they can't, which creates a contradiction, and the teacher captures the students' desire to know to enter the inquiry session.

Design Intent: By reviewing old knowledge as a classroom pre-assessment, on the one hand, it helps to consolidate what was learned in the previous class; on the other hand, according to constructivism, students can grow new knowledge based on old knowledge, and through some practical problems, students become curious about new knowledge, and this thirst for knowledge has a positive significance in improving students' learning effect in mathematics as well as the good atmosphere in the classroom. In addition, we need to pay attention to several issues in conducting classroom pre-assessment, for example, we should respect individual differences, reflect openness as much as possible, let differences become effective classroom resources, and it is best to expose the unknown, reflect the problematic, so that students understand what knowledge they know and what knowledge is needed to be nudged by the teacher before they can accept it, and at the same time the pre-assessment can reflect the fun, making it a lively and interesting The pre-assessment can be interesting, making it a lively and interesting session.

3.4. Participatory Learning (Participatory-Learning)

Exploration 1: What are the cases of one condition, starting from the least one condition to explore?

Students: Two triangles with a set of sides or a set of angles corresponding to equality.

Teacher: In small groups, draw two triangles on paper in these two cases, choose the same set of sides or angles, and draw a triangle at each table to see if they are necessarily congruent? Later, ask students to show on the board.

Student: Two triangles with an equal set of sides are not necessarily congruent, as shown in Figure 3 below. Two triangles whose sets of angles correspond to each other are also not guaranteed to be congruent, as shown in Figure 4:
Exploration 2: One condition is too few, will two conditions work? How many cases are there under two conditions?

Student: Two sets of sides, two sets of angles, and one set of sides and one set of angles correspond to each other as equal.

Teacher: Continue to work in groups to draw diagrams to explore each of these three cases and see if the two triangles are necessarily congruent. Ask students to show on the blackboard.

Student: Two triangles that have two sets of sides that correspond to each other are not necessarily congruent, as shown in Figure 5 below. Two triangles with two sets of equal angles are also not necessarily congruent, as shown in Figure 6 below. Two triangles with one set of sides and one set of angles that are equal are also not guaranteed to be congruent, as shown in Figure 7 below:

Exploration 3: Two conditions also do not guarantee that the two triangles are congruent, which means that two conditions are still not enough, so let's look at the three conditions next. First of all, let's talk about what are the cases of three conditions.

Student: Three sets of angles, three sets of sides, one set of angles and two sets of sides and one set of sides and two sets of angles correspond to each other as equal.

Teacher: Are the two triangles that correspond to each of the three sets of angles equal congruent? After the group discussion, show them on the blackboard.

Student: Two triangles with three sets of equal angles are not guaranteed to be congruent. As shown in Figure 8:

Teacher: Are the two triangles with three sets of equal sides congruent? How to make two triangles with three sets of sides corresponding to each other equal? Try it: Draw a triangle $\triangle ABC$ and a triangle $\triangle DEF$, so that $AB = DE$, $AC = DF$, $BC = EF$, and cut out two triangles to see if they are congruent.

Student: A ruler's rule is needed to make the diagram, and the steps are divided into three steps:

First draw $B'C' = BC$;

Then draw circles with $b'$ and $c'$ as centers and the lengths of lines $AB$ and $AC$ as radius, respectively, with the two arcs intersecting at point $A'$;

Finally, connect line $A'B'$ and $A'C'$. Students then find that they can overlap. As shown in Figure 9:

Teacher: How can you describe the above in mathematical terms?

Let students summarize on their own first, and the teacher will then give the content of the theorem.

Teacher: Two triangles with three equal sides are congruent. That is, the "side by side" decision theorem studied in this lesson, (can be abbreviated as "side by side" or "SSS"). Explain in mathematical language:

In triangles $\triangle ABC$ and $\triangle DEF$

\[
\begin{align*}
AB &= DE \\
AC &= EF \\
BC &= DF
\end{align*}
\]

$\therefore \triangle ABC \cong \triangle DEF \ (SSS)$
Pay attention to the principle of correspondence in writing, which is the part we emphasized in the previous lesson on the properties of congruent triangles. It must be \( AB = DE \), not \( AB = ED \).

Design Intent: On the one hand, students are more interested in learning through manipulation, so using this feature to organize can attract students to actively participate in learning by doing, understanding by doing, and mastering knowledge by doing; on the other hand, enhancing the interest in learning is conducive to grasping the intrinsic and essential connection of knowledge, inspiring thinking, and gaining perceptual understanding, which not only develops students’ spatial concept, but also promotes the formation of students’ core literacy. The more students participate in classroom activities, the more active they will be in learning, the more intense the perceptual experience of subjectivity will be, and the more enhanced their subjective consciousness will be, thus prompting them to develop in the direction of rationality, which will in turn further increase the enthusiasm and consciousness of participation immediately afterwards. In short, teachers need to always remember that "student-centered", the students are the master of the classroom, the classroom back to the students, is a good lesson should have the appearance.

3.5. Classroom Post-Test (Post-Assessment)

Exercise 1: As shown in Figure 10, \( \triangle ABC \) is a steel frame, \( AD \) is a bracket that connecting \( A \) and \( BC \) midpoint \( D \), find the proof.

![Figure 10. Exercise 1.](image)

Exercise 2: Figure 11 below shows that \( AD = BC \) and \( AC = BD \). (Hint: Connect \( AB \))

![Figure 11. Exercise 2.](image)

Students think and then respond, allowing time for independent thinking. Teachers should focus on the step-by-step specifications for student writing: prepare the conditions (Prove that all the conditions needed are in place), specify the range (Indicate which two triangles are congruent to prove), lay out the basis (Put the three required conditions together in curly brackets), and write the conclusion (Write down two triangles as congruent and label which decision theorem is used).

Teacher: The learning objectives also require students to be able to make an angle equal to a known angle, so how can you use the knowledge from this lesson to draw a diagram? Can you draw it accurately on paper?

Student: You need to use the "side by side" decision theorem to make two congruent triangles, and then the corresponding angles are equal. The steps are as follows:

1. Draw an arc with point \( O \) as the center of the circle and any length as the radius and intersect \( OA, OB \) at points \( C \) and \( D \), respectively;
2. Draw a ray \( O\dot{A} \), with point \( O' \) as the center and the length of \( OC \) as the radius of the arc, intersecting \( O\dot{A} \)’ at point \( C' \);
3. Draw an arc with point \( C' \) as the center and the length of \( CD \) as the radius, intersecting with the arc drawn in step 2 at point \( D' \);
4. Draw ray \( O\dot{B} \)’ through point \( D' \), then \( \angle AOB = \angle A'O'B' \).

Teacher: After learning this lesson, can you help Xiao Ming solve the problem of triangular glass?

Student: You only need to measure the lengths of the three sides of the triangular glass.

Design Intent: The post-assessment echoes the pre-assessment. The post-test is a test at the end of the class, and the accompanying exercises used in this lesson are one of them, which can help students understand their mastery in time and help promote students' self-evaluation, while the teacher can also clarify students' acceptance of knowledge points through questions in time, have a general understanding of the accomplishment of the teaching objectives of this lesson, analyze the problems and desirability of teaching, and then reasonably arrange the subsequent teaching schedule and teaching contents.

3.6. Classroom Summary (Summary)

First, we review what we have learned in this lesson by asking questions, the determination theorem of "side by side" (two triangles with three equal sides are congruent) and making an angle equal to a known angle (using the determination theorem of "side by side”). The whole lesson is completed by students' active participation, and the teaching activities are increased from one condition to three conditions one by one, and through the operation of drawing, the students can explore the "side by side" theorem to determine the congruence of two triangles. Students experience the idea of categorization and discussion as well as the development of core literacies such as intuition and reasoning ability. The students are then asked to make a self-evaluation of their own mastery by solving practice problems and listening to the lecture, and to estimate their next learning goals [10]. At the end of the day, the students will be able to consolidate what they have learned and incorporate the knowledge of the lesson into their own knowledge system by leaving reflection questions and post-lesson assignments.

Thinking Questions: We have explored the first two of the
three conditions together, but can we determine the congruence of two triangles in the remaining two cases, i.e., two triangles with one set of sides and two sets of angles that are equal and two triangles with one set of angles and two sets of sides that are equal? In class, you should work in small groups to draw diagrams to verify your guesses, and complete Exercises 1 and 2 at the end of class as well as find a problem in the exercise book to prove that the triangles are congruent, and swap tables to complete the assignment.

Design Intent: Classroom summarization is at the end of the class when the teacher summarizes the content of the lesson and the ideas and skills used. This process condenses the essence of a lesson and allows the content of the lesson to be presented systematically to the students, and also allows for a moderate extension of the content to guide the students to continue their exploration in the class. Summarizing can be done by the students themselves, which will leave a deeper impression in their minds and develop their ability to summarize while further promoting the formation of core literacy, with the teacher only responsible for revising and supplementing. Summarization is an important part of classroom teaching optimization and an essential step to enhance students' abilities. In addition, we set up post-lesson reflection questions to pave the way for the next lesson, while cultivating students' ability to think independently and develop the habit of cooperative communication.

4. Reflection on the Application of the BOPPPS model

Teachers need to reflect on the following in their use of the BOPPPS model:

1) Do learning objectives necessarily need to be clearly listed [11]? The author believes that they need to be appropriately assigned according to the level of difficulty of the content. If the content is simple and easy for students to understand, our teaching objectives can be permeated in the introduction content, not necessarily explicitly listed out for display; if the content is difficult, the teacher needs to analyze the teaching objectives systematically to help students to clarify what they are learning in this lesson and have an overall grasp of the whole lesson, so as to reduce the confusion brought about by the new knowledge points. This will reduce the confusion of new knowledge.

2) The means of participatory learning. The means of student participation in learning should be as diverse as possible, but not generalized. Some courses may have a single means of student participation, such as explaining the primary function, students are more involved in classroom teaching through practice and discussion to answer questions, but when explaining the part of graphical geometry, students can participate in classroom teaching through hands-on operation and watching videos, etc. The videos can be collected by students in class and shown to their classmates in class, and the students who show them need to the videos can be collected by the students in class, presented and exchanged with their classmates in class, and the students who present them need to explain them to themselves so that they can exercise their self-learning skills. However, no matter which means of participation, the basic guideline is to clearly communicate the knowledge points as the core, and according to the nature of the course, try to use the students' favorite way to teach.

3) Are pre-tests and post-tests separate components? In the BOPPPS model, both pre-tests and post-tests play a very important role. The pre-test is linked to the previous knowledge and is also closely related to the introduction; the post-test is linked to the new knowledge and is closely related to the classroom summary. The two are not absolutely independent, because mathematical knowledge itself is a big whole, the content is linked to one another, and the knowledge points are related to each other, so they cannot be completely separated from each other, but need to be designed according to the content of the lecture, not mechanically rigid.

In addition, the BOPPPS model can be combined with other teaching models [12], such as the dyadic classroom, and the combination of the two provides teachers with a complete framework and theoretical support covering all aspects of classroom teaching, while at the same time giving full play to students' initiative in the link of participatory learning, improving students' learning efficiency in the classroom, and promoting the formation of a virtuous cycle of good ethos in this hybrid in such a blended classroom, teachers should pay attention to the ratio between explanation and white space for students on the basis of clear teaching objectives and teaching structure, so as to jointly promote students' growth and development through a blended classroom [13]. In the process of practicing the BOPPPS model, we must adhere to the dialectical principle that not all teaching contents are suitable for the BOPPPS model, and the difficulty of applying this model varies from course to course, and the teaching effect may also be different, so we must not apply it rigidly, which may be counterproductive. Teachers should use different teaching models reasonably according to different contents to ensure that the classroom works to the maximum extent and to promote the overall development and personality development of students.

5. Conclusion

In middle school mathematics teaching, the BOPPPS model is applied to instructional design to effectively improve classroom teaching efficiency [14]. This model is an effective teaching model that first introduces students through interesting cases, then conducts pre-tests to firmly grasp students' attention, after which teachers should be goal-oriented and stimulate students to take the initiative to think, cooperate and communicate and operate through participatory learning, then test students' learning outcomes through post-tests, and finally help students have a clearer understanding of the content of a lesson through classroom summaries. Teachers need to flexibly arrange all aspects of
classroom teaching, drive students to participate in all aspects of classroom teaching, and give full play to students' initiative [15], pay attention to student self-assessment, and develop students' abstraction ability, geometric intuition and other core literacy. At the same time carefully design the teaching content, grasp the essence, and adjust the links according to different class periods to promote a more standardized and refined teaching of middle school mathematics.

**References**


