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Chemically reacting unsteady magnetohydrodynamic oscillatory Slip flow of a micropolar fluid in a planer channel with varying concentration

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Abstract: The study examines the problem of unsteady MHD mixed convection in a micropolar fluid with oscillatory flow of an electrically conducting optically thin fluid through a planer channel filled with saturated porous medium. The effect of buoyancy, heat source, thermal radiation and chemical reaction are taken into account embedded with slip boundary condition, varying temperature and concentration. The closed-form analytical solutions are obtained for the momentum, angular momentum, energy and concentration equations. The influence of the various parameters entering into the problem in the velocity, temperature and concentration fields are discussed with the help of graphs. Finally the effects of the pertinent parameters on the skin friction, couple stress and the rate of heat transfer coefficient at the plate are discussed.

Keywords: Micropolar Fluid, Oscillatory Flow, Thermal Radiation, Chemical Reaction, Heat and Mass Transfer, Planer Channel

1. Introduction

Oscillatory flows has known to result in higher rates of heat and mass transfer, many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors. Cooper et al.[1] have made a detailed study on fluid mechanics of oscillatory and modulated flows and associated applications in heat and mass transfer. Fusegi [2] have numerically studied the influence of convective heat transfer from periodic open cavities in a channel with oscillatory flow. Cheng et al. [3] and Hamadah and Wirtz [4] have studied the mixed convection in a vertical channel with symmetric and asymmetric heating of the walls. Barletta [5] has studied the fully developed combined free and forced convection flow in a vertical channel with viscous dissipation. Umavathi et al. [6] have numerically investigated the problem of mixed convection in a vertical channel filled with a porous medium including the effect of inertial forces is studied by taking into account the effect of viscous and Darcy dissipation. Prathap kumar et al. [7] have analyzed the problem of fully developed combined free and forced convective flow in a fluid saturated porous medium channel bounded by two vertical parallel plates. Pop et al. [8] have investigated the steady fully developed mixed convection flow in a vertical channel with constant temperature walls when there is a heat generated by an exothermic reaction inside the channel. Gomaa and Taweel [9] have examined the effect of oscillatory motion on heat transfer about vertical flat surfaces. The heat transfer enhancement of oscillatory flow in channel with periodically upper and lower walls mounted obstacles has been analyzed by Abdelkader and Lounes [10]. MHD has attracted the attention of many researchers and industrialists due to its rich applications in cosmic fluid dynamics, meteorology, motion of Earth's core and solar physics. El-Hakiem [11] has examined the influence of MHD oscillatory flow on free convectionradiation through a porous medium with constant suction velocity. Makinde and Mhone [12] have investigated the problem of heat transfer to MHD oscillatory flow in a channel filled with porous medium. The effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel has been analyzed by Mehmood and Ali [13]. The wide range of technological and industrial applications have stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Gholizadeh [14] has investigated the MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source. The work of Makinde [15] is of particular interest since it demonstrated the possibility of achieving significant unsteady incompressible flow in a porous channel. Makinde and Aziz [16] have analyzed the MHD mixed convection from a vertical plate embedded in porous medium with convective boundary condition.

The role of thermal radiation is of major importance in engineering areas occurring at high temperatures and knowledge of radiative heat transfer becomes very important in nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles. Hakeem and Sathiyanathan [17] have examined the radiation effect of an oscillatory flow through a porous medium. Srinivas and Muthuraj [18] have studied the effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel. Pal and Talukdar [19] have analyzed the unsteady MHD convective heat and mass transfer in a vertical permeable plate with thermal radiation. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Ibrahim et al. [20] analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Bakr [21] have studied the effects of chemical reaction on oscillatory plate velocity and constant heat source in a rotating frame of reference. Bakr and Raizah [22] have analyzed the unsteady MHD mixed convection flow of a viscous dissipating micropolar fluids in a boundary layer slip flow regime with Joule heating. The influence of chemical reaction on unsteady MHD mixed convective flow over a moving vertical porous plate has been examined by Prakash et al. [23]. To the best of the author's knowledge, studies pertaining to oscillatory flow of a micropolar fluid investigations in a planer channel with variable temperature and concentration have not received much attention. Therefore, the main goal here is to study the chemical reaction effects on unsteady MHD oscillatory slip flow in an optically thin fluid through a planer channel in the presence of a temperature-dependent heat source. The closed form solutions for velocity, temperature, skin friction, concentration, Nusselt number, and Sherwood number are presented. The effects of pertinent parameters on fluid flow and heat and mass transfer characteristics are studied in detail. This work is presented as follows. First, the problem is formulated, and then the solution of the problem is presented. Following are results and discussion, and finally, conclusions are summarized.

2. Mathematical Formulation of the Problem

We consider the unsteady mixed convection flow of a micropolar fluid, two dimensional slip flow of an electrically conducting, heat generating, optically thin and chemically reacting oscillatory fluid flow in a planer channel filled with porous medium in the presence of thermal radiation with temperature and concentration variation. Take a Cartesian coordinate system (X,Y) where X − axis is taken along the flow and Y − axis is taken normal to the flow direction. A uniform transverse magnetic field of magnitude B0 is applied in the presence of thermal and solutal buoyancy effects in the direction of Y − axis. Under these assumptions, the governing equations of the problem become:

$$\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\begin{aligned}\frac{\partial U}{\partial t^*}&=-\frac{1}{\rho}\frac{\partial P^*}{\partial X}+(\upsilon+\upsilon_r)\frac{\partial^2 U}{\partial Y^2}+g_0\beta_T(T-T_1)\\&+g_0\beta_C(C-C_1)-\left(\sigma\frac{B_0^2}{\rho}+\frac{\nu}{K^*}\right)U-\upsilon_r\frac{\partial W}{\partial Y}\end{aligned} \tag{2}$$

$$\frac{\partial W}{\partial t^*}=\frac{\gamma}{\rho\lambda^*}\frac{\partial^2 W}{\partial Y^2} \tag{3}$$

$$\frac{\partial T^*}{\partial t^*}=\frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}-\frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}+\frac{Q(T-T_1)}{\rho C_p} \tag{4}$$

$$\frac{\partial C^*}{\partial t^*}=D_m\frac{\partial^2 C^*}{\partial y^{*2}}-K_R(C-C_1) \tag{5}$$

$$\begin{aligned}&U=L_1\frac{\partial U}{\partial Y},W=-n\frac{\partial U}{\partial Y},\quad T=T_1+\delta_T^*\frac{\partial T}{\partial Y},\\&C=C_1+\delta_C^*\frac{\partial C}{\partial Y}\qquad at\quad Y=0\\&U=W=0,\,T=T_2+\delta_T^*\frac{\partial T}{\partial Y},C=C_2+\delta_C^*\frac{\partial C}{\partial Y}\;at\;Y=d\end{aligned} \tag{6}$$

Thus the radiative heat flux term (Cogley et al. [22]) is given by

$$\frac{\partial q^*}{\partial y^*}=4(T_1-T)I' \tag{7}$$

Introducing the following dimensionless variables

$$
\begin{aligned}
&y=\frac{Y}{d}, x=\frac{X}{d}, u=\frac{U}{U_0}, t=\frac{t^* U_0}{d}, \delta_C=\frac{\delta_C^*}{d} \\
&\theta=\frac{T-T_1}{T_2-T_1}, \phi=\frac{C-C_1}{C_2-C_1}, \mathrm{Pr}=\frac{\mu c_p}{k},\ P=\frac{dP^*}{\mu U_0}, \\
&Gc=\frac{g\beta_C(C_2-C_1)d^2}{\upsilon U_0}, Sc=\frac{D}{U_0 d}, M^2=\frac{\sigma_e B_0^2 d^3}{\mu}, \\
&K=\frac{K^*}{d^2}, \mathrm{Re}=\frac{U_0 d}{\upsilon}, Pe=\frac{\rho C_P U_0 d}{k}, \alpha=\frac{Qd^2}{k}, \\
&F=\frac{4I'd^2}{k}, K_r=\frac{K_R d}{U_0}, \gamma=\frac{\gamma^*}{d}, \omega=\frac{k^2}{\upsilon Grh^2}\omega \\
&\delta_T=\frac{\delta_T^*}{d}, \Delta=\frac{\upsilon_r}{\upsilon}\ \eta=\frac{\rho\lambda\upsilon}{\gamma}, Gr=\frac{g\beta_T(T_2-T_1)d^2}{\upsilon U_0}.
\end{aligned}
\tag{8}
$$

In view of the above non-dimensional variables, the basic field Eqs. (2)–(7) can be expressed in non dimensional form as

$$
\begin{aligned}
&\mathrm{Re}\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial x}+(1+\Delta)\frac{\partial^2 u}{\partial y^2}+Gr\,\theta+Gc\,\phi- \\
&(M+\frac{1}{K})u+2\Delta\frac{\partial\omega}{\partial y}
\end{aligned}
\tag{9}
$$

$$
\frac{\partial\omega}{\partial t}=\frac{1}{\eta}\frac{\partial^2\omega}{\partial y^2}
\tag{10}
$$

$$
Pe\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}-(F+\alpha)\theta
\tag{11}
$$

$$
\frac{\partial\phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-R\phi
\tag{12}
$$

The boundary conditions (6) can be written in non-dimensional forms as:

$$
\begin{aligned}
&u=\gamma\frac{\partial u}{\partial Y}, \omega=-n\frac{\partial u}{\partial Y}, \theta=\delta_T\frac{\partial\theta}{\partial Y}, \phi=\delta_C\frac{\partial\phi}{\partial Y}\quad at\quad y=0 \\
&u=\omega=0, \theta=1+\delta_T\frac{\partial\theta}{\partial Y}, \phi=1+\delta_C\frac{\partial\phi}{\partial Y}\quad at\quad y=1
\end{aligned}
\tag{13}
$$

3. Method of Solution

In this section we present the analytical solution for Eqs. (9) and (12) with boundary conditions (13) for purely oscillatory flow, let us take

$$
\begin{aligned}
&-\frac{\partial P}{\partial x}=Ae^{iwt}, u(y,t)=u_0(y)e^{iwt}, \omega(y,t)=\omega_0(y)e^{iwt} \\
&\theta(y,t)=\theta_0(y)e^{iwt}, \phi(y,t)=\phi_0(y)e^{nt}
\end{aligned}
\tag{14}
$$

Substituting Egs. (14) into the Eqs. (9)-(13) and comparing the harmonic and non-harmonic terms, we get:

$$
(1+\Delta)u''-A_1^2 u_0=-\lambda-Gr\,\theta_0-Gc\phi_0-2\Delta\omega_0'
\tag{15}
$$

$$
\omega_0''-A_6^2\omega_0=0
\tag{16}
$$

$$
\theta_0''-A_2^2\theta_0=A_4\phi_0''
\tag{17}
$$

$$
\phi_0''-A_3^2\phi_0=A_4\theta_0''
\tag{18}
$$

Here primes denote differentiation with respect to y. However, this expansion of the solution is meaningful only if the reduced equations are ordinary differential equations of the independent variable y. In addition, the corresponding boundary conditions can be written as

$$
\begin{aligned}
&u_0=\gamma u_0', \omega_0=-\frac{1}{2}u_0', \theta_0=\delta_T\theta_0', \phi_0=\delta_C\phi_0'\quad at\quad y=0 \\
&u_0=0, \omega_0=0, \theta_0=1+\delta_T\theta_0', \phi_0=1+\delta_C\phi_0'\quad as\quad y=1
\end{aligned}
\tag{19}
$$

Solving Eqs. (15)-(18) subject to the boundary conditions (19) we obtain

$$
\begin{aligned}
u=(&h_7+h_8e^{-A_2y}+h_9e^{A_2y}+h_{10}e^{-A_3y} \\
&+h_{11}e^{A_3y}+A_5e^{-N_3y}-h_{12}\,h_6e^{A_4y} \\
&+h_{13}\,h_6e^{A_4y}+h_{14}e^{-A_5y}+h_{15}e^{A_5y})e^{iwt}
\end{aligned}
\tag{20}
$$

$$
\omega=A_7e^{iwt}(h_6e^{A_4y}-h_6e^{-A_4y})
\tag{21}
$$

$$
\theta=(h_3e^{-A_2y}+h_4e^{A_2y})e^{iwt}
\tag{22}
$$

$$
\phi=(h_1e^{-A_3y}+h_2e^{A_3y})e^{iwt}
\tag{23}
$$

where the constants are given in Appendix. The physical quantities of engineering interest are shear stress, couple stress coefficient, the coefficient of the rate of heat transfer and the rate of mass transfer at any point in the fluid can be characterized by

$$
\begin{aligned}
&\tau=\frac{\tau^*}{\mu U_0}=[1+\Delta(1+\frac{i}{2})]u'(0);\ C_w=\frac{\partial\omega}{\partial y}; \\
&\mathrm{Nu}=\frac{Nu^*\,d}{T_1-T_0}=-\theta';\ Sh=\left(\frac{Sh^*d}{C_1-C_0}\right)=-\phi'
\end{aligned}
\tag{24}
$$

The skin friction, the Nusselt number and the Sherwood number at the walls $y=0$ and $y=1$ are given by couple stress coefficient at the wall C_w is given by

$$
\begin{aligned}
&\tau_0=-u'\big|_{y=0} && \tau_1=-u'\big|_{y=1} \\
&C_{w0}=\omega'\big|_{y=0} && C_{w1}=\omega'\big|_{y=1} \\
&\mathrm{Nu}_0=-\theta'\big|_{y=0} && Nu_1=-\theta'\big|_{y=1} \\
&\mathrm{Sh}_0=-\phi'\big|_{y=0} && Sh_1=-\phi'\big\|_{y=1}
\end{aligned}
\tag{25}
$$

4. Results and Discussion

System of equations (16) to (19) subject to the boundary conditions (20) are highly coupled and solved analytically. In order to understand the physical solution, the numerical values of concentration, transverse velocity, angular velocity and temperature are presented. Numerical evaluation for the analytical solution of this problem is performed and the results are illustrated graphically in Figs. 1–16 to show the interesting features of significant parameters on velocity, temperature and concentration distributions in the planer channel. Throughout the computations we employ $t = 1, \lambda = 1, w = 0.5, M = 2, K = 2, Re = 0.2, Gr = 10, Gm = 4, F = 2, \alpha = 3, Pe = 2, Kr = 2, Sc = 1, \gamma = 0.1, \delta T = 0.002, \Delta = 0.1$ and $\delta C = 0.002$ unless otherwise stated. In Figure 1, the effect of Δ on the translational velocity u and angular velocity ω for a stationary porous plate is shown. It is observed that, as the viscosity ratio parameter Δ is increased, u is decreases and ω is increased. Figure 2 illustrates that the presence of transverse magnetic field produces a resistive force on the fluid flow. This force is called the Lorentz force, which slows down the motion of the fluid. It is obvious that the increases in the frequencies of oscillation decrease the velocity and this is presented in Fig. 3. Figure 4 displays that the increases in the permeability coefficient of porous medium act against the porosity of the porous medium which increase the fluid velocity. Figure 5 illustrates that increase in the radiation parameter increases the temperature distribution because large values of radiation parameter oppose the conduction over radiation, thereby which increases the buoyancy force and increases the thickness of the thermal boundary layer. Figure 6 represents that the increase in the heat source parameter significantly increase the thermal buoyancy effects which raise fluid temperature. Increase in temperature variation parameter coincides with the decrease of heat transfer and the curves could be seen in Fig. 7 It is observed from Fig. 8 that the effect of raising Peclet number develop the thermal conductivities and therefore heat is able to diffuse away and the heat transfer falls monotonically. Figure 9 shows that we obtain a destructive type chemical reaction because the concentration decreases for increasing the chemical reaction parameter which indicates that the diffusion rates can be tremendously changed by the chemical reaction. Figs. 10 illustrate that concentration variation parameter are used to increase the mass transfer. In Figure 11-13, the effect of K, M and Pe on angular velocity ω for a stationary porous plate is shown. It is observed that, as the angular velocity ω is increased, K is decreases and M and Pe is increased. Table I shows that the effect of D, M, K and Re in skin friction at the walls. Table II shows that the effect of thermal radiation parameter, heat source parameter, Peclet number and temperature variation parameter in skin friction and Nusselt number at the walls. The influence of frequency of oscillation parameter, chemical reaction parameter, Schmidt number and concentration variation parameter in skin friction and Sherwood number at the walls are present in table III.

***Table I**. Effect of F, α, Pe and δT in skin friction at the walls.*

	M	K	Re	τ_0	τ_1
1				-0.798	2.123
5	2	2	0.2	-1.03	2.366
10				-1.151	2.428
	1			-0.987	2.33
0.1	2	2	0.2	-0.718	1.963
	3			-0.482	1.606
		0.1		-0.341	1.408
0.1	2	0.2	0.2	-0.454	1.633
		0.3		-0.507	1.735
		0.4		-0.572	1.859
			1	-0.726	2.054
0.1	2	2	2	-0.481	1.872
			3	-0.266	1.687

***Table II.** Effect of F, α, Pe and δT d in skin friction and Nusselt number at the walls*

	T	F	Pe	τ_0	τ_1	Nu0	Nu1
0				-0.797	2.055	-0.339	1.438
1	0.002	2	2	-0.778	2.014	-0.319	1.322
2				-0.76	1.974	-0.3	1.209
	1			0.318	-1.656	0.028	0.168
2	2	2	2	0.246	-1.065	0.017	-0.637
	3			0.22	-0.886	0.012	-0.836
		0		-0.604	2.037	-0.319	1.322
2		1	2	-0.614	1.997	-0.3	1.209
	0.002	2		-0.622	1.959	-0.28	1.099
		3		-0.629	1.921	-0.261	0.994
			0	-0.641	1.635	-0.191	-0.722
			0.2	-0.708	1.748	-0.289	-0.416
2	0.002	2	0.5	-0.783	1.884	-0.398	-.00223
			0.8	-0.822	1.972	-0.453	0.345

***Table III.** Effect of ω, Kr, Sc AND δC in skin friction and Sherwood number at the walls*

Sc	c	Kr		τ_0	τ_1	Sh0	Sh1
0.05				-0.531	1.65	0.00234	-4.668
0.1	0.002	2	7.5	-0.541	1.724	-0.02	-3.242
0.2				-0.573	1.813	-0.062	-2.263
	0			-0.582	1.804	-0.019	-1.761
	0.1			-0.617	1.625	0.02	-7.266
	0.2			-0.56	1.002	0.058	-7.982
		0		-0.795	2.044	0.775	-1.191
		1		-0.769	2.006	0.679	-1.005
		2		-0.746	1.971	0.595	-0.836
			0.1	-1.182	3.542	0.728	1.585
			0.3	-1.145	3.416	0.707	1.502
			0.6	-1.021	3.008	0.636	1.231

5. Appendix

$$A_1 = \sqrt{M + iw\text{Re} + \frac{1}{K}}, \; A_2 = \sqrt{F + \alpha + iwPe}$$

$$A_3 = \sqrt{\frac{iw + K_r}{Sc}}, \; A_5 = \frac{A1}{\sqrt{1+\Delta}}, \; h1 = \frac{h2\,(A3\,Sc - 1)}{(1 + A3\delta c)}$$

$$h2 = \frac{A3\,\delta c + 1}{e^{-A3}(A3\,\delta c - 1)(1 + A3\delta c) + e^{A3}(1 - A3\,\delta c)(1 + A3\delta c)}$$

$$h4 = \frac{A2\,\delta c + 1}{e^{-A2}(A2\delta_T - 1)(1 + A2\delta_T) + e^{A3}(1 - A2\delta_T)(1 + A2\delta_T)}$$

$$h7 = \frac{\lambda}{A1},\ h8 = \frac{-Gr\,h3}{(1+\Delta)A2^2 - A1^2}\ ,h9 = \frac{-Gr\,h4}{(1+\Delta)A2^2 - A1^2}$$

$$h11 = \frac{-Gc\,h2}{(1+\Delta)A3^2 - A1^2}\ ,h12 = \frac{2\Delta A4}{(1+\Delta)A4^2 - A1^2}$$

$$B1 = h7 + h8e^{-A2} + A9e^{A2} + h10e^{-A3} + A11e^{A3}$$

$$B2 = h7 + h8(1 + \gamma A2) + h9(1 - \gamma A2) + h10(1 + \gamma A3) + h11(1 - \gamma A3)$$

$$B3 = n\,(A2h8 - - A2h9 + A3h10 - A3h11)$$

$$B5 = h12\ e^{2A4}\,(1 + \gamma A4) + h13(\gamma A4 - 1)$$

$$B6 = 1 - e^{2A4}\,(1 - n\,A4h12) + n\,h13\,A4$$

$$h10 = \frac{-Gc\,h1}{(1+\Delta)A3^2 - A1^2}\qquad B7 = B4(1 + \gamma A5) - B5\,e^{-A5}$$

$$h13 = \frac{-2\Delta A4}{(1+\Delta)A4^2 - A1^2}\ ,B8 = B4(\gamma A5 - 1) + B5\,e^{A5}$$

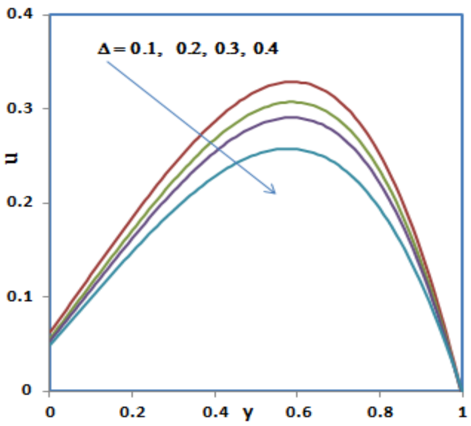
$$B9 = B1\,B5 - B2\,B4\ ,B10 = n\ B4\ A5 - B6\,e^{-A5}$$

$$B11 = n\ B4\ A5 + B6\,e^{A5}\ ,B12 = B1\ B6 - B3\,B4$$

$$h15 = \frac{B10\ B9 - B12\,B7}{B11\ B7 - B8\,B10},$$

$$h5 = -h6e^{2A4}, h6 = \frac{B1 + h14e^{-A5} + h15e^{A5}}{B4}$$

$$h14 = \frac{B9 + B8\,h15}{B7}$$



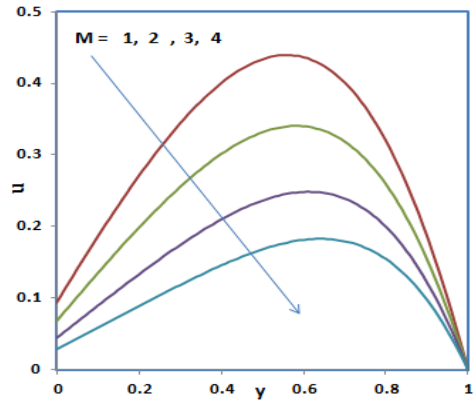
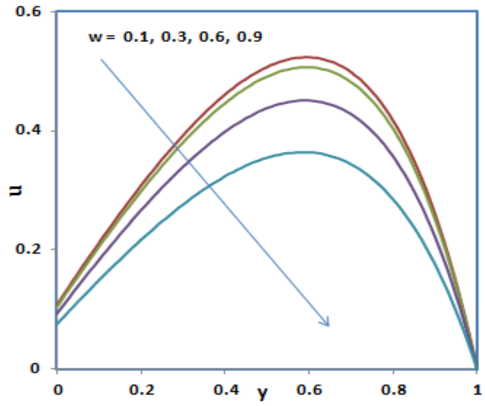


Fig (1, 2). Velocity profiles for different Δ and M.



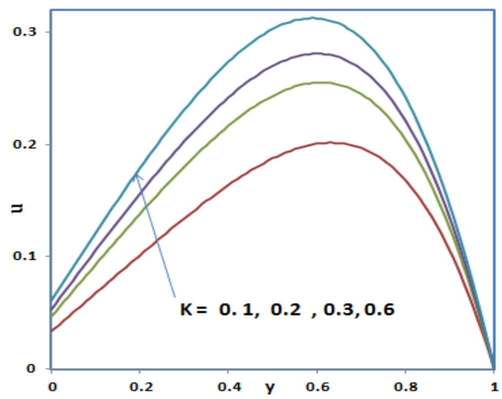
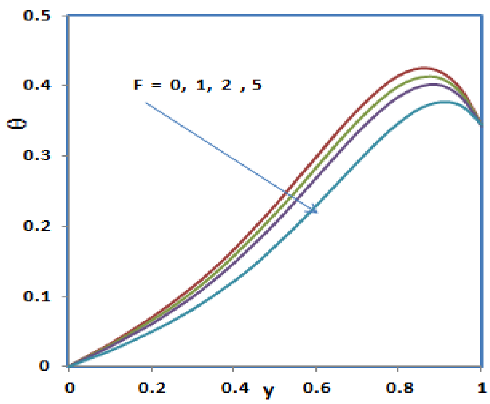


Fig (3, 4). Velocity profiles for different w and K.



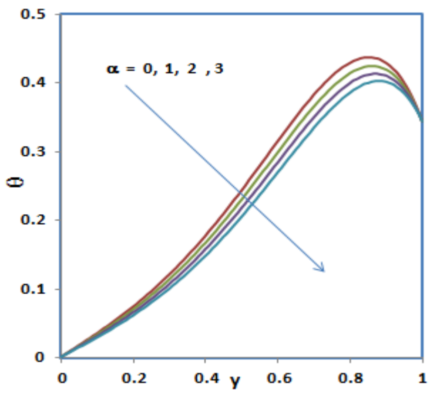
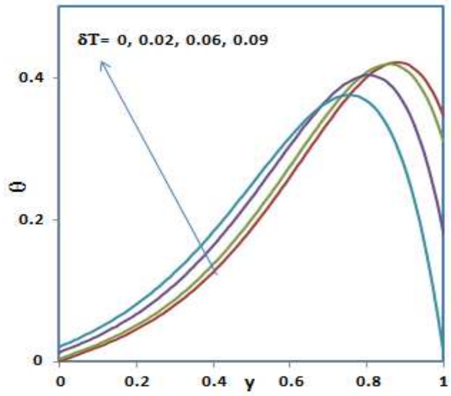


Fig (5, 6). *Temperature profiles for different F and α.*



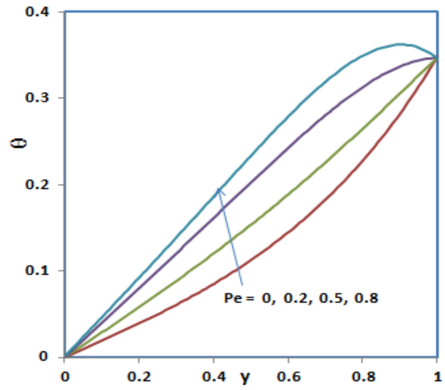
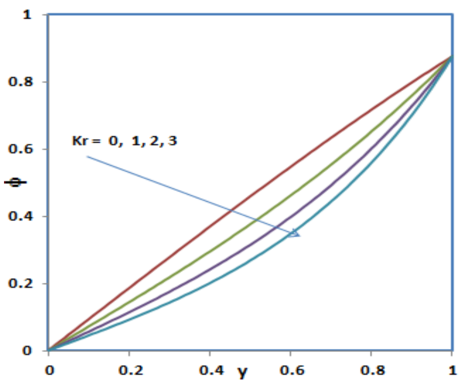


Fig (7,8). *Temperature profiles for different δT and Pe.*



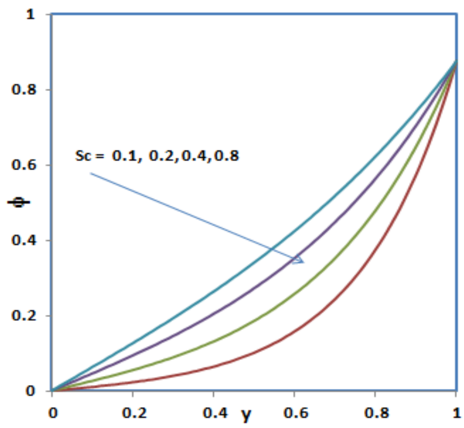
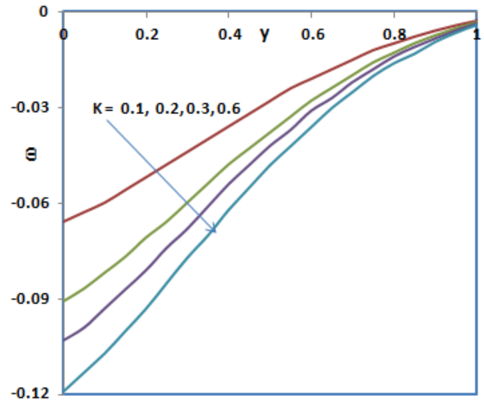
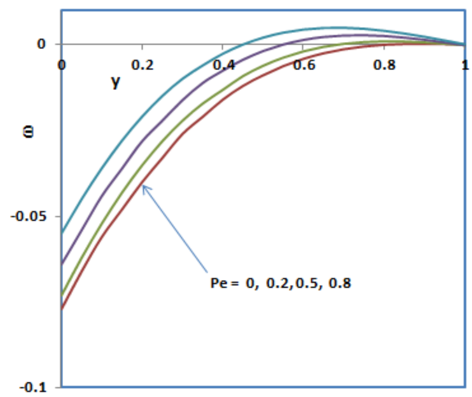


Fig (9,10). *Concentration profiles for different Kr, Sc.*





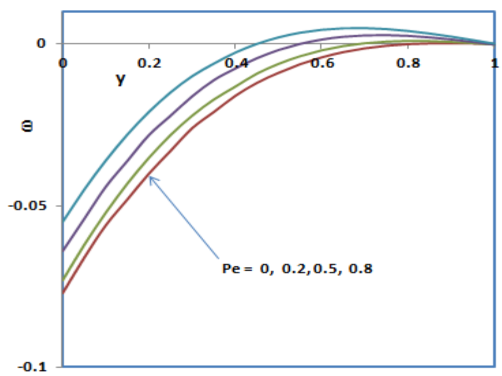


Fig (11,12,13). *Angular velocity profiles for different K, M and Pe.*

6. Limitations of the Study and Future Work

The present study has certain limitations that need to be taken into account when considering the study and its contributions. However, some of the limiting assumptions in the model can be seen as fruitful avenues for the future research. The following limitations/assumptions of the present study are taken into account

- Incompressible and laminar flow
- One dimensional and one directional flow
- A uniform magnetic field is applied in outward direction perpendicular to the flow
- Soret and Dufour effects cannot be taken into account together, otherwise energy and concentration equations will be coupled and we will be unable to find one of them explicitly and hence will be unable to solve the momentum equation for exact solution
- Electric field due to polarization of charges is not considered.

Clearly, if these limitations are considered in future research, will provide a challenging task for researcher regardless of their more specific interests in the subject. Despite of the fact that present study provides exact solutions and can be used as a bench mark by numerical analysts; the present work can also be studied for more complex phenomenon and geometrical configurations. For example cylindrical and spherical coordinate systems where such type of investigations are scarce. Motivated by the extensive applications of non-Newtonian fluids in the industrial manufacturing sector, it is of great interest to extend the present work for non-Newtonian fluids. Of course, in non-Newtonian fluids, the fluids of second grade and Maxwell form the simplest fluid models where the present analysis can be extended. However, the present study can also by analyzed for Oldroyd-B and Burger fluids apart from other non-Newtonian fluids.

7. Conclusions

This paper investigates the effect of heat and mass transfer on MHD oscillatory slip flow of a micropolar fluid in planer channel with variable temperature and concentration. The velocity, temperature and concentration distributions are obtained analytically and used to compute the wall shear stress and rate of heat and mass transfer at the channel walls. Computed results are presented to exhibit their dependence on the important physical parameters. We conclude the following from the numerical results.

- The effects of the permeability and magnetic parameters on velocity are opposite.
- Velocity decreases with increasing α, F, M,Δ, Re and w
- Velocity increases with increasing K, Gr, Gm and Pe.
- angular velocity decreases with increasing α, F and K.
- angular velocity increases with increasing Pe, M and Re.
- Temperature increases with increasing δT and decreases if α, F, Pe and w increase.
- Concentration increases with increasing δC and Decreases with increasing Sc, w and Kr.

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