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Numerical Study of the Effects of Unsteadiness on (MHD) Bioconvection of Nanofluids over a Stretching Sheet

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Abstract: Numerical solution of unsteady Mangneto Hydrodynamic (MHD) bioconvection of a nanofluid past over a stretching sheet wasinvestigated. Thegoverning nonlinear partia ldifferential equations (peds) of the flow are transformed into a system of coupled nonlinear ordinary differential equations (odes) using similarity transformations. These coupled ordinary differential equations are solved using fourth order Runge Kutta -Fehlberg integration method along with shooting technique. The effects of unsteadiness, Darcy number and magneticparameters were analyzed. It is found that the Skin friction, the reduced Nusselt number and the density of local microorganisms depend on the above parameters. It is equally found that as the darcy number increases the Skin friction reduces and inncrease in unsteadiness parameter reduces the Skin friction. Increase in the unsteadiness parameter reduces the density of local microorganism profile. Furthermore, increase in magnetic parameter increases the velocity. It is also observed that as the Nusselt number increases the temperature reduces. The present numerical results are compared with previously published results and are found to be in good agreement. Other results are presented graphally and in tables.

Keywords: Unsteadiness, Motile Organisms, Boundary Layer, Density, Bioconvection

1. Introduction

In recent decades nanofluids have become area of interests to scientists, engineers, mathematicians, medical specialists and health care branches, etc. Increasing attention on the use of nanofluids on convective heat transfer caused by continuously movement of fluid flow over a stretching sheet has gained importance because of many industrial and technological applications. These applications include the cooling of manufacturing filaments, polymer sheet on wind up roller etc. The emergence of biooconvetion attracted the attention of many scientists in their quest to explore the activities of motile organisms in nanofluids A newly developed flow, heat and mass transfer in nanofluids have been widely explored by many scholars. Choi [1] presented the definition of expression "Nanofluids at the ASME Winter Annual assembly, calling it fluid having a scattering of nanoparticles with normal length. Nanofluids have the higher properties of hotness conductivity effectiveness better than traditional liquid. Aziz [2], discussed a comparability answer for laminar warm limit layer on a horizontal sheet with a surface layer. The work, however revealed that a solution is sure if the heat transfer associated with the convective stream on the base surface of the sheet is constant. Makinde and Aziz [3], Ferkry et al. [10] Buogiormo [11], Anbuchezhian et al. [4] and Xu and Pop [12] concentrated on the progression of nanofluids brought about by lightness along an upward plate in a porous medium. Mushtaq et al. (2014) and Xu and Pop (2014) researched radiation impacts for two-layered stagnation-direct progression of gooey nanofluid due toward solar powered energy. Qian et al. [5] during the molecular dynamical simulation studies indicated considerable slipping interaction taken place between the liquid and solid. Navier limit boundary was consistent and is free of shear rate over contrary to the enormous scope of shear rates, thus there is slip. Xuand Pop [12] researched radiation impacts for two-layered stagnation-direct progression of gooey nanofluid due toward solar powered energy. Bondareva et al. [6] mentioned that MHD (Magneto-hydrodynamics) is the study of the movement of electrically carrying fluid affected by appealing powers. Jeffrey et al. [7] revealed exploratory perceptions of heat transfer in a film limit driven by both warming from beneath and in an upward direction sinusoidal vibrations and saw that convection move pace of delineated nano-liquid immersed with non-Darcy permeable fluid. Chamka and Ismael [13] numerically recreated unstable blockage point stream of nanofluids condition, it reveals that the twofold course of action thus exist at the insecurity limit was inadequate. Behseresht et al. [8] and Khanand Makinde [14] similarly showed that the hotness move related with nanoparticles relocation was unimportant contrasted and convection of flow fluid on the natural convection heat of nanofluids in an immersed fluid. Rahman et al. [9] and Khan et al. [15) analytically investigated the steady fluid flow convection of nanofluids past over a horizontal surfaces heet. Rahman et al. [9] and Aziz [2] presumed that the flimsy limit layer stream of a nanofluid over an extending sheet brought about by a rash movement or an unexpectedly extended surface utilizing the Keller-box strategy. The issue of regular convection limit layer stream about an upward conical shape in a permeable fluids soaked by a nanofluid due to motile organisms is introduced by Fekry et al. [10]; Buongiorno [11]; Makindeand Aziz, [3] prepared another relationship clarification for heat and mass transport of nanofluids thinking about the movement dissemination. Chamkhaand Ismael [13] and Anbuchezhian et al. [4] considered the reliable structure of normal convective heat move trio of nanofluids in a square porous opening and that was addressed as three-sided strong divider subject to a wide extent of diverse limit. Xu and Pop [16] and Mushtaq et al. [5] both concentrated on the blended convective stream of nanofluids brought about the outer tension and the torque force in the upward opening, so also the impacts of the Prandtl number and different boundary layers were also examined. Bondareva et al. [7] examine the effects of linear thermal stratification in a steady stationary ambient fluid on steady MHD convection of a nanofluid along a horizontal plate in mass transfer and magnetic effect. Recently, Kuznetsov and Nield [22] and Behseresht et al [8], logically concentrated on the regular convective limit layer stream of nanofluids past an upward plate. Bioconvection was described as suspension of gyrotactic motile microorganisms and is caused by swimming motile microorganisms, bringing in more density of the motile organisms at the datum fluid by Kuznetsov and Avramenko [16]. Falana et al. [17] saw that there is an enormous pull builds the nearby skin grating while huge infusion, however expands the neighborhood skin contact right away, by the by it diminishes the neighborhood skin grinding to a non-zero worth and the Temperature diminishes with the temperature bounce boundary and increments with the warm radiation boundary in the warm boundary. Clausing and Kempka [18] analyzed the effect of variable properties being investigated premise and definite that, the speed of hotness move Nu will be a component of radiation (Ra) just with reference temperature (Tf) which is taken as the ordinary temperature in the cut off layer frame work. Wang and Fan [19] suggested that nanofluids involved macro molecular and micro particles. Alharbi et al. [20] inferred that When expanding the worth of the attractive field boundary, the porosity factor speed profiles decline. The speed profile ascends with an ascent in the worth of couple-stress boundary K. The speed profile shows a rising element for more noteworthy upsides of λ. With the upgrade of the strength of the thermophoresis boundary and the Brownian dispersion boundary, the temperature profile increments. Few studies exist on nanofluids containing gyrotactic microorganisms over a convectively heated stretching sheet The purpose of this work is to investigate numerically effects of unsteadiness on MHD Bioconvection of nanofluids past over a stretching sheet.

2. Formulation of Problem

Two unsteady dimensional flow of electrically controlling magneto fluid with microorganisms over a horizontal sheet is considered, taken the velocity, temperature concentration and density of motile organisms at wall as U_w ,T_w ,C_w and n_w , respectivrly along the plate and T_∞,C_∞ and n_∞ away from the sheet. Where t is time, u and v are the velocity components, x and y are the Cartesian coordinates, T is the temperature, n is the density of motile microorganisms, υ is the kinematic viscosity,ρ_f is the density of the fluid,$\rho_p$is the density of the nanoparticles, $\rho_{m\infty}$is the microorganism density, δ is the fluid electrical conductivity,$B_0$is the strength of magnetic field, g is the acceleration due to gravity, β is the volumetric expansion coefficient, γ is the average volume of a micro-organism, k is the thermal conductivity, $C_p$is the specific heat at constant pressure, D_T is the thermophores is diffusion coefficient,$D_B$is the Brownian diffusion coefficient,$D_n$is the diffusivity of microorganisms. The presence of nanoparticles is assumed to have no effect on the direction in which microorganisms swim and on their swimming velocity. Magnetic strength B_0 is constructed parallel to the y-axis and induced magnetic field and the electric polarization charges are negligible. The presence of nanoparticles is assumed that there is no effect on microorganisms swim and the bio convection takes place in suspension of nanoparticles. Base on Oberbeck Boussinesq approximation, the governing equations are derived as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k}u+\frac{(1-c_\infty)\rho_{f\infty}g\beta(T_w-T_\infty)^\theta}{\rho_f}\left\{(\rho_f-\rho_{f_\infty})(C-C_\infty)+(n-n_\infty)\gamma\chi(\rho_{m\infty}-\rho_f\right\}\frac{g}{\rho_f}+\frac{\sigma B^2 u}{\rho} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{\partial q_r}{\partial y}\frac{1}{\rho c_p} \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$\frac{\partial n}{\partial t}+u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}=D_n\frac{\partial^2 n}{\partial y^2}-\frac{bW_C}{C_w-C_\infty}\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right) \tag{5}$$

Similarity variables as:

$$\eta=y\sqrt{\frac{a}{v(1-Ct)}};R_{a_x}=\frac{(1-C_\infty)\beta g\Delta T_w x^3}{\rho_{f\tilde{\alpha}}\mu};\ \psi=f(\eta)\sqrt{\frac{av}{1-Ct}};\chi(n)=\frac{n-n_\infty}{n_w-n_\infty};\ \theta=\frac{T-T_\infty}{T_w-T_\infty};\phi\frac{C-C_\infty}{C_w-C_\infty}; \tag{6}$$

$$T_w-T_\infty=\frac{a}{2vx^2}(1-Ct)^{3/2};C_w-C_\infty=\frac{a}{2vx^2}n_w-n_\infty=\frac{a}{2vx^2}(1-Ct)^{3/2} \tag{7}$$

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{8}$$

$$B(t)=B_0(1-Ct)^{1/2};$$

$$y=0;,u=au+N\frac{\partial u}{\partial y},,v=0,\ T=T_w,C=C_w,n=n_w,\ u\to 0,v\to 0.T\to T_\infty\ C=C_\infty n\to n_\infty asy\to\infty \tag{9}$$

Thus, transformed odes as

$$f'''+ff''-(f')^2-s\left[f'+\frac{1}{2}\eta f''\right]-\frac{1}{Da}f'-Mf'+G_r\theta-N_r\phi-R_b\chi=0 \tag{10}$$

$$(1+R)\theta''-\frac{1}{2Pr}s\left[\eta\theta'-3\theta\right]+2Prf'\theta+PrNb\theta'\phi+\ Nt(\theta)^2\ =0 \tag{11}$$

$$\phi''-s_c\left[\frac{1}{2}s\left(\eta\phi'-3\phi\right)-2f'\phi-f\phi'\right]+\frac{N_t}{N_b}\theta''\ =0 \tag{12}$$

$$\chi''+s_{mm}\left[s(\eta\chi'-3\chi)-2f'\chi-f\chi'\right]-p_e\left[\phi'\chi'+\phi''(\sigma+\chi)\right]=0 \tag{13}$$

Boundary conditions:

$$f'(0)=S+\delta_1 f''(0);f(0)=0;\ \theta(0)=1;,\phi(0)=1 \text{ and } \chi(0)=1 \tag{14}$$

$$f'(\eta)=0,f(\eta)=0,\theta(\eta)=0,\phi(\eta)=0 \text{ and } \chi(\eta)=0; \tag{15}$$

$$f=0,f'(0)=S+\delta f''(0);\theta(\eta)=1+\gamma\theta'(\eta),\chi=1 \text{ at } \eta=1 \text{ and } N_b\phi'+N_t\theta'=0 \tag{16}$$

$$f'(\eta)=0,\Rightarrow f(\eta)=0,\theta(\eta)=0,\phi(\eta)=0 \text{ and } \chi(\eta)=0 \tag{17}$$

Gammar, $\gamma=\frac{(\alpha vx)^{\frac{1}{2}}}{D_B\left((1-\emptyset_\infty)\beta g\Delta T_f\right)^{\frac{1}{2}}}k_1$ –, Unsteadiness parameter $s=\frac{c}{a}$ -, Magneticparameter, $M=\frac{\sigma B_0^2x^2}{\rho a}$ -, Grashof number, $G_r=\frac{(1-ct)^2}{a^2x}\cdot\frac{(1-c_\infty)\rho_{f\infty}g\beta(T_w-T_\infty)\theta}{\rho_f}$, Buoyancy parameter depending on the volumetric co-efficient of thermal expansion, $N_r=\frac{(1-ct)^2\cdot}{a^2x}\frac{(\rho_p-\rho_{f\infty})(c-c_\infty)\phi}{\rho_f\beta\Delta T(1-\emptyset_\infty)}$- Buoyancy ratioparameter, $R_b=\frac{(n_w-n_\infty)(\rho_p-\rho_{f\infty})g\gamma(1-ct)^2}{a^2x\rho_f}$ - Rayleighnumber, $R=\frac{16\sigma T_\infty^3}{3k^*k_0}$ -Radiationparameter, $R_b=\frac{v\rho c_p}{k_0}$, Prandtlnumberas, $P_r=\frac{v}{\alpha}$, $N_b=\frac{\tau D_B(C_w-C_\infty)}{v}$ -Brownian motion coefficient, Thermophores parameter, $N_t=\frac{\tau D_B(T_w-T_\infty)}{vT_\infty}$ -, $S_c=\frac{v}{D_B}$ -Schimdtnumber, $S_{mm}=\frac{v}{D_n}$ -Schmid tnumber for diffusing motile microorganisms, $\sigma\ \frac{n_\infty}{n_w-n_\infty}$, Motile parameter, $p_e=\frac{bw_c}{D_n}$bioconvection peclet number

$D_a=\frac{\mu}{k}\cdot\frac{ax}{1-ct}-$ Darcy, number, $\delta=\frac{N_1R_a^{1/4}}{x}-Delta$

The physical quantities of engineering interests are the Skin friction coefficient C_{fx}, local Nusselt number, Nu_x and local density motile microorganism χ' which are given as follows:

$$C_{fx}=\left(\frac{\tau_w}{\rho U^2/2}\right),\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\frac{1}{2}C_{fx}R_x^{\frac{1}{2}}=f''(0) \tag{18}$$

$$Nu_x=\frac{xq_w}{\kappa\Delta T},q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0},$$

$$Nu_x\Big/Re_x^{1/2}=-\theta'(0) \tag{19}$$

$$q_{n=}-D_n\left(\frac{\partial N}{\partial y}\right)_{y=0}, Nn_x = \frac{Lq_n}{D_n\Delta N}, \left(\frac{Nn_x}{Re_x^{1/2}}\right) - \chi'(0) \quad (20)$$

$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is surface heat flux, $dq_{n=}-D_n\left(\frac{\partial N}{\partial y}\right)_{y=0}$ is the surface motile microorganisms flux. $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is shear stress.

3. Numerical Method

Similarity transformations of the governing partial differential equations, (2) (5) with initial and boundary conditions into ordinary differential equations, (10) (13), with appropriate boundary condtions, (14). (17). A numerical method is employed for the solution of the ode with the boumdary conditions using fourth order Runge Kutta - Fehlberg integration method along with shooting technique The dimensionless equations are solved for the dependent variables velocity (U), skin friction (f''heat transfer ($-\theta'$), momentum (E) and motile organisms ($-\chi'$).

Table 1. *Comparison of reduced Nusselt number and reduced Sherwood number, Khan and Hady et al. [10, 20]. Gr=Nr=Ec=Pe=M=Lb=R=0, Sc=Pr=le=10 no slip condition with varying Nt and Nb.*

Nb=Nt	Khanettal. [14]	Hadyetal. [20]	Current Result
Nt=Nb=0.1	0.9541	0.9541395	0.9543299
Nt=Nb=0.2	0.3667	0.3559093	0.3559093
Nt=Nb=0.3	0.1359	0.1386993	0.1386991

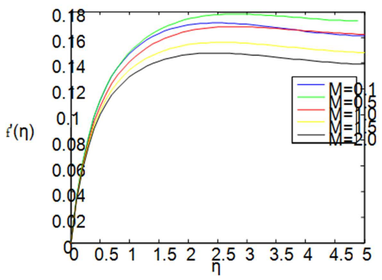

Figure 1. *Shows Magnetic Effects on the velocity profile when s=1 profile Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

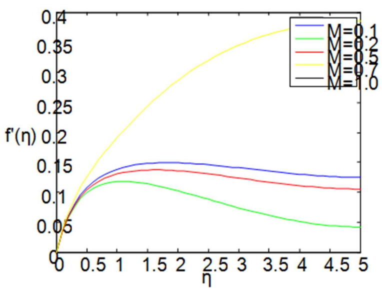

Figure 2. *Magnetic Effects on the velocity profile when s=0.5 profile Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

Figures 1-2 showed the comparative magnetic effects and unsteadiness on the velocity distribution profiles. Increase in the magnetic effects and unsteadiness aid increase in the thickness of boundary layers but decrease in unsteadiness and enhances decrease in the velocity gradient.

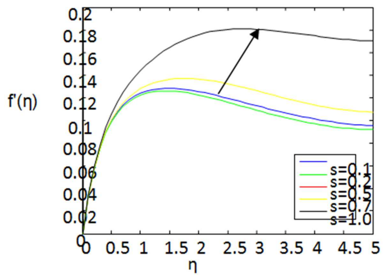

Figure 3. *Shows the effects of Unsteadiness on velocity profile profile Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

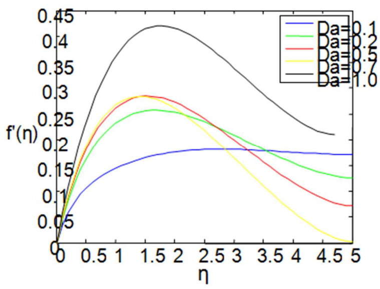

Figure 4. *Effects of Darcy number on velocity profiles profile Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

Figures 3-4 depict the graphical illustration and the comparative effects of unsteadiness parameter and Darcy number. In effect, the graphs show that both the unsteadiness parameter and darcy number behaved in different manners. However, figures 5-6 imply that the effects of unsteadiness parameter and darcy numbers on velocity profiles and skin friction. Though behave in different manners, and tend towards constant values.

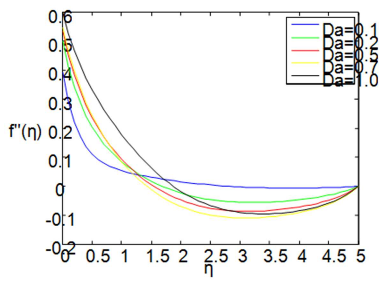

Figure 5. *Effects of Darcy number on the Skin friction s=1 Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

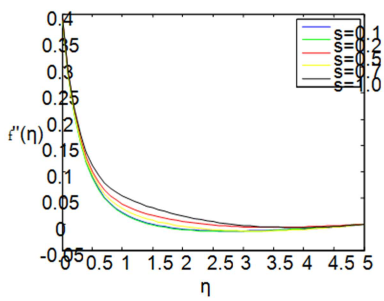

Figure 6. *Effects of unsteadiness on the Skin Friction Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

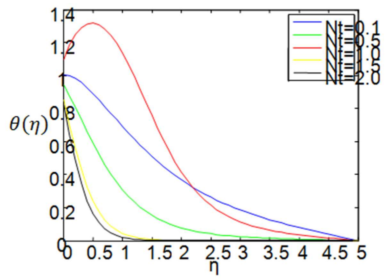

Figure 7. *Effects of Nusselt number on the temperature profiles Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

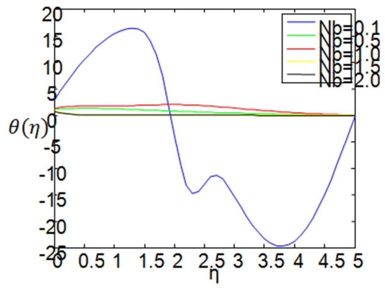

Figure 8. *Effects of Brownian motion on temperature profiles s=1 Gr=1; R=2; Pr=6.2; Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

Figure 7 toys the lines of increase in Nusselt number reduces the thermal layers on the temperature profiles for mass and heat transfer and cools to non zero values. Figure 8 depicts effects of Brownian motion on temperature profiles, increase in the parameter enhances increase in the heat transfer rate Ferkry et al. [10] and as such it enhances the cooling rate.

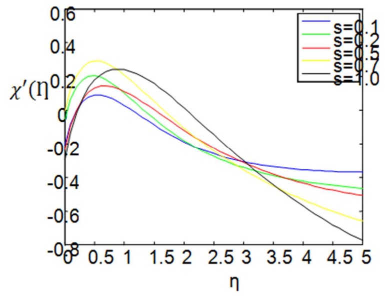

Figure 9. *Effect of unsteadiness on density of local microorganisms profile when s=1 Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; Snm=1; Da=0.1; Sc=0.1.*

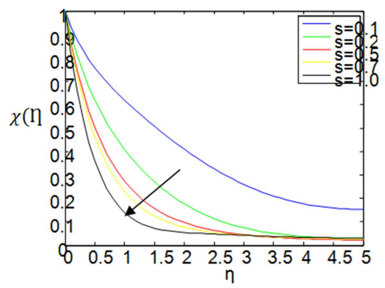

Figure 10. *Effects of unsteadiness on the rate of Density of Microorganisms profile s=1 Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

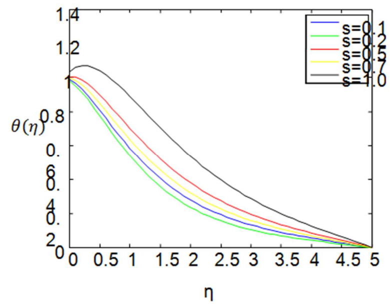

Figure 11. *Effects of unsteadiness on Temperature Profile Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Sc=0.1.*

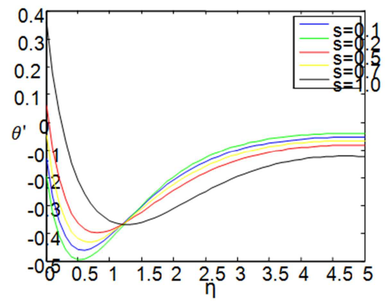

Figure 12. *Effects of Unsteadiness on Heat Transfer profile s=1 Gr=1; R=2; Pr=6.2; Nb=Rb=Nr=0.3; Pe=0.5; s=1.0; Snm=1; Da=0.1; Sc=0.1.*

Figures 9 and 10 illustrate effects of unsteadiness on density of local microorganisms and the rate of density microorganism profiles as a result of effects of its enhancement tending towards constant values which depicts the steady heat and mass transfer. The distribution profiles rise to increase in hydrodynamic thickness layer at a constant value. Figure 10 illustrates effects of unsteadiness on density of local microorganism profiles as a result of effects of its enhancement towards constant values. It depicts the steady heat and mass transfer. Figure 11 illustrates the increase in boundary layer to a constant value. Figure 12 shows the effects of unsteadiness parameter on heat transfer profile distribution. Boundary layer increases as the unsteadiness parameter increase sand the distribution profiles moves towards non zero value. The distribution tends towards constant value.

4. Conclusion

The results obtained were analyzed and computed. It was found that the Skin friction and the reduced Nusselt number and the density of local microorganisms were depending on the buoyancy, nanofluids and bioconvection parameters. It is found that the Skin friction, the reduced Nusselt number and the density of local microorganisms depend on the above parameters. It is equally found that a the darcy number increases the Skin friction reduces and increase in unsteadiness parameter reduces the skin friction. Increase in the unsteadiness parameter reduces the density of local microorganism profile. Furthermore, increase in maganetic parameter increases the velocity. It is also observed that as the Nusselt number increases the temperature reduces. The effects on unsteadiness in bio convection system enhances the quality of heat and mass transfer. The present numerical results were compared with available data and are found to be in good agreement. Other results are presented in graphical and tabular forms; are discussed accordingly to varying parameters.

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