Distance Version of Forgotten Topological Index of Some Graph Products

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Abstract: In this paper, we have been found the exact values of distance version of Forgotten Topological index of various types of products such as strong, corona and tensor products of simple and connected graphs. Also we have been calculate the exact values of distance version of Forgotten Topological index of tensor product of path, cycle and complete graphs.

Keywords: Topological Indices, Strong Product, Corona Product and Tensor Product

1. Introduction

We express the vertex and the edge sets of a graph G by $V(G)$ and $E(G)$, respectively. $d_G(v)$ denotes the degree of a vertex v in G.

For any $u, v \in V(G)$, the distance between u and v in G, symbolized by $d_G(u, v)$, is the length of a shortest (u, v)-path in G.

The *strong product* [2] of the graphs G_1 and G_2, symbolized by $G_1 \boxtimes G_2$, is the graph with vertex set $V(G_1) \times V(G_2)$ and two vertices (u_1, u_2) and (v_1, v_2) are adjacent whenever $(i) u_1 = v_1$ and $u_2 v_2 \in E(G_2)$, or, $(ii) u_2 = v_2$ and $u_1 v_1 \in E(G_1)$, or, $(iii) u_1 v_1 \in E(G_1)$ and $u_2 v_2 \in E(G_2)$.

The *tensor product* of the graphs G_1 and G_2, symbolized by $G_1 \times G_2$, is the graph with vertex set $V(G_1 \times G_2)$ and $E(G_1 \times G_2) = \{(u_1, v_1)(u_2, v_2) | u_1 u_2 \in E(G_1)$ and $v_1 v_2 \in E(G_2)\}$.

The *corona product* [4] of the graphs G_1 and G_2, symbolized by $G_1 \odot G_2$, is the graph attained by taking one copy of G_1 and $|V(G_1|$ disjoint copies of G_2, and then joining the ith vertex of G_1 to every vertex in i^{th} copy of G_2.

In [14], degree distance and Gutman index of corona product of graphs are attained.

A topological index is a real number associated with the structural graph of a molecule, it is independent of representation of the structure graph. H. Wiener [1] in 1947, introduced a topological index based on the distance $d_G(u, v)$ which is named as Wiener index and it is established as

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u, v) = \frac{1}{2} \sum_{u,v \in V(G)} d_G(u, v).$$

The Wiener index based on distance between any two vertices of G is extensively studies for establishing relationships between structure and properties of molecules. Also it is used to predict biological activity of chemical compounds, see [11, 16].

There are some degree based topological indices of a graph which are known as Zagreb indices, established by Gutman et al. in [5]. The first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ of a graph G are established respectively:

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)] = \sum_{v \in V(G)} d_G^2(v)$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v)$$

The first and second Zagreb indices of a graph G are obtained in [7].

The degree distance was proposed by Dobrynin and Kochetova [1] and Gutman [6] as a weighted version of the Wiener index. The degree distance of G, symbolized by $DD(G)$, is established as

$$DD(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u,v)[d_G(u) + d_G(v)] = \frac{1}{2} \sum_{u,v \in V(G)} d_G(u,v)[d_G(u) + d_G(v)].$$

In 6, The sum of cubes of vertex degrees was involved in the investigation of the total $\pi-$ electron energy and it was again investigated by B.Furtula et. al. in [5] as "Forgotten Topological index" or "F-index" . It is established for a graph G as

$$F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

Nilanjan De et. al. in [10] derived the exact values for $F-$ index of cetain graph operations.

In [9], Muruganandam et al. have introduced the concept of distance version of F- index which is symbolized by $DF(G)$ and it is established as

$$\begin{aligned} DF(G) &= \sum_{\{u,v\} \subseteq V(G)} d_G(u,v)[d_G(u)^2 + d_G(v)^2] \\ &= \frac{1}{2} \sum_{u,v \in V(G)} d_G(u,v)[d_G(u)^2 + d_G(v)^2]. \end{aligned}$$

In this present work, we attain the absolute values of distance version of forgotten topological index of strong, corona and tensor products of graphs.

2. Basic Properties of Distance Version of Forgotten Topological Index

Lemma 2.1. [13] For $n \geq 2$,

$$\begin{aligned} &(i) \quad W(P_n) = \frac{1}{6}n(n^2 - 1) \text{ and for } n \geq 3 \\ &(ii) \quad W(C_n) = \begin{cases} \frac{n^3}{8}, & \text{if } n \text{ is even} \\ \frac{n(n^2-1)}{8}, & \text{if } n \text{ is odd} \end{cases} \end{aligned}$$

Lemma 2.2. [3, 15] For $n \geq 2$

$$\begin{aligned} &(i) \quad DD(P_n) = \frac{1}{3}n(n - 1)(2n - 1) \text{ and for } n \geq 3 \\ &(ii) \quad DD(C_n) = \begin{cases} \frac{n^3}{2}, & \text{if } n \text{ is even} \\ \frac{n(n^2-1)}{2}, & \text{if } n \text{ is odd} \end{cases} \end{aligned}$$

Lemma 2.3. [8]

$$\begin{aligned} &(i) \quad \text{For } n \geq 2 \text{ , } M_1(P_n) = 4n - 6 \\ &(ii) \quad \text{For } n \geq 3 \text{ , } M_1(C_n) = 4n \\ &(iii) \quad \text{For } n \geq 2 \text{ , } M_1(K_n) = n(n - 1)^2 \end{aligned}$$

Lemma 2.4. From the definition, the distance based of F-index for some special graphs such as complete graphs, path, cycle and complete bipartite graphs on n vertices can be easily attained as follows.

$$(i) \quad DF(K_n) = n(n-1)^3$$

$$(ii) \quad DF(P_n) = \frac{1}{3}n(n-1)(4n-5)$$

$$(iii) \quad DF(C_n) = \begin{cases} n^3, & \text{if } n \text{ is even} \\ n(n^2-1), & \text{if } n \text{ is odd} \end{cases}$$

$$(iv) \quad DF(K_{m,n}) = 2n^2m(m-1) + 2m^2n(n-1) + (n^2+m^2)nm.$$

3. Distance Version of Forgotten Topological Index of Strong Product of Graphs

In this section, we analyze the exact value of the distance distance version of forgotten topological index of strong product $G_1 \boxtimes G_2$.

Lemma 3.1. [12] The degree of the vertex (u_i, v_l) of $V(G_1 \boxtimes G_2)$ is $d_{G_1}(u_i) + d_{G_2}(v_l) + d_{G_1}(u_i)d_{G_2}(v_l)$, That is $d_{G_1 \boxtimes G_2}(u_i, v_l) = d_{G_1}(u_i) + d_{G_2}(v_l) + d_{G_1}(u_i)d_{G_2}(v_l)$,

Proof It can be easily derived from the definition of the strong product of G_1 and G_2.

Lemma 3.2. [12] Let $w_{il} = (u_i, v_l)$ and $w_{mn} = (u_m, v_n)$ be in $V(G_1 \boxtimes G_2)$. Then the distance between w_{il} and w_{mn} is

$$d_{G_1 \boxtimes G_2}(w_{il}, w_{mn}) = \begin{cases} d_{G_2}(v_l, v_n), & i = m, l \neq n \\ d_{G_1}(u_i, u_m), & i \neq m, l = n \\ d_{G_2}(v_l, v_n), & i \neq m, l \neq n \end{cases}$$

Proof It can be easily derived from the definition of the strong product of G_1 and G_2.

Theorem 3.1. Let $G_i, i = 1, 2$, be a (p_i, q_i)- graph. Then

$$\begin{aligned} 2 \times DF(G_1 \boxtimes G_2) = {} & 2p_2 DF(G_1) + 4M_1(G_2)W(G_1) + 2DF(G_1)M_1(G_2) + 8DD(G_1)q_2 \\ & + 4DD(G_1)M_1(G_2) + 8DF(G_1)q_2 + 2p_1 DF(G_2) + 4M_1(G_1)W(G_2) \\ & + 2DF(G_2)M_1(G_1) + 8DD(G_2)q_1 + 4DD(G_2)M_1(G_1) + 8DF(G_2)q_1 \\ & + 2p_1(p_1-1)DF(G_2) + 4(p_1-1)M_1(G_1)W(G_2) + 8M_1(p_1-1)DD(G_2) \\ & + 2(p_1-1)M_1(G_1)DF(G_2) + 8q_1(p_1-1)DF(G_2) \\ & + 4(p_1-1)M_1(G_1)DD(G_2) \end{aligned}$$

Proof: Let $G = G_1 \boxtimes G_2$. Then,

$$\begin{aligned} 2 \times DF(G) = {} & \sum_{w_{il}, w_{mn} \in V(G)} d_G(w_{il}, w_{mn})[d_G^2(w_{il}) + d_G^2(w_{mn})] \\ = {} & \sum_{l=0}^{p_2-1} \sum_{i,m=0, i \neq m}^{p_1-1} d_G(w_{il}, w_{ml})[d_G^2(w_{il}) + d_G^2(w_{ml})] \\ & + \sum_{i=0}^{p_1-1} \sum_{l,n=0, l \neq m}^{p_2-1} d_G(w_{il}, w_{in})[d_G^2(w_{il}) + d_G^2(w_{in})] \\ & + \sum_{l,n=0, l \neq n}^{p_2-1} \sum_{i,m=0, i \neq m}^{p_1-1} d_G(w_{il}, w_{mn})[d_G^2(w_{il}) + d_G^2(w_{mn})] \\ = {} & S_1 + S_2 + S_3, \end{aligned} \quad (1)$$

Where S_1, S_2, S_3 are the sums of the above terms in order.

We calculate S_1, S_2, and S_3 separately.

First we calculate S_1.

$$
\begin{aligned}
S_1 &= \sum_{l=0}^{p_2-1} \sum_{i,m=0,i\neq m}^{p_1-1} d_G(w_{il},w_{ml})[d_G^2(w_{il})+d_G^2(w_{ml})] \\
&= \sum_{l=0}^{p_2-1} \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)\Big[\big[d_{G_1}(u_i)+d_{G_2}(v_l)+d_{G_1}(u_i)d_{G_2}(v_l)\big]^2 \\
&\quad + \big[d_{G_1}(u_m)+d_{G_2}(v_l)+d_{G_1}(u_m)d_{G_2}(v_l)\big]^2\Big] \\
&= \sum_{l=0}^{p_2-1} \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)\Big\{\big[d_{G_1}^2(u_i)+d_{G_2}^2(v_l)+d_{G_1}^2(u_i)d_{G_2}^2(v_l)+2d_{G_1}(u_i)d_{G_2}(v_l) \\
&\quad + 2d_{G_1}(u_i)d_{G_2}^2(v_l)+2d_{G_1}^2(u_i)d_{G_2}(v_l)\big] + \big[d_{G_1}^2(u_m)+d_{G_2}^2(v_l)+d_{G_1}^2(u_m)d_{G_2}^2(v_l) \\
&\quad + 2d_{G_1}(u_m)d_{G_2}(v_l)+2d_{G_1}(u_m)d_{G_2}^2(v_l)+2d_{G_1}^2(u_m)d_{G_2}(v_l)\big]\Big\} \\
&= \sum_{l=0}^{p_2-1} \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)\Big\{[d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]+2d_{G_2}^2(v_l) \\
&\quad + [d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]d_{G_2}^2(v_l)+2[d_{G_1}(u_i)+d_{G_1}(u_m)]d_{G_2}(v_l) \\
&\quad + 2[d_{G_1}(u_i)+d_{G_1}(u_m)]d_{G_2}^2(v_l)+2[d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]d_{G_2}(v_l)\Big\} \\
&= \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)[d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]\sum_{l=0}^{p_2-1}1+2\sum_{i,m=0,i\neq m}^{p_1-1}d_{G_1}(u_i,u_m)\sum_{l=0}^{p_2-1}d_{G_2}^2(v_l) \\
&\quad + \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)[d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]\sum_{l=0}^{p_2-1}d_{G_2}^2(v_l) \\
&\quad + 2\sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)[d_{G_1}(u_i)+d_{G_1}(u_m)]\sum_{l=0}^{p_2-1}d_{G_2}(v_l) \\
&\quad + 2\sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)[d_{G_1}(u_i)+d_{G_1}(u_m)]\sum_{l=0}^{p_2-1}d_{G_2}^2(v_l) \\
&\quad + 2\sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i,u_m)[d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]\sum_{l=0}^{p_2-1}d_{G_2}(v_l) \\
&= 2p_2 DF(G_1)+4M_1(G_2)W(G_1)+2DF(G_1)M_1(G_2)+8DD(G_1)q_2+4DD(G_1)M_1(G_2)+8DF(G_1)q_2 \quad (2)
\end{aligned}
$$

Next we compute S_2.

$$
\begin{aligned}
S_2 &= \sum_{i=0}^{p_1-1} \sum_{l,n=0,l\neq n}^{p_2-1} d_G(w_{il},w_{in})[d_G(w_{il})^2+d_G(w_{in})^2] \\
&= \sum_{i=0}^{p_1-1} \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)\Big\{\big[d_{G_1}(u_i)+d_{G_2}(v_l) \\
&\quad + d_{G_1}(u_i)d_{G_2}(v_l)\big]^2 \\
&\quad + \big[d_{G_1}(u_i)+d_{G_2}(v_n)+d_{G_1}(u_i)d_{G_2}(v_n)\big]^2\Big\} \\
&= \sum_{i=0}^{p_1-1} \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)\Big\{\big[d_{G_1}^2(u_i)+d_{G_2}^2(v_l)+d_{G_1}^2(u_i)d_{G_2}^2(v_l)+2d_{G_1}(u_i)d_{G_2}(v_l) \\
&\quad + 2d_{G_1}(u_i)d_{G_2}^2(v_l)+2d_{G_1}^2(u_i)d_{G_2}(v_l)\big] + \big[d_{G_1}^2(u_i)+d_{G_2}^2(v_n)
\end{aligned}
$$

$$
\begin{aligned}
&+\quad d_{G_1}^2(u_i)d_{G_2}^2(v_n) + 2d_{G_1}(u_i)d_{G_2}(v_n) + 2d_{G_1}(u_i)d_{G_2}^2(v_n) \\
&+\quad \left.\left.2d_{G_1}^2(u_i)d_{G_2}(v_n)\right]\right\} \\
&=\quad \sum_{i=0}^{p_1-1}\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)\Big\{[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)]+2d_{G_1}^2(u_i) \\
&+\quad [d_{G_2}^2(v_l)+d_{G_2}^2(v_n)]d_{G_1}^2(u_i)+2[d_{G_2}(v_l)+d_{G_2}(v_n)]d_{G_1}(u_i) \\
&+\quad 2[d_{G_2}(v_l)+d_{G_2}(v_n)]d_{G_1}^2(u_i)+2[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)]d_{G_1}(u_i)\Big\} \\
&=\quad \sum_{i=0}^{p_1-1}1\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)] \\
&+\quad 2\sum_{i=0}^{p_1-1}d_{G_1}^2(u_i)\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n) \\
&+\quad \sum_{i=0}^{p_1-1}d_{G_1}^2(u_i)\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)] \\
&+\quad 2\sum_{i=0}^{p_1-1}d_{G_1}(u_i)\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)[d_{G_2}(v_l)+d_{G_2}(v_n)] \\
&+\quad 2\sum_{i=0}^{p_1-1}d_{G_1}^2(u_i)\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)[d_{G_2}(v_l)+d_{G_2}(v_n)] \\
&+\quad \sum_{i=0}^{p_1-1}d_{G_1}(u_i)\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}^2(v_l,v_n)[d_{G_2}^2(v_l)+d_{G_2}(v_n)] \\
&=\quad 2p_1DF(G_2)+4M_1(G_1)W(G_2)+2DF(G_2)M_1(G_1) \\
&+\quad 8DD(G_2)q_1+4DD(G_2)M_1(G_1)+8DF(G_2)q_1 \qquad\qquad (3)
\end{aligned}
$$

Finally we compute S_3.

$$
\begin{aligned}
S_3 &=\quad \sum_{l,n=0,l\neq n}^{p_2-1}\sum_{i,m=0,i\neq m}^{p_1-1} d_G(w_{il},w_{mn})[d_G(w_{il})^2+d_G(w_{mn})^2] \\
&=\quad \sum_{l,n=0,l\neq n}^{p_2-1}\sum_{i,m=0,i\neq m}^{p_1-1} d_{G_2}(v_l,v_n)\Big\{\Big[d_{G_1}(u_i)+d_{G_2}(v_l)d_{G_1}(u_i)d_{G_2}(v_l)\Big]^2 \\
&+\quad \Big[d_{G_1}(u_m)+d_{G_2}(v_n)+d_{G_1}(u_m)d_{G_2}(v_n)\Big]^2\Big\} \\
&=\quad \sum_{l,n=0,l\neq n}^{p_2-1}\sum_{i,m=0,i\neq m}^{p_1-1} d_{G_2}(v_l,v_n)\Big\{d_{G_1}^2(u_i)+d_{G_2}^2(v_l)+d_{G_1}^2(u_i)d_{G_2}^2(v_l) \\
&+\quad 2d_{G_1}(u_i)d_{G_2}(v_l)+2d_{G_1}^2(u_i)d_{G_2}(v_l)+2d_{G_1}(u_i)d_{G_2}^2(v_l)+d_{G_1}^2(u_m)+d_{G_2}^2(v_n) \\
&+\quad d_{G_1}^2(u_m)d_{G_2}^2(v_n)+2d_{G_1}(u_m)d_{G_2}(v_n)+2d_{G_1}^2(u_m)d_{G_2}(v_n) \\
&+\quad 2d_{G_1}(u_m)d_{G_2}^2(v_n)\Big\} \\
&=\quad \sum_{i,m=0,i\neq m}^{p_1-1}1\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)] \\
&+\quad \sum_{i,m=0,i\neq m}^{p_1-1}[d_{G_1}^2(u_i)+d_{G_1}^2(u_m)]\sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)
\end{aligned}
$$

$$+ \quad 2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}(v_l)$$

$$+ \quad 2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_m) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}(v_n)$$

$$+ \quad \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}^2(u_i) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}^2(v_l)$$

$$+ \quad \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}^2(u_m) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}^2(v_n)$$

$$+ \quad 2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}^2(v_l)$$

$$+ \quad 2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_m) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}^2(v_n)$$

$$+ \quad 2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}^2(u_i) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}(v_l)$$

$$+ \quad 2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}^2(u_m) \sum_{l,n=0,l\neq n}^{p_2-1} d_{G_2}(v_l,v_n)d_{G_2}(v_n)$$

$$\begin{aligned}
= \quad & 2p_1(p_1-1)DF(G_2) + 4(p_1-1)M_1(G_1)W(G_2) \\
+ \quad & 8q_1(p_1-1)DD(G_2) + 2(p_1-1)M_1(G_1)DF(G_2) \\
+ \quad & 8q_1(p_1-1)DF(G_2) + 4(p_1-1)M_1(G_1)DD(G_2) \qquad\qquad (4)
\end{aligned}$$

Using (2), (3) and (4), in (1) we get,

$$\begin{aligned}
2 \times DF(G_1 \boxtimes G_2) = \quad & 2p_2DF(G_1) + 4M_1(G_2)W(G_1) + 2DF(G_1)M_1(G_2) + 8DD(G_1)q_2 \\
+ \quad & 4DD(G_1)M_1(G_2) + 8DF(G_1)q_2 + 2p_1DF(G_2) + 4M_1(G_1)W(G_2) \\
+ \quad & 2DF(G_2)M_1(G_1) + 8DD(G_2)q_1 + 4DD(G_2)M_1(G_1) + 8DF(G_2)q_1 \\
+ \quad & 2p_1(p_1-1)DF(G_2) + 4(p_1-1)M_1(G_1)W(G_2) + 8M_1(p_1-1)DD(G_2) \\
+ \quad & 2(p_1-1)M_1(G_1)DF(G_2) + 8q_1(p_1-1)DF(G_2) \\
+ \quad & 4(p_1-1)M_1(G_1)DD(G_2)
\end{aligned}$$

4.   Distance Version of of Forgotten Toplogical Index of Corona Product of Graphs

In this section, we analyze the exactvalue of the distance distance version of forgotten topological index of corona product $G_1 \odot G_2$.

Let $V(G_1) = \{u_0, u_1, ..., u_{p_1-1}\}$ and $V(G_2) = \{v_0, v_1, ..., v_{p_2-1}\}$. For $0 \leq i \leq p_1 - 1$, denote by G_2^i the ith copy of G_2 joined to the vertex u_i and $V(G_2^i) = \{v_{i0}, v_{i1}, ..., v_{i(p_2-1)}\}$.

Lemma 4.1. [14]The degree of $w \in V(G_1 \odot G_2)$ is

$$d_{G_1 \odot G_2}(w) = \begin{cases} d_{G_1}(w) + p_2 & \text{if} w \in V(G_1) \\ d_{G_1}(w) + p_2 & \text{if} w \in V(G_1^i) \text{for some} 0 \leq i \leq p_1 - 1. \end{cases}$$

Proof It can be easily derived from the definition of the corona product of G_1 and G_2.

Lemma 4.2. [14] Let G_1 be arbitrary graph.Let G_2^i be the ith copy of G_2 in $d_{G_1 \odot G_2}$ and let $V(G_2^i) = \{v_{i0}, v_{i1}, ..., v_{i(p_2-1)}\}$. Then

$d_{(G_1 \odot G_2)}(u_i, u_m) = d_{G_1}(u_i, u_m), \ if \ 0 \leq i, m \leq p_1 - 1,$

$d_{(G_1 \odot G_2)}(u_i, v_{mn}) = d_{G_1}(u_i, u_m) + 1, \ if \ 0 \leq i, m \leq p_1 - 1, 0 \leq n \leq p_2 - 1,$

$$d_{G_1 \odot G_2}(v_{il}, v_{mn}) = \begin{cases} d_{G_1}(u_i, u_m) + 2, & if \ i \neq m, \\ 1, & if \ i = m, \ and \ v_l v_n \in E(G_2), \\ 2, & if \ i = m, \ and \ v_l v_n \notin E(G_2). \end{cases}$$

Proof It can be easily derived from the definition of the corona product of G_1 and G_2.

Theorem 4.1. Let $G_i, i = 1, 2$, be a (p_i, q_i)- graph ,put $\overline{m}_i = e(\overline{G}_i)$. Then

$$
\begin{aligned}
2 \times DF(G_1 \odot G_2) &= 2DF(G_1) + 4p_2^2 W(G_1) + 4p_2 DD(G_1) + 2p_1 \Big[2p_2 M_1(G_2) + 8p_2 q_2 \\
&+ 2p_2(p_2 - 1) - F(G_2) - 4M_1(G_2) - 10q_2 \Big] + 2p_2 DF(G_1) + 4p_2^2 DD(G_1) \\
&+ 4W(G_1)[p_2(p_2^2 + 1)M_1(G_2) + 4q_2] + 2p_1 p_2[M_1(G_1) + 4q_1 p_2] \\
&+ 2p_1^2[M_1(G_2) + 4q_2 + p_2(p_2^2 + 1)] + 4[W(G_1) + p_1(p_1 - 1)][p_2 M_1(G_2) \\
&+ 4p_2 q_2 + p_2^2]
\end{aligned}
$$

Proof: Let $G = G_1 \odot G_2$. Then,

$$
\begin{aligned}
2 \times DF(G) &= \sum_{u,v \in V(G)} d_G(u,v)[d_G(u)^2 + d_G(v)^2] \\
&= \sum_{i,m=0,i\neq m}^{p_1-1} d_G(u_i, u_m)[d_G(u_i)^2 + d_G(u_m)^2] \\
&+ \sum_{i=0}^{p_1-1} \sum_{l,n=0,l\neq m}^{p_2-1} d_G(v_{il}, v_{in})[d_G(v_{il})^2 + d_G(v_{in})^2] \\
&+ 2\sum_{m=0}^{p_1-1} \sum_{i=0}^{p_1-1} \sum_{n=0}^{p_2-1} [d_G(u_i, v_{mn})[d_G(u_i)^2 + d_G(v_{mn})^2] \\
&+ \sum_{i,m=0,i\neq m}^{p_1-1} \sum_{l,n=0}^{p_2-1} d_G(v_{il}, v_{mn})[d_G(v_{il}) + d_G(v_{mn})] \\
&= S_1 + S_2 + S_3 + S_4 \text{ where } S_1, \ S_2, \ S_3, \ S_4, \quad (5)
\end{aligned}
$$

are the sums of the above terms in order. We calculate $S_1, \ S_2, \ S_3$ and S_4 separately.

First we compute S_1

$$
\begin{aligned}
S_1 &= \sum_{i,m=0,i\neq m}^{p_1-1} d_G(u_i, u_m)[d_{G_1}(u_i)^2 + d_{G_1}(u_m)^2 \\
&= \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i, u_m)\Big[\big(d_{G_1}(u_i) + p_2\big)^2 + \big(d_{G_1}(u_m) + p_2\big)^2\Big] \\
&= \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i, u_m)\Big[d_{G_1}^2(u_i) + p_2^2 + 2p_2 d_{G_1}(u_i) + d_{G_1}^2(u_m) + p_2^2 + 2p_2 d_{G_1}(u_m)\Big] \\
&= \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i, u_m)\Big[d_{G_1}^2(u_i) + d_{G_1}^2(u_i)\Big] + 2p_2^2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i, u_m) \\
&+ 2p_2 \sum_{i,m=0,i\neq m}^{p_1-1} d_{G_1}(u_i, u_m)\Big[d_{G_1}(u_i) + d_{G_1}(u_m)\Big] \\
&= 2DF(G_1) + 4p_2^2 W(G_1) + 4p_2 DD(G_1) \quad (6)
\end{aligned}
$$

Next we compute S_2

$$
\begin{aligned}
S_2 &= \sum_{i=0}^{p_1-1} \sum_{l,n=0,l\neq n}^{p_2-1} d_G(v_{il},v_{in})\left[d_G^2(v_{il})^2 + d_G(v_{in})^2\right] \\
&= \sum_{i=0}^{p_1-1} \Big\{ \sum_{l,n=0,l\neq n,\ v_lv_n\in E(G_2)}^{p_2-1} d_G(v_{il},v_{in})\left[d_G(v_{il})^2 + d_G(v_{in})^2\right] \\
&\quad + \sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} d_G(v_{il},v_{in})\left[d_G(v_{il})^2 + d_G(v_{in})^2\right] \Big\} \\
&= \sum_{i=0}^{p_1-1} \Big\{ \sum_{l,n=0,l\neq n,\ v_lv_n\in E(G_2)}^{p_2-1} \left[\left(d_{G_2}(v_l)+1\right)^2 + \left(d_{G_2}(v_n)+1\right)^2\right] \\
&\quad + \sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} 2\left[\left(d_{G_2}(v_l)+1\right)^2 + \left(d_{G_2}(v_n)+1\right)^2\right] \Big\} \\
&= \sum_{i=0}^{p_1-1} \Big\{ \sum_{l,n=0,l\neq n,\ v_lv_n\in E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \\
&\quad + \sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} 2\left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \Big\} \\
&= \sum_{i=0}^{p_1-1} \Big\{ \Big[\sum_{l,n=0,l\neq n,\ v_lv_n\in E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \\
&\quad + \sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \Big] \\
&\quad + \sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \Big\} \\
&= \sum_{i=0}^{p_1-1} \Big\{ \sum_{l,n=0,l\neq n,}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \\
&\quad + \Big[\sum_{v_lv_n\in E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \\
&\quad + \sum_{v_lv_n\in E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \Big] \\
&\quad - \sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \Big\} \\
&= \sum_{i=0}^{p_1-1} \Big\{ 2\sum_{l,n=0,l\neq n,}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \\
&\quad - 2\sum_{l,n=0,l\neq n,\ v_lv_n\notin E(G_2)}^{p_2-1} \left[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\left(d_{G_2}(v_l)+d_{G_2}(v_n)\right)+2\right] \Big\} \\
&= \sum_{i=0}^{p_1-1} \left[4(p_2-1)q_1(G_2)+16(p_2-1)q_2+8p_2C_2-2F(G_2)-4q_1(G_2)-4q_2\right] \\
&= \sum_{i=0}^{p_1-1} \left[4p_2M_1(G_2)+16p_2q_2+4p_2(p_2-1)-2F(G_2)-8M_1(G_2)-20q_2\right] \\
&= 2p_1\left[2p_2M_1(G_2)+8p_2q_2+2p_2(p_2-1)-F(G_2)-4M_1(G_2)-10q_2\right] \quad (7)
\end{aligned}
$$

Next we compute S_3.

$$
\begin{aligned}
S_3 &= 2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1}\sum_{n=0}^{p_2-1} d_G(u_i,v_{mn})\Big[d_G(u_i)^2 + d_G(v_{mn})^2\Big] \\
&= 2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1}\sum_{n=0}^{p_2-1}\big[d_{G_1}(u_i,u_m)+1\big]\Big[\big(d_{G_1}(u_i)+p_2\big)^2 + \big(d_{G_2}(v_n)+1\big)^2\Big] \\
&= 2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1}\sum_{n=0}^{p_2-1}\Big\{ d_{G_1}(u_i,u_m)\Big[d_{G_1}^2(u_i)+2p_2 d_{G_1}(u_i)+(p_2^2+1)+d_{G_2}^2(v_n)+2d_{G_2}(v_n)\Big] \\
&\quad + \Big[d_G^2(u_i)+2p_2 d_{G_1}(u_i)+(p_2^2+1)+d_{G_2}^2(v_n)+2d_{G_2}(v_n)\Big]\Big\} \\
&= 2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1}\Big[p_2 d_{G_1}(u_i,u_m)d_{G_1}^2(u_i)+2p_2^2 d_{G_1}(u_i,u_m)d_{G_1}(u_i)+d_{G_1}(u_i,u_m)p_2(p_2^2+1) \\
&\quad + d_{G_1}(u_i,u_m)q_1(G_2)+4q_2 d_{G_1}(u_i,u_m)+p_2 d_{G_1}^2(u_i)+2p_2^2 d_{G_1}(u_i)+p_2(p_2^2+1)+q_1(G_2)+4q_2\Big] \\
&= 2\Big\{ \sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} p_2 d_{G_1}(u_i,u_m)d_{G_1}^2(u_i)+2p_2^2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} d_{G_1}(u_i,u_m)d_{G_1}(u_i) \\
&\quad + p_2(p_2^2+1)\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} d_{G_1}(u_i,u_m)+M_1(G_2)\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} d_{G_1}(u_i,u_m) \\
&\quad + 4q_2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} d_{G_1}(u_i,u_m)+p_2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} d_{G_1}^2(u_i)+2p_2^2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} d_{G_1}(u_i)+p_2(p_2^2+1)\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} 1 \\
&\quad + M_1(G_2)\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} 1 +4q_2\sum_{m=0}^{p_1-1}\sum_{i=0}^{p_1-1} 1 \Big\} \\
&= 2p_2 DF(G_1)+4p_2^2 DD(G_1)+4W(G_1)\big[p_2(p_2^2+1)M_1(G_2)+4q_2\big] \\
&\quad + 2p_1 p_2[M_1(G_1)+4q_1 p_2]+2p_1^2\big[M_1(G_2)+4q_2+p_2(p_2^2+1)\big]
\end{aligned}
$$
(8)

Finally, we calculate S_4

$$
\begin{aligned}
S_4 &= \sum_{i,m=0,i\neq m}^{p_1-1}\sum_{l,n=0}^{p_2-1} d_G(v_{il},v_{mn})\Big(d_G^2(v_{il})+d_G^2(v_{mn})\Big) \\
&= \sum_{i,m=0,i\neq m}^{p_1-1}\sum_{l,n=0}^{p_2-1}\Big(d_{G_1}(u_i,u_m)+2\Big)\Big[\big(d_{G_2}(v_l)+1\big)^2+\big(d_{G_2}(v_n)+1\big)^2\Big] \\
&= \sum_{i,m=0,i\neq m}^{p_1-1}\Big(d_{G_1}(u_i,u_m)+2\Big)\sum_{l,n=0}^{p_2-1}\Big[d_{G_2}^2(v_l)+d_{G_2}^2(v_n)+2\big(d_{G_2}(v_l)+d_{G_2}(v_n)\big)+2\Big] \\
&= \sum_{i,m=0,i\neq m}^{p_1-1}\Big(d_{G_1}(u_i,u_m)+2\Big)\,2\sum_{l,n=0}^{p_2-1}\Big[d_{G_2}^2(v_l)+2d_{G_2}(v_l)+1\Big] \\
&= 4\Big[W(G_1)+p_1(p_1-1)\Big]\Big[p_2 M_1(G_2)+4p_2 q_2+p_2^2\Big]
\end{aligned}
$$
(9)

Using $(6),(7),(8)$ and $(9),$ in (5) we get

$$
\begin{aligned}
2\times DF(G_1\odot G_2) &= 2DF(G_1)+4p_2^2 W(G_1)+4p_2 DD(G_1)+2p_1\Big[2p_2 M_1(G_2)+8p_2 q_2 \\
&\quad + 2p_2(p_2-1)-F(G_2)-4M_1(G_2)-10q_2\Big]+2p_2 DF(G_1)+4p_2^2 DD(G_1) \\
&\quad + 4W(G_1)[p_2(p_2^2+1)M_1(G_2)+4q_2]+2p_1 p_2[M_1(G_1)+4q_1 p_2] \\
&\quad + 2p_1^2[M_1(G_2)+4q_2+p_2(p_2^2+1)]+4[W(G_1)+p_1(p_1-1)][p_2 M_1(G_2)+4p_2 q_2+p_2^2]
\end{aligned}
$$

5. Distance Version of Forgotten Topological Iindex of Tensor Product of Graphs

In this section, we analyze the exact value of the distance version of forgotten topological index of tensor product $G_1 \times G_2$.

Lemma 5.1. [6]

$$
\begin{aligned}
(i) |V(G_1 \times G_2)| &= |V(G_1)||V(G_2)| \\
(ii) |E(G_1 \times G_2)| &= 2|E(G_1)||E(G_2)| \\
(iii) d_{G_1 \times G_2}(u_i, v_l) &= d_{G_1}(u_i) d_{G_2}(v_l)
\end{aligned}
$$

Proof It can be easily derived from the definition of the tensor product of G_1 and G_2.

Lemma 5.2. Let $w_{il} = (u_i, v_l)$ and $w_{mn} = (u_m, v_n)$ be in $V(G_1 \times G_2)$. Then the distance between w_{il} and w_{mn} is

$$
d_{G_1 \times G_2}(w_{il}, w_{mn}) = \begin{cases} d_{G_2}(v_l, v_n), & \text{if, } i \neq m, l \neq n \\ 0, & \text{otherwise} \end{cases}
$$

Proof It can be easily derived from the definition of the tensor product of G_1 and G_2.

Theorem 5.1. Let $G_i, i = 1, 2$, be a (p_i, q_i)- graph. Then $DF(G) = (p_1 - 1)M_1(G_1)DF(G_2)$.

Proof:

$$
\begin{aligned}
2 \times DF(G) &= \sum_{w_{il}, w_{mn} \in V(G)} d_G(w_{il}, w_{mn}) \Big[d_G^2(w_{il}) + d_G^2(w_{mn}) \Big] \\
&= \sum_{i,m=0, i \neq m}^{p_1 - 1} \sum_{l,n=0, l \neq n}^{p_2 - 1} d_G(w_{il}, w_{mn}) \Big[d_G^2(w_{il}) + d_G^2(w_{mn}) \Big] \\
&= \sum_{i,m=0, i \neq m}^{p_1 - 1} \sum_{l,n=0, l \neq n}^{p_2 - 1} d_{G_2}(v_l, v_n) \Big[d_{G_1}^2(u_i) d_{G_2}^2(v_l) + d_{G_1}^2(u_m) d_{G_2}^2(v_n) \Big] \\
&= \sum_{i,m=0, i \neq m}^{p_1 - 1} d_{G_1}^2(u_i) \sum_{l,n=0, l \neq n}^{p_2 - 1} d_{G_2}(v_l, v_n) d_{G_2}^2(v_l) + \sum_{i,m=0, i \neq m}^{p_1 - 1} d_{G_1}^2(u_m) \sum_{l,n=0, l \neq n}^{p_2 - 1} d_{G_2}(v_l, v_n) d_{G_2}^2(v_n) \Big] \\
&= (p_1 - 1)q_1(G_1)DF(G_2) + (p_1 - 1)M_1(G_1)DF(G_2) \\
2 \times DF(G) &= 2(p_1 - 1)M_1(G_1)DF(G_2) \\
DF(G) &= (p_1 - 1)M_1(G_1)DF(G_2)
\end{aligned}
$$

Corollary 5.2. Using Theorem [13], Lemmas [1], [2], [3] and [4], we obtain the exact distance Version of F- index of the following graphs.

$$
(i) \quad DF(C_n \times C_m) = \begin{cases} 4(n-1)nm^3, & \text{if } m \text{ even} \\ 4n(n-1)m(m^2 - 1), & \text{if } m \text{ odd} \end{cases}
$$

$$
\begin{aligned}
(ii) \quad DF(K_n \times K_m) &= (n-1)n(n-1)^2 m(m-1)^3 \\
&= nm(n-1)^3(m-1)^3 \\
(iii) \quad DF(C_n \times K_m) &= 4(n-1)m(m-1)^3 \\
&= 4nm(n-1)(m-1)^3 \\
(iv) \quad DF(P_n \times P_m) &= (n-1)(4n-6)\frac{1}{3}m(m-1)(4m-5) \\
&= \frac{2}{3}m(n-1)(m-1)(2n-3)(4m-5) \\
(v) \quad DF(P_n \times C_m) &= \begin{cases} (n-1)(4n-6)m^3, & \text{if } m \text{ even} \\ (n-1)(4n-6)m(m^2 - 1), & \text{if } m \text{ odd} \end{cases} \\
(vi) \quad DF(P_n \times K_m) &= (n-1)(4n-6)m(m-1)^3
\end{aligned}
$$

6. Conclusion

In this paper, we mainly survey the distance version of forgotten topological index for finding the exact values for graph products namely strong, corona and tensor of simple and connected graphs. In adddition, the exact values of distance version of forgotten topological index of tensor product of path, cycle and complete graphs have found.

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