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Modeling the spread of malaria in North-Eastern Nigeria: a case study of Adamawa state

Adamu Abdul Kareem[1], Moses Vandi Tumba[1], Dang Bwebum Cleofas[2]

[1]Mathematical Sciences Department, School of Pure & Applied Sciences, Modibbo Adama University of Technology Yola, Adamawa State, Nigeria

[2]Mathematical Sciences Department, University of Jos, Plateau State, Nigeria

Email address:

mathsmanadams@yahoo.com (A. Abdul Kareem), vandimoses@yahoo.com (M. Vandi Tumba), bwebumcd@yahoo.com (D. Bwebum Cleofas),

Abstract: This paper deals with modeling the spread of Malaria in Adamawa State of Nigeria. The simple SIR model and data obtained from the World Health Organization WHO, Adamawa was used to analyze the rate of infection of malaria in the state. It was discovered that $R_o > 0$. Based on the principle of R_o which states that when $R_o < 0$, the infection will die out with a certainty, but if $R_o > 0$, there will be a proper malaria outbreak. It was observed that the force of infection of malaria in the Adamawa state is high. Some necessary recommendations for the eradication of malaria were made.

Keywords: Disease-Free Equilibrium, Force Of Infection, Infectious disease, Jacobian Matrix, Reproduction Ratio

1. Introduction

Malaria, derived from male aria (Italian for "bad air") and formerly called ague or marsh fever in English, is an infectious disease, which cause at least 300 million infections in humans and approximately 1.3-2 million deaths annually, mainly in the Tropics, Sub-Saharan Africa (Engers and Godal, 1998). The exact statistics are unknown because many cases occur in the rural areas where people do not have access to hospital and/or the means to afford health care. Consequently, many cases are treated at home and are not documented. Malaria is the common name for diseases caused by single-celled parasites of the genus *Plasmodium*. Among the parasites of the genus *Plasmodium,* four species have been identified which can cause disease in humans. These include: Plasmodium falciparum, Plasmodium vivax, Plasmodium malaria and Plasmodium ovale. Of these, Plasmodium falciparum is of greatest risk to non-immune humans. The Plasmodium falciparum variety of parasites account for 80% of cases and 90% off deaths Malaria is a major cause of morbidity and mortality. It ranks alongside acute respiratory infections, measles and diarrhea diseases as a major cause of mortality worldwide. Malaria is spread by the bite of an infected female mosquito, of the genus anopheles each time the infected insect takes a blood meal. The symptoms in an infected human include bouts of fever, headache, vomiting flu-like, anemia (destroying red blood cell) and malaria can kill by clogging the capillaries that carry blood to the brain (cerebral malaria) or other vital organs. On the average the incubation period of Plasmodium falciparum is about 12 days in humans. Malaria is endemic in tropical areas where the climatic and weather conditions allow continuous breeding of the mosquito. Malaria is one of the most important parasitic and infectious diseases especially in tropical and subtropical areas caused by protozoan parasites of the genus plasmodium. Malaria, affecting mainly children and pregnant women is one of the greatest menaces of our society in terms of morbidity and mortality and the occurrence of malaria in our part of the world correlates with poverty and ignorance (Perandin, 2003). Malaria is a major public health problem in the world. The WHO estimates that in tropical countries among the 500 million cases of malaria infection, one million deaths occur annually. Malaria affects the health and wealth of nations and individuals alike. In Africa today, malaria is understood to be both disease of poverty and causing poverty. Malaria has significantly measurable direct and indirect cost and has recently been shown to be a major constraint to economic development. The direct cost of malaria include a combination of personal and public expenditure, include individual and family spending on Insecticide, Treated Mosquito Nets (ITNS), doctor fees, anti malaria drugs, transport to health facilities,

support for patient and sometimes an accompanying family members during hospital stays. The indirect cost of malaria includes lost productivity or low income associated with illness or death. Another indirect cost of malaria is the human pain and suffering caused by the disease. It also hampers children schooling and social development through both absenteeism and permanent neurological. The simple presence of malaria in a community also hampers individual and national prosperity due to its influence on social and economic decisions. The risk of contracting malaria in endemic area can deter investment both internally and externally and affect individual and household decision making in many ways that have negative impact on economic productivity and growth.

Several researches have been carried out on malaria and studies have been reviewed, relevant publications are the work done by Aron and May (1982), Bailey (1982), Macdonal (1957), Nasell (1987), Day *et al*. (1998), Anderson and May, (1989), and Carter *et al*. (2000). However, the increase in this epidemic in the sub-African regions has created much vacuum and left much to be desired. It is in the light of the above that this study is carried out to look at modeling the epidemiology of malaria: a case study of Adamawa State in the North Eastern Region of Nigeria. This study is of great importance to Adamawa State because a slightly high rainfall pattern is recorded in the northern part of the nation of which the state is located.

1.1. Mathematical Model

Variables and parameters: We use the following notations and/or definitions for the human populations:

Susceptibles (S)t: The number of individuals who can be infected but have not yet contracted the malaria fever but may contract it if exposed to any mode of its transmission.

Infectives I(t): The number of individuals who have been infected of malaria fever.

Recoveries R(t): The number of individuals who are recovered after treatment and are immune to the disease.

Assuming in the give population at time say t, S(t), I(t) and R(t) denote susceptible, infective carriers and recoveries respectively.

We define the following parameters as follows

N(t): The total human population size at time t.

ß: The contact rate, which takes into account the probability of getting the disease in contact between a susceptible and an infectious object.

V: The rate of recovery from the disease by an individual.

F: The force of infection (coefficient of interaction between the susceptibles and the infectious).

Once a human being recovers from the disease, the person acquires immunity against the disease. The immunity may be temporal or permanent. The natural process of reproduction also occurs in each of the compartments. Natural death (due to other diseases) also occurs in each of the compartments.

1.2. Assumptions of the Model

The disease is short lived and rarely fatal.

The disease is spread by contact between parasites and the individual host.

Individuals who recover develop immunity.

2. Equations of the Model

In view of the above assumptions and inter-relations between the variables and parameters as described in the compartmentalized model in Fig. 1, we have the following system of differential equation.

$$\frac{dS}{dt} = -\beta IS \qquad (2.1)$$

$$\frac{dI}{dt} = \beta IS - VI \qquad (2.2)$$

$$\frac{dR}{dt} = VI \qquad (2.3)$$

Analysis of the Model

Here present the results of the existence and stability analysis of the likely steady states in a human individual as far as the parasites at different stages are concerned and the state of immunity of the individual likewise the equilibrium point.

The force of infection (F)

In the above model, the function F = ßI models the transition rate from the compartment of susceptible individuals to the compartment of infectious individuals. This is the force of infection of the malaria parasites in the population. However since the population under consideration is large, it is more realistic to consider a force of infection that does not depend on the absolute number of infectious subjects, but rather on their fraction (with respect to the total population size N)

Thus

$$F = \beta \frac{1}{N}$$

The basic reproduction ratio (R_o)

The basic reproduction ratio is given by:

$$R_0 = \frac{\beta}{V}$$

This ratio is derived as the number of new infections from a single infection in a population where all subjects are susceptible. The role of the basic reproduction number in determining the rate of infection is extremely important. In fact upon rewriting the equation of the rate of infection as follows:

$$\frac{dI}{dt} = \frac{\beta}{V} \cdot VIS - VI$$

$$= R_o \, VIS - VI$$

$$= (R_O S - I)VI \qquad (2.4)$$

It yields that for $R_o > 0$ in Eq. (4) above:

$$\frac{dI}{dt} \, \square \, 0$$

Hence there will be a proper malarial outbreak in the state with an increase in the number of the infected individuals in the population (which can reach a considerable fraction of the population).The disease free equilibrium is unstable and the host will not recover from the disease

On the contrary, if $R_o < 0$,

$$\frac{dI}{dt} \, \square \, 0$$

That is independently from the initial size of the susceptible population, the disease can never cause a proper epidemic outbreak. This is the case where the human host will always recover from the disease

Equilibrium analysis:

At equilibrium:

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

$$\text{i.} \frac{dS}{dt} = \frac{dI}{dt}$$

$$-\text{ßIS} = \text{ßIS} - \text{VI}$$

$$-2\text{ßIs} = -\text{VI}$$

$$\text{ßS} = \frac{1}{2}(V)$$

$$\text{S} = \frac{V}{2\beta}$$

$$\text{S} = \left(\frac{1}{2}\right)\left(\frac{1}{R_o}\right)$$

$$\text{S} \propto \frac{K}{R_o}$$

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

$$\text{ii.} \frac{dI}{dt} = \frac{dR}{dt}$$

$$\beta \, IS - VI = VI$$

$$\beta \, IS = 2VI$$

$$\beta \, \text{S} = 2V$$

$$\text{S} = \frac{2V}{\beta}$$

$$\text{S} = 2\left(\frac{1}{R_o}\right)$$

$$\text{S} \propto \frac{K}{R_o}$$

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

$$\text{iii.} \frac{dS}{dt} = \frac{dR}{dt}$$

$$-\beta \, IS = VI$$

$$\beta \, \text{S} = -V$$

$$\text{S} = (-1)\left(\frac{1}{R_o}\right)$$

$$\text{S} \propto \frac{K}{R_o}$$

Thus, the number of susceptible increases as the basic reproduction ratio decreases and vice versa.

Existence of steady states: Let E(S*,I*,R*) be steady states such that ,

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

Then Equation (1), (2) and (3) becomes:

$$-\beta \, \text{I*S*} = 0 \qquad (2.4)$$

$$\beta \, \text{I*S*} - \text{VI*} = 0 \qquad (2.5)$$

$$\text{VI*} = 0 \qquad (2.6)$$

On existence of trivial equilibrium:

Solving Eq. (2.4), (2.5) and (2.6) at (S*, I*, R*) = (0, 0, 0) and with all parameters greater than zero, β,v >0 then β,v≠ 0. Moreover when the human host is not infected with the malaria, the individual is susceptible to the disease. This implies that there is no trivial equation and thus we cannot have (S*, I*, R*) = (0, 0, 0)

Disease free equilibrium:

In this state the individual has no malaria parasites in the body. Thus we take I* = 0. Hence from Eq. (2.5) we have:

$$\beta \, \text{I*S*} - \text{VI*} = 0$$

$$\text{S*} = \frac{V}{\beta}$$

$$S^* = \frac{1}{R_o}$$

Thus the disease free equilibrium is given as $E^*(\frac{v}{\beta}, 0, 0)$

Stability of steady states: We establish the stability of equilibrium points from the Jacobian matrix of Eq. (2.1) to (2.3) which is given as:

$$J = \begin{bmatrix} -\beta I & 0 & 0 \\ \beta I & -V & 0 \\ 0 & V & 0 \end{bmatrix} \begin{bmatrix} S \\ I \\ R \end{bmatrix}$$

$$= \begin{bmatrix} -\beta I - \lambda & 0 & 0 \\ \beta I & -V - \lambda & 0 \\ 0 & V & 0 - \lambda \end{bmatrix}$$

where λ are the eigenvalues of J

$$\text{Det}(J) = (-\beta I - \lambda) \begin{bmatrix} -V - \lambda & 0 \\ V & -\lambda \end{bmatrix} + 0$$

$$= (-\beta I - \lambda)(-V - \lambda)(-\lambda) = 0$$

$$\lambda_1 = -\beta I, \lambda_2 = -V, \lambda_3 = 0$$

It is clear that Det (J) < 0, which implies that the system is unstable, This contradicts with the initial assumption that individuals are permanently immune, Hence there is no permanent immunity in the state

3. Results and Discussion

3.1. Data collected from the World Health Organization, Adamawa State

Table 2,1 shows the statistical analysis of the state population with the target groups 2011 and Table 2,2 shows Malaria cases from 2006 to 2011 in the state, More-over, the Fig, 3,1 to 3,3 is a narrative to the basic reproductive ratio.

Table 2.1. *Adamawa state population with target groups: 2011*

Local Government	Population 1991	Population 2006	Projection 2011
Demsa	95,149	180,251	210,722
Fufore	158,137	207,287	242,329
Ganye	146,835	164,087	191,826
Gombi	88,635	146,429	171,183
Guyuk	70,528	177,785	207,839
Hong	117,240	169,126	197,717
Maiha	83,192	111,215	130,016
Mayo-belwa	125,073	153,129	179,015
Michika	117,684	155,302	181,556
Numan	130,450	90,723	106,060
Song	198,474	192,697	225,272
Jada	124,858	168,473	196,953
Madagali	90,159	134,827	157,619
Shelleng	52,477	149,069	174,269
Girei	Not created	129,995	151,970
Lamurde	Not created	112,803	131,877
Toungo	Not created	052,040	060,837
Yola North	Not created	198,247	231,760
Yola South	Not created	194,607	227,505
Mubi North	Not created	128,937	150,734
Mubi South	Not created	151,072	176,611
Total		3,168,101	3,703,664

Source: National Population Commission, Abuja, Nigeria

Table 2. 2. *Malaria Cases in Adamawa state, North-Eastern Nigeria*

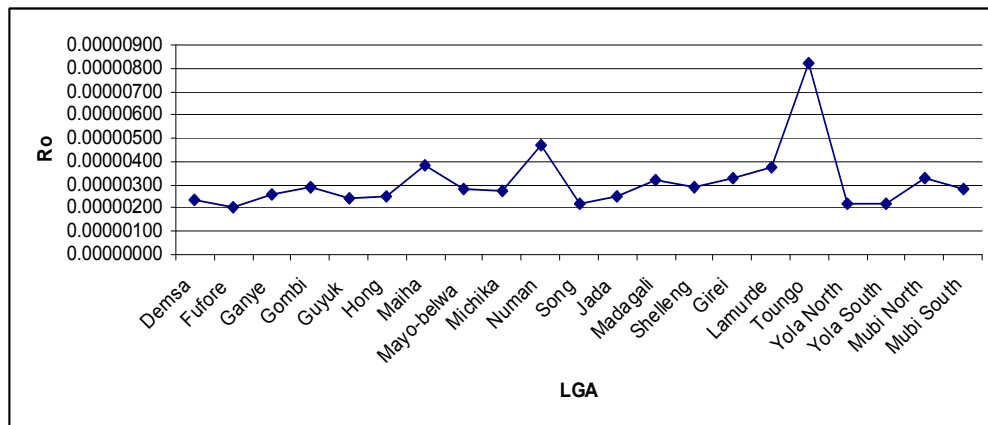| Years	cases
2006	753,752
2007	881,895
2008	1,083,109
2009	1,476,383
2010	1,859,978
2011	2,200,270

Fig. 3.1. *Graph of the basic Reproduction Ratio (R_o) in the state when K = $\overline{2}$,*
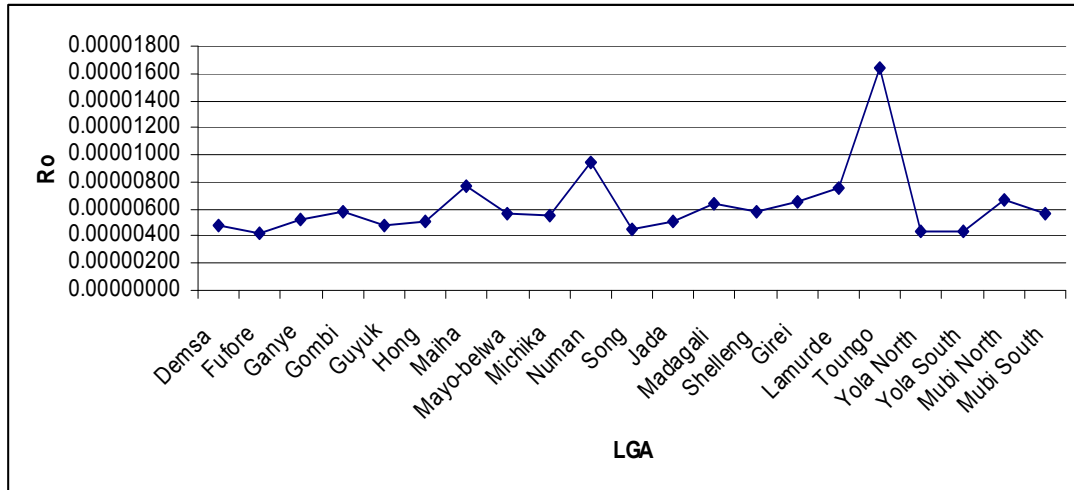
Fig. 3.2. *Graph of the basic Reproduction Ratio (R_o) in the state when K = 1,*
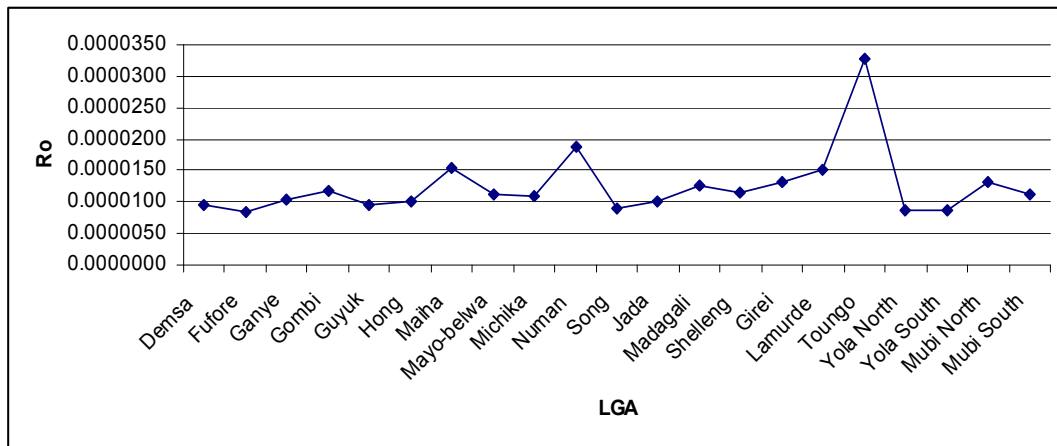

Fig. 3.3. *The basic Reproduction Ratio (R_o) in the state when K = 2.*

3.2. Calculation of the Basic Reproduction Ratio (R_0) with Target Groups: 2011

From the equilibrium analysis, it was deduced that S $\propto$ $\dfrac{K}{R_o}$, thus $R_o = \dfrac{K}{S(t)}$, for all k > 0 we have:

R_o for Demsa

$$R_o = \frac{K}{210722} = 0{,}00000475K$$

For all K = ½, 1and 2, R_o = 0,00000238, 0,00000475 and 0,0000095, respectively, $R_o > 0$

R_o for Fufore

$$R_o = \frac{K}{242329} = 0{,}00000413K$$

For all K = ½, 1and 2, R_o = 0,00000207, 0,00000413 and 0,00000826, respectively, $R_o > 0$

R_o for Ganye

$$R_o = \frac{K}{191826} = 0{,}00000521K$$

For all K = ½, 1and 2, R_o = 0,00000261, 0,00000521 and 0,0000104, respectively, $R_o > 0$

R_o for Gombi

$$R_o = \frac{K}{171183} = 0{,}00000584K$$

For all K = ½, 1and 2, R_o = 0,00000292, 0,00000584 and 0,0000117, respectively, $R_o > 0$

R_o for Guyuk

$$R_o = \frac{K}{207839} = 0{,}00000481K$$

For all K = ½, 1and 2, R_o = 0,00000241, 0,00000481 and 0,00000962, respectively, $R_o > 0$

R_o for Hong

$$R_o = \frac{K}{197717} = 0{,}00000506K$$

For all K = ½, 1and 2, R_o = 0,00000253, 0,00000506 and 0,0000101, respectively, $R_o > 0$

R_o for Maiha

$$R_o = \frac{K}{130016} = 0{,}00000769K$$

For all K = ½, 1and 2, R_o = 0,00000385, 0,00000769 and 0,0000154, respectively, $R_o > 0$

R_o for Mayo-belwa

$$R_o = \frac{K}{179015} = 0{,}00000559K$$

For all K = ½, 1and 2, R_o = 0,00000280, 0,00000559 and 0,0000112, respectively, $R_o > 0$

R_o for Michika

$$R_o = \frac{K}{181556} = 0{,}00000551K$$

For all K = ½, 1and 2, R_o = 0,00000276, 0,00000551 and 0,0000110, respectively, $R_o > 0$

R_o for Numan

$$R_o = \frac{K}{106060} = 0{,}00000943K$$

For all K = ½, 1and 2, R_o = 0,00000472, 0,00000943 and 0,0000189, respectively, $R_o > 0$

R_o for Song

$$R_o = \frac{K}{225272} = 0{,}00000444K$$

For all K = ½, 1and 2, R_o = 0,00000222, 0,00000444 and 0,00000888, respectively, $R_o > 0$

R_o for Jada

$$R_o = \frac{K}{196953} = 0{,}00000508K$$

For all K = ½, 1and 2, R_o = 0,00000254, 0,00000508 and 0,0000102, respectively, $R_o > 0$

R_o for Madagali

$$R_o = \frac{K}{157619} = 0{,}00000634K$$

For all K = ½, 1and 2, R_o = 0,00000317, 0,00000634 and 0,0000127, respectively, $R_o > 0$

R_o for Shelleng

$$R_o = \frac{K}{174269} = 0{,}00000574K$$

For all K = ½, 1and 2, R_o = 0,00000287, 0,00000574 and 0,0000115, respectively, $R_o > 0$

R_o for Girei

$$R_o = \frac{K}{151970} = 0{,}00000658K$$

For all K = ½, 1and 2, R_o = 0,00000329, 0,00000658 and 0,0000132, respectively, $R_o > 0$

R_o for Lamurde

$$R_o = \frac{K}{131877} = 0{,}00000758K$$

For all K = ½, 1and 2, R_o = 0,00000379, 0,00000758 and 0,0000152, respectively, $R_o > 0$

xvii, R_o for Toungo

$$R_o = \frac{K}{060837} = 0{,}0000164K$$

For all K = ½, 1and 2, R_o = 0,00000820, 0,0000164 and 0,0000328, respectively, $R_o > 0$

R_o for Yola North

$$R_o = \frac{K}{231760} = 0{,}00000431K$$

For all K = ½, 1and 2, R_o = 0,00000216, 0,00000431 and 0,00000862, respectively, $R_o > 0$

R_o for Yola South

$$R_o = \frac{K}{227505} = 0{,}00000440K$$

For all K = ½, 1and 2, R_o = 0,00000220, 0,00000440 and 0,00000880, respectively, $R_o > 0$

R_o for Mubi North

$$R_o = \frac{K}{150734} = 0{,}00000663K$$

For all K = ½, 1 and 2, R_o = 0,00000332, 0,00000663 and 0,0000133, respectively, $R_o > 0$

R_O for Mubi South

$$R_o = \frac{K}{176611} = 0{,}00000566K$$

For all K = ½, 1 and 2, R_o = 0,00000283, 0,00000566 and 0,0000113, respectively, $R_o > 0$

Hence there is a proper malarial outbreak in the state with an increase in the number of the infected individuals in the population (which can reach a considerable fraction of the population), The disease free equilibrium is unstable and the host will not recover from the disease.

4. Interpretation of Result

The number of new infections increases over the years in this population of which every individual is susceptible and thus the basic reproduction ratio is positive (that is $R_o > 0$), Hence there will be a proper malaria outbreak in the state with an increase in the number of the infected individuals in the population (Which can reach a considerable fraction of the population).

The force of infection increases over the years and thus the number of susceptible individuals in the population decreases making the relation S $\propto \frac{K}{R_o}$ valid.

It is seen that from 2006 to 2011, the number of infected individuals keep increasing verifying the fact that as t →∞, S(t) → 0 and I(t) → N, Considering 2011, out of a total population of 3,703,664, 2,200,270 people contracted the disease which is about 68,3% of the population, This shows a high level of infection in the population and testifies the fact that every individual in the population is susceptible.

4.1. Conclusion

In conclusion, I will like to put forward the following to help check the rate of infection if not to eradicate the malaria in the state:

As a major cause of morbidity and mortality in North Eastern part of Nigeria and the Adamawa state to be precise, malaria must be addressed.

Given the high prevalence of malaria in the state at large, all patients should always be treated for malaria first.

Early treatment is the key, Early recognition of symptoms by patients and early diagnosis by physicians will prevent complications.

4.2. Recommendation

To eradicate malaria fever from Adamawa state, the following recommendations should be taken into consideration:

Malaria prevention must be a public health priority.

The government should provide inhabitants of the state a clean environment and clean drainage system.

Vaccines against malaria fever should be made available by the World Health Organization and should be used everywhere in the state.

The battle against malaria can only be won through prevention.

The public should undertake practices which can limit malaria transmissions by trying to reduce the number of mosquitoes and mosquito bites through the use of bed nets, spraying and wearing long sleeves shirts and trousers to reduce the amount of exposed skin.

The public should remove potential mosquito breeding sites and so reduce the number of mosquitoes.

References

[1] Anderson, R,M, and R,M, May, 1981, The population dynamics of micro parasites and their invertebrate hosts, Phil, Trans, R, Soc, Lond, B,, 291: 451-524.

[2] Aron, J,L, and R,M, May, 1982, The Population Dynamics of Malaria, In: Anderson, R,M, (Ed,), The Population Dynamics of Infectious Diseases: The theory and Applications, London, pp: 139-179, Bailey, N,T,J,, 1982, The Biomathematics of Malaria, Charles Griffin and Company Ltd,, London.

[3] Carter, R,, K,N, Mendis, L,H, Miller, L, Monlineaux and A,L, Soul, 2000, Malaria transmission – blocking vaccines - how can their development be supported? Nature, 6: 241-244.

[4] Day, K,P,, R,E, Hayward and M, Dyer, 1998, The biology of plasmodium falcparum transmission stages, Parasitology, 116(Suppl): S95-S109.

[5] Engers, H,D, and T, Godal, 1998, Malaria Vaccine Development; Current Status, Parasitol, Today, Trend, Parasitol,, 14: 56-64.

[6] Macdonal, G,, 1957, The Epidemiology and Control of Malaria, Oxford University Press, Oxford.

[7] Nasell, I,, 1987, Malaria Infection with Relapses and Misdiagnosis, In: Gabriel, J,P,, C, Lequere and P, Picard, (Eds,), Stochastic Process in Epidemic Theory, Lecture Notes Biomath, 86, Springer Verlag, New York.

[8] Perandin F, (2003), Development of a Real-time PCR assay for detection of plasmodium falciparum, plsmodium vivax, and plasmodium ovale for routine clinical diagnosis, Journal of clinical microbiology, 42 (3), 1214-1219, A Moody, Rapid diagnostic tests for malaria parasites, Clin Microbiol Rev 15 (2002), pp, 66–78.

[9] World Health Organisation (2011), Routine Yearly Notification Form: IDSR 2006-2011.