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Several Kinds of Chromatic Numbers of Multi-fan Graphs

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Abstract: Coloring problem is a classical difficult problem of graph theory. It is a fundamental problem in scientific computation and engineering design. In recent years, a variety of graph coloring problems frequently appeared and solved many problems in production. It is a difficult problem to discuss the chromatic number of a given graph class. In the paper, we introduce several kinds of chromatic numbers of graphs such as adjacent-vertex-distinguishing total chromatic number, adjacent-vertex-distinguishing proper edge chromatic number, smarandachely-adjacent-vertex-distinguishing edge chromatic number, and the multi-fan graphs are considered.

Keywords: Multi-fan Graphs, Adjacent-vertex-distinguishing Total Chromatic Number, Adjacent-vertex-distinguishing Proper Edge Chromatic Number, Smarandachely-adjacent-vertex-distinguishing Edge Chromatic Number

1. Introduction

Graph theory is an important branch of Applied Mathematics. Colouring problems originated in the four colour conjecture 150 years ago. In recent years, many interesting and useful results have been obtained on the study of colouring problems. It is widely used in chemistry, computer, communication and other fields. There for they are widely discussed in graph theory. In this paper, we introduce several kinds of chromatic numbers of graphs such as the adjacent -vertex -distinguishing total chromatic number, the adjacent –vertex-distinguishing proper edge chromatic number, the smarandachely-adjacent-vertex-distinguishing edge chromatic number. And the Multi-fan graphs are considered in this paper. In the end, the paper abtained the chromatic numbers of graphs the paper considered.

The graphs considered in this paper are connected, finite, undirected and simple graphs. The multi-fan graphs are joint graphs that jointed by P_1 and $P_{n_1} \cup P_{n_2} \cup \ldots \cup P_{n_l}$, which P_{n_k} denotes the path graphs with order n_k ($n_k \geq 1$). We denote for all $1 \leq k \leq l$, $V(P_{n_k}) = \{v_1^{(k)}, v_2^{(k)}, v_{n_k}^{(k)}\}$ $v_j^{(k)} v_{j+1}^{(k)} \in E(P_{n_k})$ $(1 \leq j \leq n_k - 1)$, and P_1 denotes the graph which has only one vertex w. The symbol Δ is the maximum degree of the graph we discussed.

The paper use apagoge, construction method and direct proving method.

2. Adjacent-vertex-distinguishing Proper Total Coloring Number

Defenition 1 [1] A k-proper total colouring of a graph G is a mapping f from $V(G) \cup E(G)$ to $\{1,2,\ldots, k\}$ such that:

1) $\forall u, v \in V(G)$, if $uv \in E(G)$, then $f(u) \neq f(v)$;

2) $\forall e_i, e_j \in E(G)$, $e_i \neq e_j$, if e_i, e_j have a common end vertex, then $f(e_i) \neq f(e_j)$;

3) $\forall u \in V(G), e \in E(G)$, if u is the end vertex of e, then $f(u) \neq f(e)$.

Let f be a k-proper-total-colouring of G .Denote $C(u) = \{f(u)\} \bigcup \{f(uv) \mid v \in V(G) \wedge uv \in E(G)\}$ for every $u \in V(G)$, if $\forall u, v \in V(G), uv \in E(G)$, we have $C(u) \neq C(v)$, then f is called a k-proper-adjacent-vertex-distinguishing proper total coloring, short for k-AVDTC.

The number $\min\{k \mid G$ has a k-proper-adjacent vertex-distinguishing total coloring} is called the adjacent –vertex-distinguishing proper total chromatic number and denoted by $\chi_{at}(G)$.

The adjacent –vertex-distinguishing proper total chromatic number was first put forward by Zhang Zhong-fu, and he show a conjecture such that:

Conjecture 1 [11] For every connected graph G with order at least 2, we have $\chi_{at}(G) \le \Delta + 3$.

Lemma 1 If two arbitrary distinct vertices of maximum degree in G are not adjacent, then $\chi_{at}(G) \ge \Delta + 1$; If G has two distinct vertices of maximum degree which are adjacent, then $\chi_{at}(G) \ge \Delta + 2$

Theorem 1: $\chi_{at}(P_1 \vee (P_{n_1} \cup P_{n_2})) = n_1 + n_2 + 1$

Proof. Because there is only one vertex w whose degree($= n_1 + n_2$) is the maximum degree, so concluded by Lemma 1, we get the result such that

$$\chi_{at}(P_1 \vee (P_{n_1} \cup P_{n_2})) \ge n_1 + n_2 + 1.$$

Then we let f be a mapping from $V(P_1 \vee (P_{n_1} \cup P_{n_2})) \cup E(P_1 \vee (P_{n_1} \cup P_{n_2}))$ to $\{1,2,3,\ldots, n_1 + n_2 + 1\}$ as follows :

$f(w) = n_1 + n_2 + 1 \quad f(v_{n_1}{}^{(1)}) = 1 \quad f(v_{n_2}{}^{(2)}) = n_1 + 1$

$f(v_j{}^{(1)}) = j + 1 \quad (1 \le j \le n_1 - 1)$

$f(v_i{}^{(2)}) = n_1 + i + 1 \quad (1 \le i \le n_2 - 1)$

$f(wv_j{}^{(1)}) = j \quad (1 \le j \le n_1) \quad f(v_1{}^{(1)}v_2{}^{(1)}) = n_1$

$f(wv_i{}^{(2)}) = n_1 + i \quad (1 \le j \le n_2)$

$f(v_1{}^{(2)}v_2{}^{(2)}) = n_1 + n_2$

$f(v_j{}^{(1)}v_{j+1}{}^{(1)}) = j \quad (2 \le j \le n_1 - 1)$

$f(v_i{}^{(2)}v_{i+1}{}^{(2)}) = n_1 + i \quad (2 \le i \le n_2 - 1)$

At this time, we have

$C(w) = \{1,2,3,\ldots, n_1, n_1 + 1,\ldots, n_1 + n_2 + 1\}$

$C(v_1{}^{(1)}) = \{1,2,, n_1\}$

$C(v_2{}^{(1)}) = \{1,2,3, n_1\}$

$C(v_j{}^{(1)}) = \{j-2, j-1, j, j+1\} \quad (3 \le j \le n_1 - 1)$

$C(v_{n_1}{}^{(1)}) = \{n_1 - 2, n_1, 1\}$

$C(v_1{}^{(2)}) = \{n_1 + 1, n_1 + 2,, n_1 + n_2\}$

$C(v_2{}^{(2)}) = \{n_1 + 1, n_1 + 2, n_1 + 3, n_1 + n_2\}$

$C(v_i{}^{(2)}) = \{n_1 + i - 2, n_1 + i - 1, n_1 + i, n_1 + i + 1\}$
$(3 \le i \le n_2 - 1)$

$C(v_{n_2}{}^{(2)}) = \{n_1 + n_2 - 2, n_1 + n_2, n_1 + 1\}$

As defined in definition 1, obviously, f is a $n_1 + n_2 + 1$ -AVDTC.

There for

$$\chi_{at}(P_1 \vee (P_{n_1} \cup P_{n_2})) = n_1 + n_2 + 1$$

Corollary 1

$\chi_{at}(P_1 \vee (P_{n_1} \cup P_{n_2} \cup \ldots \cup P_{n_l})) = n_1 + n_1 + \ldots n_l + 1$

The proof of Corollary 1 can be easy done.

3. Adjacent-vertex-distinguishing Proper Edge Chromatic Number

Defenition 2 [2] A k-proper-edge-colouring of a graph G is a mapping f from $E(G)$ to $\{1,2,\ldots, k\}$ such that:

1) $\forall e_i, e_j \in E(G), e_i \ne e_j$, if e_i, e_j have a common end vertex, then $f(e_i) \ne f(e_j)$;

Let f be a k-proper-edge-colouring of G .Denote $C(u) = \{\{f(uv) \mid v \in V(G) \wedge uv \in E(G)\}$ for every $u \in V(G)$, if $\forall u, v \in V(G), uv \in E(G)$, we have $C(u) \ne C(v)$, then f is called a k-proper-adjacent-vertex-distinguishing-edge coloring, short for k-AVDPEC.

The number $\min\{k \mid G$ has a k-proper-adjacent-vertex-distinguishing edge colouring} is called the adjacent –vertex-distinguishing edge chromatic number and denoted by $\chi_a'(G)$.

For graphs G, n_i denote the number of the vertex whose degree= i, δ, Δ denote the minimum degree and the maximum degree of the graph. Then, define number $\upsilon(G)$ such that

$$\upsilon(G) = \max\{\min\{\lambda \mid \binom{\lambda}{2} \ge n_i\}, \delta \le i \le \Delta\}$$

Then a conjecture was put forward by [12]

Conjecture 2 For graphs without isolated edge and the number of the isolated vertex is no more than one, then

$\upsilon(G) \le \chi_a'(G) \le \upsilon(G) + 1$

Lemma 2: For all graphs G, $\chi_a'(G) \ge \Delta$.

Theorem 2: $\chi_a'(P_1 \vee (P_{n_1} \cup P_{n_2})) = n_1 + n_2$

Proof. There is only one vertex w whose degree($= n_1 + n_2$) is the maximum degree, so concluded by Lemma 2, we get the result such that

$$\chi_a'(P_1 \vee (P_{n_1} \cup P_{n_2})) \ge n_1 + n_2$$

Then we let f be a mapping from

$E(P_1 \vee (P_{n_1} \cup P_{n_2}))$ to $\{1,2,3,\ldots, n_1+n_2\}$ as follows :

$f(wv_j^{(1)}) = j \quad (1 \le j \le n_1) \quad f(v_1^{(1)}v_2^{(1)}) = n_1$

$f(wv_i^{(2)}) = n_1 + i \quad (1 \le j \le n_2)$

$f(v_1^{(2)}v_2^{(2)}) = n_1 + n_2$

$f(v_j^{(1)}v_{j+1}^{(1)}) = j \quad (2 \le j \le n_1 - 1)$

$f(v_i^{(2)}v_{i+1}^{(2)}) = n_1 + i \quad (2 \le i \le n_2 - 1)$

At this time, we have

$C(w) = \{1,2,3,\ldots, n_1, n_1+1,\ldots, n_1+n_2\}$

$C(v_1^{(1)}) = \{1, n_1\}$

$C(v_2^{(1)}) = \{1,2,n_1\}$

$C(v_j^{(1)}) = \{j-2, j-1, j\} \quad (3 \le j \le n_1 - 1)$

$C(v_{n_1}^{(1)}) = \{n_1 - 2, n_1\}$

$C(v_1^{(2)}) = \{n_1+1, n_1+n_2\}$

$C(v_2^{(2)}) = \{n_1+1, n_1+2, n_1+n_2\}$

$C(v_i^{(2)}) = \{n_1+i-2, n_1+i-1, n_1+i\} \quad (3 \le i \le n_2 - 1)$

$C(v_{n_2}^{(2)}) = \{n_1+n_2 - 2, n_1+n_2\}$

As defined in definition 2, obviously, f is a n_1+n_2 -AVDPEC.

There for

$$\chi_a'(P_1 \vee (P_{n_1} \cup P_{n_2})) = n_1 + n_2.$$

Corollary 2

$$\chi_a'(P_1 \vee (P_{n_1} \cup P_{n_2} \cup \ldots \cup P_{n_l})) = n_1 + n_1 + \ldots n_l.$$

The proof can be easy copied from the proof of Theorem 2.

4. Smarandachely Adjacent-vertex-distinguishing Proper Edge Chromatic Number

Defenition 3 [3] Let f be a k-proper-edge-colouring of G .Denote $C(u) = \{\{f(uv) \mid v \in V(G) \wedge uv \in E(G)\}$ for every $u \in V(G)$, if $\forall u, v \in V(G), uv \in E(G)$, we have $C(u) \not\subset C(v)$ and $C(v) \not\subset C(u)$,then f is called a smarandachely adjacent-vertex-distinguishing proper edge colouring, short for k-SA.

The number $\min\{k \mid G$ has a k smarandachely adjacent-vertex-distinguishing proper edge coloring} is called the smarandachely adjacent –vertex-distinguishing proper edge chromatic number and denoted by $\chi_{as}'(G)$.

Lemma 3: If G is a graph without one degree vertex, then $\chi_{as}'(G) \ge \Delta + 1$.

Theorem 3:

i) If $n_1 \equiv 0 \pmod 2$ and $n_2 \equiv 0 \pmod 2$,

Then $\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) = n_1 + n_2 + 1$.

ii) If $n_1 \equiv 1 (\mathrm{mod}\, 2)$ or $n_2 \equiv 1 (\mathrm{mod}\, 2)$, then

$$\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) = n_1 + n_2 + 2$$

Proof. i) Obviously, the maximum degree of $P_1 \vee (P_{n_1} \cup P_{n_2})$ denotes by Δ , then $\Delta = n_1 + n_2$, so

$$\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) \ge n_1 + n_2 + 1.$$

Then we give the mapping f from

$$E(P_1 \vee (P_{n_1} \cup P_{n_2})$$

to

$$\{1,2,\ldots, n_1+n_2\}$$

as below:

$$f(wv_j^{(1)}) = j \quad (1 \le j \le n_1)$$

$$f(wv_i^{(2)}) = n_1 + i \quad (1 \le j \le n_2)$$

$$f(v_2^{(1)}v_3^{(1)}) = n_1 + 1$$

$$f(v_{2i-1}^{(1)}v_{2i}^{(1)}) = n_1 + n_2 + 1 \quad \left(1 \le i \le \frac{n_1}{2}\right)$$

$$f(v_{2i+2}^{(1)}v_{2i+3}^{(1)}) = i \quad \left(1 \le i \le \frac{n_1}{2}\right)$$

$$f(v_{2i-1}^{(2)}v_{2i}^{(2)}) = n_1 + n_2 + 1 \quad \left(1 \le i \le \frac{n_2}{2}\right)$$

$$f(v_{2i+2}^{(2)}v_{2i+3}^{(2)}) = i \quad \left(1 \le i \le \frac{n_2}{2}\right)$$

$$f(v_{2i+2}^{(2)}v_{2i+3}^{(2)}) = n_1 + i \quad \left(1 \le i \le \frac{n_2}{2}\right)$$

$$f(v_2^{(2)}v_3^{(2)}) = 1$$

By listing $C(u)$ of every vertex of the graph, we can see that the mapping we give is a smarandachely adjacent-vertex-distinguishing proper edge coloring of $P_1 \vee (P_{n_1} \cup P_{n_2})$, so

$$\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) = n_1 + n_2 + 1 .$$

ii) First, we must illustrate that there have no $n_1 + n_2 + 1$ -SA for graph $P_1 \vee (P_{n_1} \cup P_{n_2})$.

Assume that $n_1 \equiv 1(\mathrm{mod}\, 2)$:

Suppose that $n_1 + n_2 + 1$ -SA for graph $P_1 \vee (P_{n_1} \cup P_{n_2})$ exist. And $|C(w)| = n_1 + n_2$, so we denote $C(w) = \{1, 2, 3, \ldots, n_1 + n_2\}$, $f(wv_j^{(1)}) = j$, $f(wv_i^{(2)}) = n_i + i$.

Because for all i, j,

$$c(v_j^{(1)}) / c(w) \ge 1 \quad c(v_i^{(2)}) / c(w) \ge 1 ,$$

$n_1 + n_2 + 1 \in c(v_j^{(1)}) \quad (1 \le j \le n_1)$, there is n_1 times for $c(v_j^{(1)})$, but n_1 is and odd number, the result is a contradiction to handshaking lemma.

So $\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) \ge n_1 + n_2 + 2$.

Then we define a mapping from $E(P_1 \vee (P_{n_1} \cup P_{n_2}))$ to $\{1, 2, \ldots, n_1 + n_2 + 2\}$ like this:

$$f(wv_j^{(1)}) = j \quad (1 \le j \le n_1)$$

$$f(wv_i^{(2)}) = n_1 + i \quad (1 \le j \le n_2)$$

$$f(v_{2i-1}^{(1)} v_{2i}^{(1)}) = n_1 + n_2 + 1 \quad \left(1 \le i \le [\frac{n_1}{2}]\right)$$

$$f(v_{2i}^{(2)} v_{2i+1}^{(2)}) = n_1 + n_2 + 2 \quad \left(1 \le i \le \frac{n_1}{2}\right)$$

$$f(v_{2i-1}^{(2)} v_{2i}^{(2)}) = n_1 + n_2 + 1 \quad \left(1 \le i \le [\frac{n_2}{2}]\right)$$

$$f(v_{2i}^{(2)} v_{2i+1}^{(2)}) = n_1 + n_2 + 2 \quad \left(1 \le i \le \frac{n_2}{2}\right)$$

By listing $C(u)$ of every vertex of the graph, we can see that the mapping we give is a smarandachely adjacent-vertex-distinguishing proper edge colouring of $P_1 \vee (P_{n_1} \cup P_{n_2})$.

So $\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) = n_1 + n_2 + 2$.

5. Conclusion

Through the paper's research, conclusions are follows:

Theorem 1: $\chi_{at}(P_1 \vee (P_{n_1} \cup P_{n_2})) = n_1 + n_2 + 1$

Theorem 2: $\chi_a'(P_1 \vee (P_{n_1} \cup P_{n_2})) = n_1 + n_2$

Theorem 3:

i) If $n_1 \equiv 0 \pmod 2$ and $n_2 \equiv 0 \pmod 2$,

Then $\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) = n_1 + n_2 + 1$.

ii) If $n_1 \equiv 1(\mathrm{mod}\, 2)$ or $n_2 \equiv 1(\mathrm{mod}\, 2)$, then

$$\chi_{as}'(P_1 \vee (P_{n_1} \cup P_{n_2}) = n_1 + n_2 + 2$$

Theorem 1 and Theorem 2 are consistent with the Conjecture 1 and Conjecture 2. Then in future we can also study the upper limit of the smarandachely adjacent – vertex-distinguishing proper edge chromatic number.

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