

# Statistical Theory of Four-Point Distribution Functions in MHD Turbulent Flow

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**Abstract:** In this paper, the four-point distribution functions for simultaneous velocity, magnetic, temperature and concentration fields in MHD turbulent flow have been studied. It is tried to derive the transport equation for four-point distribution function in MHD turbulent flow. The obtained equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

**Keywords:** Magnetic Temperature, Concentration, Four-Point Distribution Functions, MHD Turbulent Flow

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## 1. Introduction

In physics, a particle's distribution function is a function of several variables. Such as velocity, Magnetic temperature, concentration etc. Particle distribution functions are mainly used in plasma physics to describe wave-particle interactions and velocity-space instabilities. It is also used in fluid mechanics, statistical mechanics and nuclear physics. It is specialized by a particular set of dimensions. Distribution functions may also feature non-isotropic temperatures, in which each term in the exponent is divided by a different temperature. In the past Hopf [1] Kraichnan [2], Edward [3] and Herring [4] studied the several analytical theories in the statistical theory turbulent and MHD turbulent flow. Further Lundgren [5,7] a great pioneer who established the uses of distribution function in turbulence and derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-You [6] in the kinetic theory of gasses. Later Kishore [8] studied the Distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope [9] studied the statistical theory of turbulence flames. The transport equation for the joint probability density function of velocity and scalars in turbulent flow is derived by pope [10]. Kollman and Janicka [11] derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient-flux model. Kishore and Singh [12] derived the transport equation for the bivariate

joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh [13] have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Dixit and Upadhyay [14] Sarker and Kishore [15, 16], Sarker and Islam [17], Azad and Sarker [18], Sarker and Azad [19], Aziz et al [20], Azad et al [21] discussed on distribution functions in the statistical theory of MHD turbulent flow. Recently Azad et al [22] studied the statistical theory of certain Distribution Functions in MHD turbulent flow undergoing a first order reaction in presence of dust particles and rotating system. Azad et al [23] derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow in presence of dust particles. Very Recently Molla et al [24], Azad et al [25], further have been studied the transport equation for the joint distribution functions in convective turbulent flow in presence of Coriolis force and dust particles undergoing a first order reaction respectively. The above researchers had carried out their research for one, two and three point distribution functions. In next Azad et al [26] derived the transport equations of certain distribution function in MHD Turbulent flow for velocity, Magnetic temperature and concentration. In recent times, Bkar Pk. et al [29] considering the effects of first-order reactant on MHD turbulence at four-point correlation. Azad et al [30] derived a transport equation for the joint distribution functions of certain variables in convective dusty fluid turbulent flow in a rotating system undergoing a first order reaction. Some of

researchers as Bkar Pk et al [28], Azad et al [31, 32, 33 and 34] have done their research on MHD turbulent flow considering 1<sup>st</sup> order chemical reaction for three- point distribution function. Bkar Pk [35] extended the above problem considering Coriolis force. Molla et al [36] derived transport equation for the joint distribution functions of velocity, temperature and concentration in convective turbulent flow in a rotating system in presence of dust particles. Mostly the above researchers had carried out their research works for three-point distribution functions.

By analyzing the above works, in this paper the statistical theory for four-point distribution functions for simultaneous velocity, magnetic temperature and concentration fields in MHD turbulent flow is studied. Through this work we have tried to derive the transport equations for four point distribution functions in MHD turbulent flow. Important properties of the distribution function have been discussed in this paper.

## 2. Formulation of the Problem

The equations of motion and continuity for viscous incompressible fluid in MHD turbulent flow, the diffusion equations for the temperature and concentration are given by

$$\frac{\partial u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (u_\alpha u_\beta - h_\alpha h_\beta) = -\frac{\partial w}{\partial x_\alpha} + \nu \nabla^2 u_\alpha \quad (1)$$

$$\frac{\partial h_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (h_\alpha u_\beta - u_\alpha h_\beta) = \lambda \nabla^2 h_\alpha \quad (2)$$

$$\frac{\partial \theta}{\partial t} + u_\beta \frac{\partial \theta}{\partial x_\beta} = \gamma \nabla^2 \theta, \quad (3)$$

$$\frac{\partial u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (u_\alpha u_\beta - h_\alpha h_\beta) = -\frac{1}{4\pi} \frac{\partial}{\partial x_\alpha} \int \left[ \frac{\partial u'_\alpha}{\partial x'_\beta} \frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta} \frac{\partial h'_\beta}{\partial x'_\alpha} \right] \frac{d\vec{x}'}{|\vec{x}' - \vec{x}|} + \nu \nabla^2 u_\alpha \quad (8)$$

$$\frac{\partial h_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} (h_\alpha u_\beta - u_\alpha h_\beta) = \lambda \nabla^2 h_\alpha, \quad (9)$$

$$\frac{\partial \theta}{\partial t} + u_\beta \frac{\partial \theta}{\partial x_\beta} = \gamma \nabla^2 \theta, \quad (10)$$

$$\frac{\partial c}{\partial t} + u_\beta \frac{\partial c}{\partial x_\beta} = D \nabla^2 c, \quad (11)$$

We consider the chemical reaction and the local mass transfer have no effect on the velocity field we also consider the turbulence and the concentration fields are homogeneous. The reaction rate and the diffusivity are constant. Considering a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity  $u$ , Alfven velocity  $h$ , temperature  $\theta$  and concentration  $C$  are

$$\frac{\partial c}{\partial t} + u_\beta \frac{\partial c}{\partial x_\beta} = D \nabla^2 c \quad (4)$$

$$\text{With } \frac{\partial u_\alpha}{\partial x_\alpha} = \frac{\partial v_\alpha}{\partial x_\alpha} = \frac{\partial h_\alpha}{\partial x_\alpha} = 0 \quad (5)$$

where,  $u_\alpha(x, t)$ ,  $\alpha$  – component of turbulent velocity,  $h_\alpha(x, t)$ ;  $\alpha$  – component of magnetic field;  $\theta(x, t)$ , temperature fluctuation;  $c$ , concentration of contaminants;  $w(\hat{x}, t) = P/\rho + \frac{1}{2}|\vec{h}|^2 + \frac{1}{2}|\vec{\Omega} \times \hat{x}|^2$ , total pressure;  $P(\hat{x}, t)$ , hydrodynamic pressure;  $\rho$ , fluid density;  $\nu$ , Kinetic viscosity;  $\lambda = (4\pi\mu\sigma)^{-1}$ , magnetic diffusivity;  $\gamma = \frac{k_T}{\rho c_p}$ , thermal diffusivity;  $c_p$ , = specific heat at constant pressure;  $k_T$ , thermal conductivity;  $\sigma$ , electrical conductivity;  $\mu$ , magnetic permeability;  $D$ , diffusive co-efficient for contaminants.

The repeated suffices are assumed over the values 1, 2 and 3. The unrepeated suffices may take any of these values. Here  $u$ ,  $h$  and  $x$  are vector quantities in the whole process.

To eliminate the total pressure  $w$ , taking the divergence of equation (1), we get

$$\nabla^2 w = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta} (u_\alpha u_\beta - h_\alpha h_\beta) = -\left[ \frac{\partial u_\alpha}{\partial x_\beta} \frac{\partial u_\beta}{\partial x_\alpha} - \frac{\partial h_\alpha}{\partial x_\beta} \frac{\partial h_\beta}{\partial x_\alpha} \right] \quad (6)$$

For a conducting infinite fluid,

$$w = \frac{1}{4\pi} \int \left[ \frac{\partial u'_\alpha}{\partial x'_\beta} \frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta} \frac{\partial h'_\beta}{\partial x'_\alpha} \right] \frac{d\vec{x}'}{|\vec{x}' - \vec{x}|} \quad (7)$$

Hence equation (1) to (4) becomes,

randomly distributed functions of position and time and satisfy their field.

Some microscopic properties of conducting fluids such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct the distribution functions, study its properties and derive an equation for its evolution of the four point distribution functions.

## 3. Definition of the Various Distribution Function in MHD Turbulence and Their Properties

In MHD turbulence, we may consider the fluid velocity  $u$ ,

Alfven velocity  $h$ , temperature  $\theta$  and concentration  $c$  at each point of the flow field. We have four measurable characteristics corresponding to each point of the flow field  $v$ ,  $g$ ,  $\phi$  and  $\psi$  are the four variables.

$(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)})$ ,  $(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)})$ , .....,  $(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)})$  Represent the pair of variables at the points  $\bar{x}^{(1)}, \bar{x}^{(2)}, \dots, \bar{x}^{(n)}$  at a fixed instant of time.

It is more possible that the same pair may be occurring more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as  $\{ (\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}); (\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}); \dots, (\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}) \}$

If the distribution is not discrete points in the flow field, then we consider the continuous distribution of the variables and  $\psi$  over the entire flow field, statistically behavior of the

fluid may be described by the distribution function  $F(\bar{v}, \bar{g}, \phi, \psi)$  which is normalized so that,

$$\int F(\bar{v}, \bar{g}, \phi, \psi) d\bar{v} d\bar{g} d\phi d\psi = 1$$

where, the integration ranges over all the possible values of  $v$ ,  $g$ ,  $\phi$  and  $\psi$ . We shall make use of the same normalization condition for the discrete distributions also. The distribution functions of the above quantities can be defined in terms of Dirac delta function. The one-point distribution function  $F_1^{(1)}(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)})$ , defined so that  $F_1^{(1)}(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}) dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$  is the probability that the fluid velocity, Alfven velocity, temperature and concentration at a time  $t$  are in the element  $dv^{(1)}$  about  $v^{(1)}$ ,  $dg^{(1)}$  about  $g^{(1)}$ ,  $d\phi^{(1)}$  about  $\phi^{(1)}$  and  $d\psi^{(1)}$  about  $\psi^{(1)}$  respectively and is given by

$$F_1^{(1)}(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}) = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \rangle \quad (12)$$

Where  $\delta$  is the Dirac delta-function defined as,

$$\int \delta(\bar{u} - \bar{v}) d\bar{v} = \begin{cases} 1 & \text{at the point } \bar{u} = \bar{v} \\ 0 & \text{elsewhere} \end{cases}$$

Two-point distribution function is given by,

$$F_2^{(1,2)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle \quad (13)$$

Three-point distribution function is given by,

$$F_3^{(1,2,3)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \rangle \quad (14)$$

and four point distribution function is given by,

$$F_4^{(1,2,3,4)} = \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle \quad (15)$$

Similarly, we can define an infinite numbers of multi-point distribution functions  $F_4^{(1,2,3,4,5)}$ ,  $F_5^{(1,2,3,4,5,6)}$  and so on. The following properties of the constructed distribution functions

can be deduced from the above definitions:

(A). *Reduction Properties.*

Integration with respect to pair of variables at one-point

lowers the order of distribution function by one. For example,

$$\int F_1^{(1)} dv^{(1)} dg^{(1)} d\varphi^{(1)} d\psi^{(1)} = 1,$$

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\varphi^{(2)} d\psi^{(2)} = F_1^{(1)},$$

$$\int F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\varphi^{(3)} d\psi^{(3)} = F_2^{(1,2)}$$

Similarly, we can define an infinite numbers of multi-point distribution functions  $F_4^{(1,2,3,4,5)}$ ,  $F_5^{(1,2,3,4,5,6)}$  and so on. Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as,

$$\int F_1^{(1)} dv^{(1)} = \left\langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\rangle$$

$$\int F_1^{(1)} dg^{(1)} = \left\langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\rangle, \int F_1^{(1)} d\varphi^{(1)} = \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right\rangle,$$

and,

$$\int F_2^{(1,2)} dv^{(2)} = \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \right.$$

$$\left. \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \right\rangle$$

#### (B). Separation Properties.

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

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$$\left| \bar{x}^{(2)} - \bar{x}^{(1)} \right| \rightarrow \infty \quad F_2^{(1,2)} = F_1^{(1)} F_1^{(2)} \quad \text{similarly,}$$

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$$\left| \bar{x}^{(3)} - \bar{x}^{(2)} \right| \rightarrow \infty \quad F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)} \quad \text{and} \quad \left| \bar{x}^{(4)} - \bar{x}^{(3)} \right| \rightarrow \infty \quad F_4^{(1,2,3,4)} = F_3^{(1,2,3)} F_1^{(4)}$$

#### (C). Co-incidence Property

When two points coincide in the flow field, the components at these points should be obviously the same that is  $F_2^{(1,2)}$  must be zero. Thus  $\bar{v}^{(2)} = \bar{v}^{(1)}$ ,  $g^{(2)} = g^{(1)}$ ,  $\varphi^{(2)} = \varphi^{(1)}$  and  $\psi^{(2)} = \psi^{(1)}$ , but  $F_2^{(1,2)}$  must also have the property.

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\varphi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

and hence it follows that,

Lim

$$\left| \bar{x}^{(2)} - \bar{x}^{(1)} \right| \rightarrow \infty \quad \int F_2^{(1,2)} = F_1^{(1)} \delta(v^{(2)} - v^{(1)}) \delta(g^{(2)} - g^{(1)}) \delta(\varphi^{(2)} - \varphi^{(1)}) \delta(\psi^{(2)} - \psi^{(1)})$$

Similarly,

Lim

$$\left| \bar{x}^{(3)} - \bar{x}^{(2)} \right| \rightarrow \infty \quad \int F_3^{(1,2,3)} = F_2^{(1,2)} \delta(v^{(3)} - v^{(1)}) \delta(g^{(3)} - g^{(1)}) \delta(\varphi^{(3)} - \varphi^{(1)}) \delta(\psi^{(3)} - \psi^{(1)})$$

and,

Lim

$$\left| \bar{x}^{(4)} - \bar{x}^{(3)} \right| \rightarrow \infty \quad \int F_4^{(1,2,3,4)} = F_3^{(1,2,3)} \delta(v^{(4)} - v^{(1)}) \delta(g^{(4)} - g^{(1)}) \delta(\varphi^{(4)} - \varphi^{(1)}) \delta(\psi^{(4)} - \psi^{(1)})$$

etc.

(D). Symmetric Conditions

$$F_n^{(1,2,r,\dots,s,\dots,n)} = F_n^{(1,2,\dots,s,\dots,r,\dots,n)}.$$

(E). Incompressibility Conditions

$$(i) \int \frac{\partial F_n^{(1,2,\dots,n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0;$$

$$(ii) \int \frac{\partial F_n^{(1,2,\dots,n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \langle u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \langle u_\alpha^{(1)} \rangle \langle F_1^{(1)} \rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (16)$$

#### 4. Continuity Equation in Terms of Distribution Functions

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective MHD turbulent flow and are obtained directly by using  $\text{div } u = 0$ , taking

ensemble average of  $\frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}$ , we get,

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

and similarly,

$$0 = \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (17)$$

which are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)})$$

and if we take the ensemble average, we obtain

$$\begin{aligned} 0 &= \left\langle \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \left[ \int \langle u_\alpha^{(1)} \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \right. \\ &\quad \times \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \left. \right] \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \end{aligned} \quad (18)$$

and similarly,

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int g_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (19)$$

The Nth – order continuity equations are,

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_N^{(1,2,\dots,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (20)$$

and

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int g_\alpha^{(1)} F_N^{(1,2,\dots,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (21)$$

The continuity equations are symmetric in their arguments i.e.

$$\frac{\partial}{\partial x_{\alpha}^{(r)}} \left( v_{\alpha}^{(r)} F_N^{(1,2,\dots,r,N)} dv^{(r)} dg^{(r)} d\varphi^{(r)} d\psi^{(r)} \right) = \frac{\partial}{\partial x_{\alpha}^{(s)}} \int v_{\alpha}^{(s)} F_N^{(1,2,\dots,r,s,\dots,N)} dv^{(s)} dg^{(s)} d\varphi^{(s)} d\psi^{(s)} \quad (22)$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_{\alpha}^{(1)}} \int v_{\alpha}^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\varphi^{(1)} d\psi^{(1)} \frac{\partial}{\partial x_{\alpha}^{(1)}} \langle u_{\alpha}^{(1)} \rangle = \langle \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\alpha}^{(1)}} \rangle = 0 \quad (23)$$

and all the properties of the distribution function obtained in section (3) can also be verified.

## 5. Equations for Four-Point Distribution Function $F_4^{(1234)}$

Differentiating equation (15) partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of  $u, h, \theta$  and  $c$  from the equation to (8) to (11)

$$\begin{aligned} \frac{\partial F_4^{(1,2,3,4)}}{\partial t} &= \frac{\partial}{\partial t} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\ &\delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle \\ &= \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\ &\delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(u^{(1)} - v^{(1)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \\ &\delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \\ &\delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(h^{(1)} - g^{(1)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \\ &\delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\ &\delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(\theta^{(1)} - \varphi^{(1)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \\ &\delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \\ &\delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(c^{(1)} - \psi^{(1)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\ &\delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \\ &\delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(u^{(2)} - v^{(2)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ &\delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \\ &\delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(h^{(2)} - g^{(2)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ &\delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \\ &\delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial}{\partial t} \delta(\theta^{(2)} - \varphi^{(2)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\ &\delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \end{aligned}$$

[illegible]

$$\begin{aligned}
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial c^{(2)}}{\partial t} \frac{\partial}{\partial \psi^{(2)}} \\
& \delta(c^{(2)} - \psi^{(2)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial u^{(3)}}{\partial t} \frac{\partial}{\partial v^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
& + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial h^{(3)}}{\partial t} \frac{\partial}{\partial g^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \\
& \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial \theta^{(3)}}{\partial t} \frac{\partial}{\partial \varphi^{(3)}} \delta(\theta^{(3)} - \varphi^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \\
& \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \\
& \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial c^{(3)}}{\partial t} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \\
& \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
& \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial u^{(4)}}{\partial t} \frac{\partial}{\partial v^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
& \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \\
& \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \frac{\partial h^{(4)}}{\partial t} \frac{\partial}{\partial g^{(4)}} \delta(h^{(4)} - g^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\
& \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \\
& \delta(c^{(4)} - \psi^{(4)}) \frac{\partial \theta^{(4)}}{\partial t} \frac{\partial}{\partial \varphi^{(4)}} \delta(\theta^{(4)} - \varphi^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
& \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \\
& \frac{\partial c^{(4)}}{\partial t} \frac{\partial}{\partial \psi^{(4)}} \delta(c^{(4)} - \psi^{(4)}) \rangle
\end{aligned}$$

Using equations (8) to (11), we get,

$$\begin{aligned}
& = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \{ -\frac{\partial}{\partial x_{\beta}^{(1)}} (u_{\alpha}^{(1)} u_{\beta}^{(1)} - h_{\alpha}^{(1)} h_{\beta}^{(1)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(1)}} \\
& \int [ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} - \frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} ] \frac{d\vec{x}^{\vee}}{|\vec{x}^{\vee} - \vec{x}|} + v \nabla^2 u_{\alpha}^{(1)} \} \times \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
& \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)})
\end{aligned}$$



$$\begin{aligned}
& \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -\frac{\partial}{\partial x_\beta^{(1)}} (h_\alpha^{(1)} u_\beta^{(1)} - u_\alpha^{(1)} h_\beta^{(1)}) + \lambda \nabla^2 h_\alpha^{(1)} \right\} \times \frac{\partial}{\partial g_\alpha^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \\
& \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \\
& \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \left\{ -u_\beta^{(1)} \frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}} + \lambda \nabla^2 \theta^{(1)} \right\} \times \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
& + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -u_\beta^{(1)} \frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} + D \nabla^2 c^{(1)} \right\} \times \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
& + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -\frac{\partial}{\partial x_\beta^{(2)}} (u_\alpha^{(2)} u_\beta^{(2)} - h_\alpha^{(2)} h_\beta^{(2)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(2)}} \right. \\
& \left. \int \left[ \frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}} - \frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}} \frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}} \right] \frac{d\vec{x}^\vee}{|\vec{x}^\vee - \vec{x}|} + \lambda \nabla^2 u_\alpha^{(2)} \right\} \times \frac{\partial}{\partial v_\alpha^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \\
& \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \\
& \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -\frac{\partial}{\partial x_\beta^{(2)}} (h_\alpha^{(2)} u_\beta^{(2)} - u_\alpha^{(2)} h_\beta^{(2)}) + \lambda \nabla^2 h_\alpha^{(2)} \right\} \times \frac{\partial}{\partial g_\alpha^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \\
& \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \\
& \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \left\{ -u_\beta^{(2)} \frac{\partial \theta^{(2)}}{\partial x_\beta^{(2)}} + \lambda \nabla^2 \theta^{(2)} \right\} \times \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
& + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -u_\beta^{(2)} \frac{\partial c^{(2)}}{\partial x_\beta^{(2)}} + D \nabla^2 c^{(2)} \right\} \\
& \times \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
& \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -\frac{\partial}{\partial x_\beta^{(3)}} \right. \\
& \left. (u_\alpha^{(3)} u_\beta^{(3)} - h_\alpha^{(3)} h_\beta^{(3)}) - \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(3)}} \int \left[ \frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial u_\beta^{(3)}}{\partial x_\alpha^{(3)}} - \frac{\partial h_\alpha^{(3)}}{\partial x_\beta^{(3)}} \frac{\partial h_\beta^{(3)}}{\partial x_\alpha^{(3)}} \right] \frac{d\vec{x}^\vee}{|\vec{x}^\vee - \vec{x}|} + \lambda \nabla^2 u_\alpha^{(3)} \right\} \times \frac{\partial}{\partial v_\alpha^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \\
& \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \\
& \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -\frac{\partial}{\partial x_\beta^{(3)}} (h_\alpha^{(3)} u_\beta^{(3)} - u_\alpha^{(3)} h_\beta^{(3)}) + \lambda \nabla^2 h_\alpha^{(3)} \right\} \times \frac{\partial}{\partial g_\alpha^{(3)}} \\
& \delta(h^{(3)} - g^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
& \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)})
\end{aligned}$$

$$\begin{aligned}
& \left\{ -\frac{\partial}{\partial x_\beta^{(3)}} \left( h_\alpha^{(3)} u_\beta^{(3)} - u_\alpha^{(3)} h_\beta^{(3)} \right) + \lambda \nabla^2 h_\alpha^{(3)} \right\} \frac{\partial}{\partial g_\alpha^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \\
& \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \\
& \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \\
& \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -u_\beta^{(3)} \frac{\partial c^{(3)}}{\partial x_\beta^{(3)}} \right. \\
& \left. + D\nabla^2 c^{(3)} \right\} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
& \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \\
& \left\{ -\frac{\partial}{\partial x_\beta^{(4)}} \left( u_\alpha^{(4)} u_\beta^{(4)} - h_\alpha^{(4)} h_\beta^{(4)} \right) - \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(4)}} \int \left[ \frac{\partial u_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial u_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial h_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial h_\beta^{(4)}}{\partial x_\alpha^{(4)}} \right] \frac{d\vec{x}'^\vee}{|\vec{x}^\vee - \vec{x}''|} + \nu \nabla^2 u_\alpha^{(4)} \right\} \times \frac{\partial}{\partial v_\alpha^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle \\
& + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \left\{ -\frac{\partial}{\partial x_\beta^{(4)}} \left( h_\alpha^{(4)} u_\beta^{(4)} - u_\alpha^{(4)} h_\beta^{(4)} \right) \right. \\
& \left. + \lambda \nabla^2 h_\alpha^{(4)} \right\} \times \frac{\partial}{\partial g_\alpha^{(4)}} \delta(h^{(4)} - g^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \\
& \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \\
& \times \left\{ -u_\beta^{(4)} \frac{\partial \theta^{(4)}}{\partial x_\beta^{(4)}} + \mathcal{N}^2 \theta^{(4)} \right\} \times \frac{\partial}{\partial \phi^{(4)}} \delta(\theta^{(4)} - \phi^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \\
& + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \left\{ -u_\beta^{(4)} \frac{\partial c^{(4)}}{\partial x_\beta^{(4)}} + D\nabla^2 c^{(4)} \right\} \\
& \times \frac{\partial}{\partial \psi^{(4)}} \delta(c^{(4)} - \psi^{(4)}) \rangle \\
& = \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
& + \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle
\end{aligned}$$



[illegible]



$$\begin{aligned}
& \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \\
& \times \nabla^2 \theta^{(4)} \frac{\partial}{\partial \varphi^{(4)}} \delta(\theta^{(4)} - \varphi^{(4)}) \rangle + \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \\
& \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial c^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial \psi^{(4)}} \quad (24) \\
& \delta(c^{(4)} - \psi^{(4)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \times D \nabla^2 c^{(4)} \frac{\partial}{\partial \psi^{(4)}} \delta(c^{(4)} - \psi^{(4)}) \rangle
\end{aligned}$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one, two, three and four point distribution functions.

The 1<sup>st</sup> term in the above equation is simplified as follows

$$\begin{aligned}
& \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)} \partial v_{\alpha}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
& = \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \quad (25)
\end{aligned}$$

Similarly, 5<sup>th</sup>, 8<sup>th</sup> and 9<sup>th</sup> terms of right hand-sides of equation (24) can be simplified as follows,  
5<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(1)} u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)} \partial g_{\alpha}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \quad (26)
\end{aligned}$$

8<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial \theta^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \varphi^{(1)}} \delta(\theta^{(1)} - \varphi^{(1)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \varphi^{(1)}) \rangle \quad (27)
\end{aligned}$$

And 9<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \\
& \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \quad (28)
\end{aligned}$$

Adding these equations from (26) to (28), we get,

$$\begin{aligned}
& \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \\
& \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \\
& \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \\
& \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \\
& \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(\theta^{(1)} - \varphi^{(1)}) \rangle + \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \\
& \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \\
& \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle. \\
& = -\frac{\partial}{\partial x_{\beta}^{(1)}} \langle u_{\beta}^{(1)} \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle \\
& = -v_{\beta}^{(1)} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(1)}} [\text{Applying the properties of distribution functions}] \quad (29)
\end{aligned}$$

Similarly 12<sup>th</sup>, 16<sup>th</sup>, 19<sup>th</sup> and 21<sup>st</sup> terms of right hand-side of equation (24) can be simplified as follows:

12<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \quad (30)
\end{aligned}$$

16<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(2)} u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \quad (31)
\end{aligned}$$

19<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(2)} \frac{\partial \theta^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \varphi^{(2)}} \delta(\theta^{(2)} - \varphi^{(2)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(\theta^{(2)} - \varphi^{(2)}) \rangle \quad (32)
\end{aligned}$$

And 21<sup>st</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(2)} \frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(2)} \frac{\partial}{\partial x_{\beta}^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \quad (33)
\end{aligned}$$

Adding these equations from (30) to (33), we get

$$\begin{aligned}
& -\frac{\partial}{\partial x_{\beta}^{(2)}} \langle u_{\beta}^{(2)} \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle \\
& = -v_{\beta}^{(2)} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(2)}} \quad (34)
\end{aligned}$$

Similarly, 23<sup>rd</sup>, 27<sup>th</sup>, 30<sup>th</sup> and 32<sup>nd</sup> terms of right hand-side of equation(24) can be simplified as follows;

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(3)} u_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \quad (35)
\end{aligned}$$

27<sup>th</sup> term,



$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(3)} u_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \quad (36)
\end{aligned}$$

30<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(3)} \frac{\partial \theta^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial \phi^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(\theta^{(3)} - \phi^{(3)}) \rangle \quad (37)
\end{aligned}$$

And 32<sup>nd</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(3)} \frac{\partial c^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(3)} \frac{\partial}{\partial x_{\beta}^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \quad (38)
\end{aligned}$$

Adding these equations from (35) to (38), we get,

$$\begin{aligned}
& -\frac{\partial}{\partial x_{\beta}^{(3)}} \langle u_{\beta}^{(3)} \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle \rangle = -v_{\beta}^{(3)} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(3)}} \quad (39)
\end{aligned}$$

Similarly, 34<sup>th</sup>, 38<sup>th</sup>, 41<sup>th</sup> and 43<sup>th</sup> terms of right hand-side of equation (24) can be simplified as follows;

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(4)} u_{\beta}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial v_{\alpha}^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial}{\partial x_{\beta}^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle \quad (40)
\end{aligned}$$

38<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(4)} u_{\beta}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial g_{\alpha}^{(4)}} \delta(h^{(4)} - g^{(4)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial}{\partial x_{\beta}^{(4)}} \delta(h^{(4)} - g^{(4)}) \rangle \quad (41)
\end{aligned}$$

41<sup>th</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial \theta^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial \varphi^{(4)}} \delta(\theta^{(4)} - \varphi^{(4)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial}{\partial x_{\beta}^{(4)}} \delta(\theta^{(4)} - \varphi^{(4)}) \rangle \quad (42)
\end{aligned}$$

And 43<sup>nd</sup> term,

$$\begin{aligned}
& \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial c^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial \psi^{(4)}} \delta(c^{(4)} - \psi^{(4)}) \rangle \\
& = \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \times u_{\beta}^{(4)} \frac{\partial}{\partial x_{\beta}^{(4)}} \delta(c^{(4)} - \psi^{(4)}) \rangle \quad (43)
\end{aligned}$$

Adding these equations from (40) to (43), we get

$$\begin{aligned}
& -\frac{\partial}{\partial x_{\beta}^{(4)}} \langle u_{\beta}^{(4)} \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \rangle \rangle = -v_{\beta}^{(4)} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(4)}} \quad (44)
\end{aligned}$$

Similarly, 2<sup>nd</sup>, 6<sup>th</sup>, 13<sup>th</sup>, 17<sup>th</sup>, 24<sup>th</sup>, 28<sup>th</sup>, 35<sup>th</sup> and 39<sup>th</sup> terms of right hand-side of equation (24) can be simplified as follows;

2<sup>nd</sup> term,

$$\begin{aligned}
& \langle -\delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
& = -g_{\beta}^{(1)} \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(1)}} \quad (45)
\end{aligned}$$

6<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(1)} h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial}{\partial g_{\alpha}} \delta(h^{(1)} - g^{(1)}) \rangle \\
& = -g_{\beta}^{(1)} \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(1)}}
\end{aligned} \tag{46}$$

13<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(2)} h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
& = -g_{\beta}^{(2)} \frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(2)}}
\end{aligned} \tag{47}$$

17<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(2)} h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
& = -g_{\beta}^{(2)} \frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(2)}}
\end{aligned} \tag{48}$$

24<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(3)} h_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
& = -g_{\beta}^{(3)} \frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(3)}}
\end{aligned} \tag{49}$$

and 28<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(3)} h_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
& = -g_{\beta}^{(3)} \frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(3)}}
\end{aligned} \tag{50}$$

35<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial h_{\alpha}^{(4)} h_{\beta}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial v_{\alpha}^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle \\
& = -g_{\beta}^{(4)} \frac{\partial g_{\alpha}^{(4)}}{\partial v_{\alpha}^{(4)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(4)}}
\end{aligned} \tag{51}$$

And 39<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \frac{\partial u_{\alpha}^{(4)} h_{\beta}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial}{\partial g_{\alpha}^{(4)}} \delta(h^{(4)} - g^{(4)}) \rangle \\
& = -g_{\beta}^{(4)} \frac{\partial v_{\alpha}^{(4)}}{\partial g_{\alpha}^{(4)}} \frac{\partial F_4^{(1,2,3,4)}}{\partial x_{\beta}^{(4)}}
\end{aligned} \tag{52}$$

4<sup>th</sup> term can be reduced in the following,

$$\begin{aligned}
& \langle -\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times v \nabla^2 u_{\alpha}^{(1)} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
& = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \langle \nabla^2 u_{\alpha}^{(1)} [ \delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)}) \\
& \delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) ] \rangle \\
& = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(4)} \partial x_{\beta}^{(4)}} \left\langle \int u_{\alpha}^{(5)} \delta(u^{(5)} - v^{(5)})\delta(h^{(5)} - g^{(5)})\delta(\theta^{(5)} - \varphi^{(5)})\delta(c^{(5)} - \psi^{(5)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)}) \right. \\
& \delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)}) \\
& \left. \delta(c^{(2)} - \psi^{(2)})\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)}) dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \right\rangle \\
& = -v \frac{\partial}{\partial v_{\alpha}^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int v_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)}
\end{aligned} \tag{53}$$

Similarly, 7<sup>th</sup>, 9<sup>th</sup>, 11<sup>th</sup>, 15<sup>th</sup>, 18<sup>th</sup>, 20<sup>th</sup>, 22<sup>nd</sup>, 26<sup>th</sup>, 29<sup>th</sup>, 31<sup>st</sup>, 33<sup>rd</sup>, 37<sup>rd</sup>, 40<sup>th</sup>, 42<sup>th</sup> and 44<sup>th</sup> terms of right hand-side of equation (24) can be simplified as follows;

7<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \lambda \nabla^2 h_{\alpha}^{(1)} \frac{\partial}{\partial g_{\alpha}^{(1)}} \delta(h^{(1)} - g^{(1)}) \rangle \\
& = -\lambda \frac{\partial}{\partial g_{\alpha}^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int g_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)}
\end{aligned} \tag{54}$$

9<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta(\theta^{(1)} - \phi^{(1)}) \rangle \\
 & = -\gamma \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \phi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \quad (55)
 \end{aligned}$$

11<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta(c^{(1)} - \psi^{(1)}) \rangle \\
 & = -D \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \psi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \quad (56)
 \end{aligned}$$

15<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \nu \nabla^2 u_{\alpha}^{(2)} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & = -\nu \frac{\partial}{\partial v_{\alpha}^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int v_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \quad (57)
 \end{aligned}$$

18<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \lambda \nabla^2 h_{\alpha}^{(2)} \frac{\partial}{\partial g_{\alpha}^{(2)}} \delta(h^{(2)} - g^{(2)}) \rangle \\
 & = -\lambda \frac{\partial}{\partial g_{\alpha}^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int g_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \quad (58)
 \end{aligned}$$

20<sup>th</sup> term,

$$\begin{aligned}
 & \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \gamma \nabla^2 \theta^{(2)} \frac{\partial}{\partial \phi^{(2)}} \delta(\theta^{(2)} - \phi^{(2)}) \rangle \\
 & = -\gamma \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \phi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \quad (59)
 \end{aligned}$$

22<sup>nd</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(u^{(3)} - v^{(3)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times D\nabla^2 c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta(c^{(2)} - \psi^{(2)}) \rangle \\
& = -D \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \psi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (60)
\end{aligned}$$

26<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \nu \nabla^2 u_{\alpha}^{(3)} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
& = -\nu \frac{\partial}{\partial v_{\alpha}^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int v_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (61)
\end{aligned}$$

29<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)})\delta(c^{(3)} - \psi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \lambda \nabla^2 h_{\alpha}^{(3)} \frac{\partial}{\partial g_{\alpha}^{(3)}} \delta(h^{(3)} - g^{(3)}) \rangle \\
& = -\lambda \frac{\partial}{\partial g_{\alpha}^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int g_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (62)
\end{aligned}$$

31<sup>st</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \varphi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times \gamma \nabla^2 \theta^{(3)} \frac{\partial}{\partial \varphi^{(3)}} \delta(\theta^{(3)} - \varphi^{(3)}) \rangle \\
& = -\gamma \frac{\partial}{\partial \varphi^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \varphi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (63)
\end{aligned}$$

33<sup>rd</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)})\delta(h^{(1)} - g^{(1)})\delta(\theta^{(1)} - \varphi^{(1)})\delta(c^{(1)} - \psi^{(1)})\delta(u^{(2)} - v^{(2)})\delta(h^{(2)} - g^{(2)})\delta(\theta^{(2)} - \varphi^{(2)})\delta(c^{(2)} - \psi^{(2)})\delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)})\delta(\theta^{(3)} - \varphi^{(3)})\delta(u^{(4)} - v^{(4)})\delta(h^{(4)} - g^{(4)})\delta(\theta^{(4)} - \varphi^{(4)})\delta(c^{(4)} - \psi^{(4)}) \times D\nabla^2 c^{(3)} \frac{\partial}{\partial \psi^{(3)}} \delta(c^{(3)} - \psi^{(3)}) \rangle \\
& = -D \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \psi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (64)
\end{aligned}$$

37<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \nu \nabla^2 u_{\alpha}^{(4)} \frac{\partial}{\partial v_{\alpha}^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle \\
& = -\nu \frac{\partial}{\partial v_{\alpha}^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int v_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (65)
\end{aligned}$$

40<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \lambda \nabla^2 h_{\alpha}^{(4)} \frac{\partial}{\partial g_{\alpha}^{(4)}} \delta(h^{(4)} - g^{(4)}) \rangle \\
& = -\lambda \frac{\partial}{\partial g_{\alpha}^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int g_{\alpha}^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (66)
\end{aligned}$$

42<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \gamma \nabla^2 \theta^{(4)} \frac{\partial}{\partial \varphi^{(4)}} \delta(\theta^{(4)} - \varphi^{(4)}) \rangle \\
& = -\gamma \frac{\partial}{\partial \varphi^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \varphi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (67)
\end{aligned}$$

44<sup>th</sup> term,

$$\begin{aligned}
& \langle -\delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \\
& \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \times D \nabla^2 c^{(4)} \frac{\partial}{\partial \psi^{(4)}} \delta(c^{(4)} - \psi^{(4)}) \rangle \\
& = -D \frac{\partial}{\partial \psi^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \frac{\partial^2}{\partial x_{\beta}^{(5)} \partial x_{\beta}^{(5)}} \int \psi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \quad (68)
\end{aligned}$$

We reduce the 3<sup>rd</sup> term of right hand side of equation (24),

$$\begin{aligned}
& \langle \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \varphi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \varphi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
& \delta(\theta^{(3)} - \varphi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \varphi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(1)}} \int \left[ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} - \frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}} \frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}} \right] \\
& \frac{d\bar{x}^{\vee}}{|\bar{x}^{\vee} - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(1)}} \delta(u^{(1)} - v^{(1)}) \rangle \\
& = \frac{\partial}{\partial v_{\alpha}^{(1)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(1)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(1)}|} \right) \left( \frac{\partial v_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial v_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} - \frac{\partial g_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial g_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} \right) F_5^{(1,2,3,4,5)} dx^{(5)} dv^{(5)} dg^{(5)} d\varphi^{(5)} d\psi^{(5)} \right] \quad (69)
\end{aligned}$$

Similarly 14<sup>th</sup>, 25<sup>th</sup> and 36<sup>th</sup> term,

14<sup>th</sup> term,

$$\begin{aligned}
 & \left\langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \right. \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(2)}} \int \left[ \frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} - \frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}} \frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}} \right] \\
 & \times \frac{d\bar{x}^{\vee}}{|\bar{x}^{\vee} - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(2)}} \delta(u^{(2)} - v^{(2)}) \rangle \\
 & = \frac{\partial}{\partial v_{\alpha}^{(2)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(2)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(2)}|} \right) \left( \frac{\partial v_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial v_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} - \frac{\partial g_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial g_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} \right) F_5^{(1,2,3,4,5)} dx^{(5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \right] \quad (70)
 \end{aligned}$$

25<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(h^{(3)} - g^{(3)}) \delta(\theta^{(3)} - \phi^{(3)}) \\
 & \delta(c^{(3)} - \psi^{(3)}) \delta(u^{(4)} - v^{(4)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(3)}} \int \left[ \frac{\partial u_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial u_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} - \frac{\partial h_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}} \frac{\partial h_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}} \right] \\
 & \frac{d\bar{x}^{\vee}}{|\bar{x}^{\vee} - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(3)}} \delta(u^{(3)} - v^{(3)}) \rangle \\
 & = \frac{\partial}{\partial v_{\alpha}^{(3)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(3)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(3)}|} \right) \left( \frac{\partial v_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial v_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} - \frac{\partial g_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial g_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} \right) F_5^{(1,2,3,4,5)} dx^{(5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \right] \quad (71)
 \end{aligned}$$

36<sup>th</sup> term,

$$\begin{aligned}
 & \langle \delta(u^{(1)} - v^{(1)}) \delta(h^{(1)} - g^{(1)}) \delta(\theta^{(1)} - \phi^{(1)}) \delta(c^{(1)} - \psi^{(1)}) \delta(u^{(2)} - v^{(2)}) \delta(h^{(2)} - g^{(2)}) \delta(\theta^{(2)} - \phi^{(2)}) \delta(c^{(2)} - \psi^{(2)}) \delta(u^{(3)} - v^{(3)}) \delta(h^{(3)} - g^{(3)}) \\
 & \delta(\theta^{(3)} - \phi^{(3)}) \delta(c^{(3)} - \psi^{(3)}) \delta(h^{(4)} - g^{(4)}) \delta(\theta^{(4)} - \phi^{(4)}) \delta(c^{(4)} - \psi^{(4)}) \times \frac{1}{4\pi} \frac{\partial}{\partial x_{\alpha}^{(4)}} \int \left[ \frac{\partial u_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial u_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} - \frac{\partial h_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}} \frac{\partial h_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}} \right] \\
 & \frac{d\bar{x}^{\vee}}{|\bar{x}^{\vee} - \bar{x}|} \frac{\partial}{\partial v_{\alpha}^{(4)}} \delta(u^{(4)} - v^{(4)}) \rangle \\
 & = \frac{\partial}{\partial v_{\alpha}^{(4)}} \left[ \frac{1}{4\pi} \int \frac{\partial}{\partial x_{\alpha}^{(4)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(4)}|} \right) \left( \frac{\partial v_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial v_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} - \frac{\partial g_{\alpha}^{(5)}}{\partial x_{\beta}^{(5)}} \frac{\partial g_{\beta}^{(5)}}{\partial x_{\alpha}^{(5)}} \right) F_5^{(1,2,3,4,5)} dx^{(5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \right] \quad (72)
 \end{aligned}$$

Substituting the results (25)-(72) in equation (24) we get the transport equation for four-point distribution function  $F_4^{(1,2,3,4)}(v, g, \phi, \psi)$  in MHD turbulent flow as



$$\begin{aligned}
& \frac{\partial F_4^{(1,2,3,4)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} + v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} + v_\beta^{(4)} \frac{\partial}{\partial x_\beta^{(4)}} \right) F_4^{(1,2,3,4)} + \left[ g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \right. \\
& \left. \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} + g_\beta^{(4)} \left( \frac{\partial g_\alpha^{(4)}}{\partial v_\alpha^{(4)}} + \frac{\partial v_\alpha^{(4)}}{\partial g_\alpha^{(4)}} \right) \frac{\partial}{\partial x_\beta^{(4)}} \right] F_4^{(1,2,3,4)} + \nu \left( \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} \right. \\
& \left. + \frac{\partial}{\partial v_\alpha^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} + \frac{\partial}{\partial v_\alpha^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \right) \times \frac{\partial^2}{\partial x_\beta^{(5)} \partial x_\beta^{(5)}} \int v_\alpha^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} + \lambda \left( \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} \right. \\
& \left. + \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial g_\alpha^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} + \frac{\partial}{\partial g_\alpha^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \right) \times \frac{\partial^2}{\partial x_\beta^{(5)} \partial x_\beta^{(5)}} \int g_\alpha^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \\
& + \gamma \left( \frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} + \frac{\partial}{\partial \phi^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \right) \times \frac{\partial^2}{\partial x_\beta^{(5)} \partial x_\beta^{(5)}} \int \phi^{(5)} F_5^{(1,2,3,4,5)} \\
& dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} + D \left( \frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(3)}} + \frac{\partial}{\partial \psi^{(4)}} \lim_{\bar{x}^{(5)} \rightarrow \bar{x}^{(4)}} \right) \\
& \times \frac{\partial^2}{\partial x_\beta^{(5)} \partial x_\beta^{(5)}} \int \psi^{(5)} F_5^{(1,2,3,4,5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} - \left[ \frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \right. \right. \\
& \left. \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(2)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(3)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(4)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(4)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(4)}|} \right) \right\} \right. \\
& \left. \times \left( \frac{\partial v_\alpha^{(5)}}{\partial x_\beta^{(5)}} \frac{\partial v_\beta^{(5)}}{\partial x_\alpha^{(5)}} - \frac{\partial g_\alpha^{(5)}}{\partial x_\beta^{(5)}} \frac{\partial g_\beta^{(5)}}{\partial x_\alpha^{(5)}} \right) F_5^{(1,2,3,4,5)} \times dx^{(5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \right] = 0
\end{aligned} \quad (73)$$

This is named the transport equation for evolution of four-point distribution function in MHD turbulent flow.

## 6. Results and Discussions

If we drop the kinetic viscosity ( $\nu$ ), magnetic diffusivity ( $\lambda$ ), thermal diffusivity ( $\gamma$ ) and concentration ( $D$ ) terms from the four-point evolution equation (73), we have

$$\begin{aligned}
& \frac{\partial F_4^{(1,2,3,4)}}{\partial t} + \left( v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} + v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} + v_\beta^{(4)} \frac{\partial}{\partial x_\beta^{(4)}} \right) F_4^{(1,2,3,4)} + \left[ g_\beta^{(1)} \left( \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} + g_\beta^{(2)} \left( \frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \right. \\
& \left. + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left( \frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} + g_\beta^{(4)} \left( \frac{\partial g_\alpha^{(4)}}{\partial v_\alpha^{(4)}} + \frac{\partial v_\alpha^{(4)}}{\partial g_\alpha^{(4)}} \right) \frac{\partial}{\partial x_\beta^{(4)}} \right] F_4^{(1,2,3,4)} - \left[ \frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \right. \right. \\
& \left. \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(1)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(2)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(3)}|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(4)}} \left\{ \frac{1}{4\pi} \right. \\
& \left. \int \frac{\partial}{\partial x_\alpha^{(4)}} \left( \frac{1}{|\bar{x}^{(5)} - \bar{x}^{(4)}|} \right) \right\} \times \left( \frac{\partial v_\alpha^{(5)}}{\partial x_\beta^{(5)}} \frac{\partial v_\beta^{(5)}}{\partial x_\alpha^{(5)}} - \frac{\partial g_\alpha^{(5)}}{\partial x_\beta^{(5)}} \frac{\partial g_\beta^{(5)}}{\partial x_\alpha^{(5)}} \right) \left. \right] F_5^{(1,2,3,4,5)} \times dx^{(5)} dv^{(5)} dg^{(5)} d\phi^{(5)} d\psi^{(5)} \quad ] = 0
\end{aligned} \quad (74)$$

The existence of the term

$$\left( \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}} + \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \right), \left( \frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}} + \frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}} \right), \left( \frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}} + \frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}} \right) \text{ and } \left( \frac{\partial g_{\alpha}^{(4)}}{\partial v_{\alpha}^{(4)}} + \frac{\partial v_{\alpha}^{(4)}}{\partial g_{\alpha}^{(4)}} \right)$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at the point  $x^{(1)}$ ,  $x^{(2)}$ ,  $x^{(3)}$  and  $x^{(4)}$ .

We can exhibit an analogy of this equation with the 1<sup>st</sup> equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given as

$$\frac{\partial F_1^{(1)}}{\partial t} + \frac{1}{m} v_{\beta}^{(1)} \frac{\partial}{\partial x_{\beta}^{(1)}} F_1^{(1)} = n \iint \frac{\partial \psi_{1,2}}{\partial x_{\alpha}^{(1)}} \frac{\partial F_2^{(1,2)}}{\partial v_{\alpha}^{(1)}} d\bar{x}^{(2)} d\bar{v}^{(2)} \quad (75)$$

Where  $\psi_{1,2} = \psi \left| v_{\alpha}^{(2)} - v_{\alpha}^{(1)} \right|$  is the intermolecular potential.

If we consider the collection of ionized particles, i.e. in plasma turbulence case, it can be provided closure form easily by decomposing  $F_2^{(1,2)}$  as  $F_1^{(1)} F_1^{(2)}$ . But such type of approximations can be possible if there is no interaction or correlation between two particles. If we decompose  $F_2^{(1,2)}$  as

$$F_2^{(1,2)} = (1+\epsilon) F_1^{(1)} F_1^{(2)} \text{ and } \\ F_3^{(1,2,3)} = (1+\epsilon)^2 F_1^{(1)} F_1^{(2)} F_1^{(3)} \text{ Also } \\ F_4^{(1,2,3,4)} = (1+\epsilon)^3 F_1^{(1)} F_1^{(2)} F_1^{(3)} F_1^{(4)}$$

Where  $\epsilon$  is the correlation coefficient between the particles. If there is no correlation between the particles,  $\epsilon$  will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximation only to provide closed form of the equation.

## 7. Conclusion

In this paper, the various properties of constructed joint distribution functions have been discussed which are used to study this work. Through this study we have try to derive the transport equation (73) for four-point distribution function in MHD turbulent flow for velocity, magnetic temperature and concentration. We have used ensemble averages at a particular time to derive the transport equation (73) including some microscopic properties of MHD turbulent flow such as total energy, total pressure and stress tensor with the help of the joint distribution functions. Continuing this way, one can derive the equations for evolution of  $F_4^{(1,2,3,4)}$ ,  $F_5^{(1,2,3,4,5)}$  and so on. Logically it is possible to have an equation for every  $F_n$  ( $n$  is an integer) but the system of equations so obtained is not closed. To close the system the above certain approximations will be required.

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