The Dynamic Relationship of the GDP Per capita Among the Three Baltic States (1990-2021)

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Abstract: The geographical situation in Europe of Estonia, Latvia and Lithuania, the Three Baltic States, forms an optimal environment for the study of the economic relationships present among them. The global magnitudes are very similar for the three States, with a little difference in favor of Lithuania regarding population and extension. The three States joined the European Union at the same time, May 1, 2004. A vector autoregressive model, a VAR model, relating the three economies in their temporal evolution is an appropriate model for this study. With the intervention of temporal lags, it is possible to formulate the dynamical relationship present in these economies regarding the percentage growth change in the respective gdp per capita. Our attention is directed to the evolution of this percentage growth rate for the period 1990-2020. The estimated VAR(2) model shows that the percentage change in the gdp per capita of Lithuania is dynamically related to the lagged growth changes of Estonia and Latvia in a direct way, with more complex dynamic relationships regarding the other two States, as explained in the Conclusion. This study is supplemented with the Impulse Response Analysis and the Forecast Error Variance Decomposition to measure the effects of random impulses in the evolution of the percentage growth change in the estimated model.

Keywords: Baltic States, GDP Per Capita, Percentage Change, VAR Models, Impulse Response Analysis, Forecast Error Variance Decomposition, Vars Statistical Package

1. Introduction

The geographic situation of the three Baltic States offers an attractive field of attention to study the economic ties among them. [4]

In this paper a model for the dynamic relationship among the GDP per capita in the three Baltic States: Estonia, Latvia and Lithuania, is presented. It is a vector autoregressive model for the three time series of the respective economies.

In the real economy, the relationship among variables is a reality. The analysis of the time series representing the different variables, allow us to better interpret the situation, as well as to get better predictions, better forecasts.

Let

\[ z_t = (z_{1t}, z_{2t}, ..., z_{kt})' \]

be a k-dimensional vector of time series, observed at equally spaced time points.

In this paper, let be

- \( z_{1t} \): GDP per capita of Estonia,
- \( z_{2t} \): GDP per capita of Latvia, and
- \( z_{3t} \): GDP per capita of Lithuania.

The data are taken from the database datosmacro of the newspaper Expansión, from 1990 to 2021, annual data.

It has to be pointed out the relevance of the order of variables in a VAR model. Here, the variables are considered in alphabetical order.

The three time series are represented in figure 1.
For easy of interpretation, and to induce stationarity, let us transform the series into percentage growth changes. Figure 2.

2. VAR Models

A VAR model \[1, 2, 8, 11, 12, 13\] for the multivariate vector of time series \(z_t\) of order \(p\) can be written as

\[
z_t = \phi_0 + \sum_{i=1}^{p} \phi_i z_{t-i} + a_t \quad (1)
\]

where \(\phi_0\) is a \(k\)-dimensional constant vector, \(\phi_i\) are \(k \times k\) matrices for \(i > 0\), \(\phi_0 \neq 0\) and \(a_t\) a sequence of independent and identically distributed random vectors, with mean \(0\), and matrix-covariance matrix \(\Sigma_a\), positive-definite. \[6, 14, 15\].

Coming to our case, we can display (1) as a VAR1

\[
\begin{bmatrix}
  z_{1,t} \\
  z_{2,t} \\
  z_{3,t}
\end{bmatrix}
= \begin{bmatrix}
  \phi_{10} \\
  \phi_{20} \\
  \phi_{30}
\end{bmatrix}
+ \begin{bmatrix}
  \phi_{11} & \phi_{12} & \phi_{13} \\
  \phi_{21} & \phi_{22} & \phi_{23} \\
  \phi_{31} & \phi_{32} & \phi_{33}
\end{bmatrix}
\begin{bmatrix}
  z_{1,t-1} \\
  z_{2,t-1} \\
  z_{3,t-1}
\end{bmatrix}
+ \begin{bmatrix}
  a_{1,t} \\
  a_{2,t} \\
  a_{3,t}
\end{bmatrix} \quad (2)
\]

2.1. Model Estimation

With the package \textit{vars} of professor Bernhard Pfaff \[9, 10\] we proceed.

After a few intents, a VAR (2) model is estimated

\[
\text{# VAR Estimation Results:}
\]

\[
\text{# Endogenous variables: eston, latv, litu}
\]

\[
\text{# Deterministic variables: const}
\]

\[
\text{# Sample size: 29}
\]

\[
\text{# Log Likelihood: -290.396}
\]

\[
\text{# Roots of the characteristic polynomial:}
\]

\[
0.6956 0.6956 0.6134 0.6134 0.3322 0.3322
\]

\[
\text{# Call:}
\]

\[
\text{# VAR(y = zt, p = 2)}
\]

\[
\text{#}
\]

\[
\text{# Estimation results for equation eston:}
\]

\[
\text{#}
\]

\[
\text{# eston = eston.l1 + latv.l1 + litu.l1 + eston.l2 + latv.l2 + litu.l2 + const}
\]

\[
\text{#}
\]

\[
\text{# Residual standard error: 11.1 on 22 degrees of freedom}
\]

\[
\text{# Multiple R-Squared: 0.8168, Adjusted R-squared: 0.7668}
\]

\[
\text{# F-statistic: 16.35 on 6 and 22 DF, p-value: 4.219e-07}
\]

\[
\text{#}
\]

\[
\text{# Estimation results for equation latv:}
\]

\[
\text{#}
\]

\[
\text{# latv = eston.l1 + latv.l1 + litu.l1 + eston.l2 + latv.l2 + litu.l2 + const}
\]

\[
\text{#}
\]

\[
\text{# Residual standard error: 11.1 on 22 degrees of freedom}
\]

\[
\text{# Multiple R-Squared: 0.8168, Adjusted R-squared: 0.7668}
\]

\[
\text{# F-statistic: 16.35 on 6 and 22 DF, p-value: 4.219e-07}
\]

\[
\text{#}
\]

\[
\text{# Estimation results for equation litu:}
\]

\[
\text{#}
\]

\[
\text{# litu = eston.l1 + latv.l1 + litu.l1 + eston.l2 + latv.l2 + litu.l2 + const}
\]

\[
\text{#}
\]

\[
\text{# Residual standard error: 11.1 on 22 degrees of freedom}
\]

\[
\text{# Multiple R-Squared: 0.8168, Adjusted R-squared: 0.7668}
\]

\[
\text{# F-statistic: 16.35 on 6 and 22 DF, p-value: 4.219e-07}
\]

\[
\text{#}
\]

\[
\text{# Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1}
\]

\[
\text{#}
\]

\[
\text{#}
\]

\[
\text{#}
\]
2.2. Model Simplification

The estimated model $VAR(2)$ presents parameters non-statistically significant for the usual significant level $\alpha = 0.05$. The simplification of a model is of interest when there is no prior knowledge to support those parameters. However, there exists no optimal method to simplify the fitted model. Cf. Tsay [15], p. 72.

Coming to our case,
# Residual standard error: 11.08 on 26 degrees of freedom
# Multiple R-Squared: 0.4958, Adjusted R-squared: 0.4376
# F-statistic: 8.521 on 3 and 26 DF, p-value: 0.0004155

# Covariance matrix of residuals:
#  eston  latv  litu
# eston 127.6 145.1 107.6
# latv  145.1 263.3 138.4
# litu  107.6 138.4 145.1

# Correlation matrix of residuals:
#  eston   latv   litu
# eston 1.0000 0.7918 0.7907
# latv  0.7918 1.0000 0.7080
# litu  0.7907 0.7080 1.0000

2.3. Model Checking

Before any considerations, let us perform the checking of model $m_2$.

Regarding the residuals, we have figure 3. The residuals, to save space, are depicted jointly in figure 3.

![Figure 3. Residuals for model m2.](image)

These residuals do not show a failure to comply with the fundamental suppositions.

Next, regarding the autocorrelations, they are depicted in figure 4.

![Figure 4. Autocorrelations of residuals for model m2.](image)

The residuals, validate the estimated model. Other tests, as well, validate this model $m_2$.

3. Interpretation

For the easy of interpretation, let us write separately the three estimated models.

For Estonia:

$$\hat{z}_{1t} = 7.189 + 1.114z_{1,t-1} - 0.794z_{2,t-1} - 0.223z_{2,t-2}$$

For Latvia:

$$\hat{z}_{2t} = 1.907z_{1,t-1} - 1.439z_{2,t-1} - 0.278z_{2,t-2} + 0.5z_{3,t-2}$$

For Lithuania:

$$\hat{z}_{3t} = 6.076 + 0.444z_{1,t-1} - 0.158z_{2,t-1}$$
In these models, all the estimated parameters are statistically significant.

Regarding Estonia, the percentage growth rate of the GDP per capita is significantly related to its own past value, and to the past values of Latvia, but it does not depend on the lagged percentage growth rate of the GDP per capita of Lithuania.

On the other hand, the percentage growth rate of the GDP per capita of Latvia is dynamically related to the percentage growth rate of its own past values and to the past values of the GDP per capita of Estonia and Lithuania.

Similarly, the percentage growth rate of the GDP per capita of Lithuania is dynamically related to the percentage growth rate of the GDP per capita of Estonia and Latvia.

All three percentage growth rate are directly dynamically correlated, as shown by the above estimated correlation matrix. Validated the model, let us use it for prediction.

4. Prediction

Let us consider the horizon of 5 years. These forecasts are depicted in figure 4. Delete

5. Impulse Response Analysis

Since all variables in a VAR model depend on each other, the estimated coefficients provide limited information on the reaction of the system to a shock. In order to get a better picture of the model’s dynamic behavior, impulse responses are used. The impulse response functions [3, 7] show the effects of shocks on the adjustment path of the variables. Using the vars package, Paff [10], we get for model m2.

### Figure 5. Forecasts of the percentage growth changes in GDP per capita; Estonia, Latvia, Lithuania: 2022-2026.
The advantage of considering the impulse response functions, and not just VAR coefficients, is that they show the size of the impact of one standard deviation shock in the variables plus the rate at which the shock dissipates, allowing for interdependencies. In figure 6, the impact is positive, and dissipates rapidly.

The interpretation of figure 6 is straightforward. An impulse (shock) to \( \text{latv} \) and to \( \text{litu} \) at time zero, has large effects the next period, but the effects become smaller and smaller as time passes. The dotted lines show the 95% interval estimates of these effects.

### 6. Forecast Error Variance Decomposition

Another way of studying the effects of shocks in the variables, is to consider the contributions of a shock in the forecast error variance. This decomposition is often expressed in proportional terms. [5, 6, 15].

Forecast error variance decomposition estimates the contribution of a shock in each variable to the response in all variables.

Coming to our case, Paff [10], we have

![Orthogonal Impulse Response from eston](image)

Result depicted in figure 6.
depicted in figure 7.

Figure 7 shows that almost 100% of the variance in *eston* is caused by *eston* itself, while around 60% in the variance of *latv* is caused by *latv* itself and similar can be said for *litu*.

### 7. Conclusion

The estimated VAR(2) model shows the existence of an overall dynamic relationship in the percentage change of the Gdp per capita in the Three Baltic States. But at the same time, and during the period 1990-2020, the model reveals the following notes of differentiation: Estonia is not dynamically related to Lithuania, Latvia is dynamically related to Estonia and Lithuania and Lithuania is dynamically related to Estonia and Latvia. With independency of these distinctions, the estimated correlation matrix of the error term of the model shows a strong dynamic relationship among all of them.

### References


