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Optimal Control of COVID-19: Examining the Incidence of Contamination and Its Recurrence in Nigeria

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Abstract: COVID-19 is an epidemic virus infection that is ravaging the world today. There are no pre-existing immunity and People were easily infected by this virus known as severe acute respiratory syndrome coronavirus (SARS-CoV-2) which caused Covid-19 (CDC, 2020). According to available data, the COVID-19 virus transmits most easily amongst people who are in proximity, typically within some feet (6) or meters. In this paper, we present the Susceptible – Exposed – Infected-Recovered (SEIR) epidemic model for the dynamics of COVID-19 outbreak and its optimal control in Nigeria. SEIR is characterized by a system of four non-linear differential equations. We established the existence and uniqueness of solutions of these equations. Using Nigeria's COVID-19 data, we computed the basic reproduction number of the system. Further, an optimal control approach is performed to study the effect of control measure against the spread of the virus, the control level which minimizes the spread and optimal value of the control which maximizes the objective function. Through the application of Pontryagin's Maximum Principle, we determined how the spread of the virus could be suppressed. The investigation shows that an effective strategy in combating the Covid-19 epidemic is adhering to the dictates of the control measures.

Keywords: SEIR Model, COVID-19, Pontryagin Maximum Principle, Basic Reproduction Number, Optimal Control

1. Introduction

The globe is currently experiencing COVID-19 (Corona virus disease 2019), the fifth pandemic since the 1918 flu pandemic [1]. This new coronavirus has been linked to millions of infections worldwide and more than 2 million fatalities. The mortality rate differs from nation to nation [2]. The initial report from a cluster of unique human pneumonia cases was detected in Wuhan City, China, in late December 2019, and experts have been tracking the infectious outbreak since late 2019 [3]. The first day of December 2019 was noted as the earliest symptom start date [4].

A Susceptible-Exposed-Infectious-Removed (SEIR) model was formulated and we went to describe the spread of the virus [5]. The SEIR model has many versions, and mathematical treatments were found, in the following studies; the mathematics of infectious diseases in [6], Mathematical Tools for understanding Disease Dynamics in [7], Mathematical epidemiology: past, present, and future in [8]. Their goal was to compute the number of individuals infected, recovered and dead. The difference between this model and the vaccination control model [10] is that the number of vaccinated individuals per day is constrained to be less than the susceptible population while the Contact rate epidemic control of COVID-19 in [10], the control input is the ratio of the vaccinated individuals per day to the average born population per day.

Adewole et al. in [24] investigated the dynamics of COVID-19. Their model was calibrated using information obtained from the Nigeria Centre for Disease Control, and Pontryagin's Maximum Principle was used to generate numerical simulations, which were then used to investigate various optimal control strategies involving both single and multiple controls in Nigeria. The optimum management of a single intervention input variable for an ordinary differential equation (ODE) model was examined [25]. In this work, we shall optimize the prescribed social distancing control order in Nigeria.

Fabric face masks, one of the control methods used to prevent the dissemination of Covid-19, are not always 100% successful [26], and there have been some instances of abuse and inadequate use of the fabric face mask [27]. The three control techniques (hand sanitizer usage, COVID-19 patient treatment, and active screening with testing and prevention against recurrence and reinfection of persons who have recovered from COVID-19) must be carefully implemented if COVID-19 is to be successfully eradicated in Nigeria [28]. The incidence of reinfection and recurrence in persons who have recovered from COVID-19 will be determined in this investigation.

In 2021, Xu, et al modified the SIR model with Shield immunity as proposed by Weitz J, et al in 2020, which was aimed at limiting the transmission of COVID-19. The model used in Control strategies for COVID-19 epidemic with vaccination, shield immunity and quarantine was modified from an SIR model [11] considered a corresponding SEIR model with shield immunity control.

This study aims to modifying the model established by Carcione et al in [5] known as Susceptible – Exposed – Infectious-Removed (SEIR) that explains the dissemination of the Covid-19 by examining the incidence of contamination and recurring of people who already have survived from Covid-19 infections. We adopt the social distancing order and Covid-19 vaccine, checkmating the number of people in the population need to be vaccinated and what happens if the deployed covid-19 vaccine (AstraZeneca/Oxford made by the Serum Institute of India) in Nigeria is not 100% effective.

2. Data Source

The COVID-19 data report is publicly available on Nigeria Centre for Disease Control (NCDC, 2020).

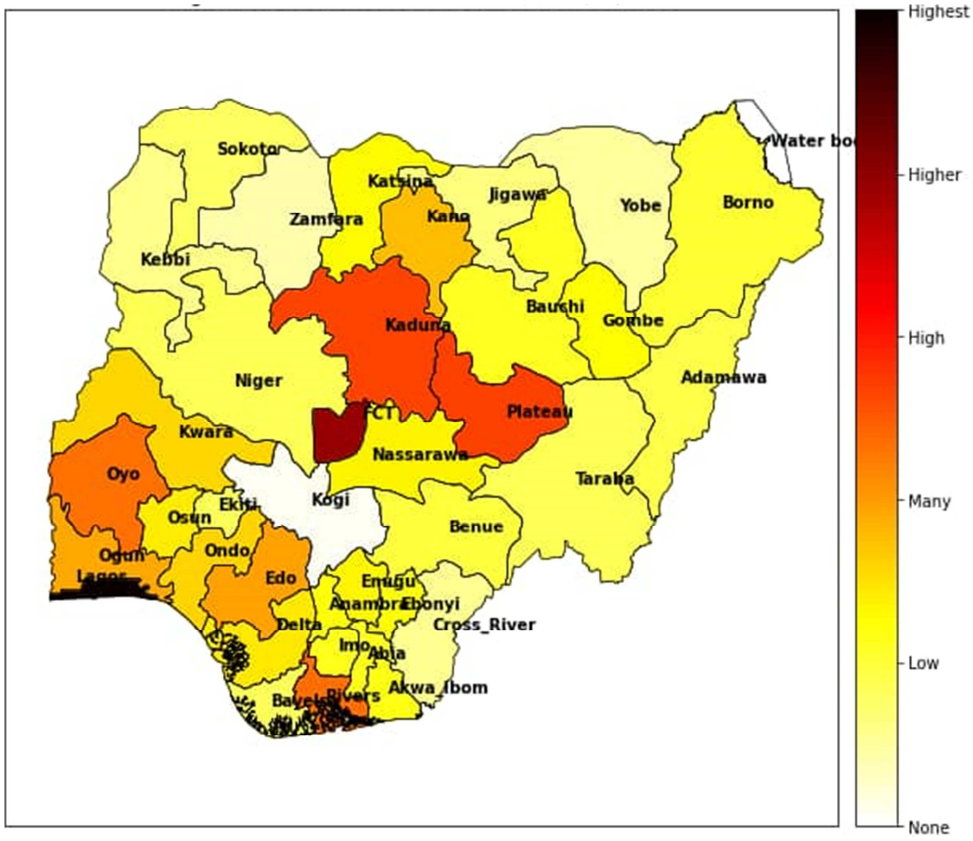

Figure 1. *Nigeria confirmed cases of covid-19 on 04/04/2021.*

Figure 1 shows the distribution of the disease across the towns in Nigeria. Lagos and Abuja ranked very high on the infection scale [13-15]. The collected COVID-19 data represents the dynamics of the disease before the enforcement of intervention strategies [16].

2.1. Properties of the Model

The well-known COVID-19 Susceptible-Infected-Recovered (SIR) model [9] has been modified to SEIR model with four compartments namely S, E, I and R. The name of these compartments represents the state variables or the number of people in each compartment at time t. Thus, $S(t)$, $E(t)$, $I(t)$ and $R(t)$ denote the susceptible, exposed, infectious and recovered population at time t. The susceptible populations are those persons who can contract the disease once they are in contact with an infectious person. The exposed populations are those that are infected but are not yet infectious. The infectious ones are those who contracted the disease and can infect others. The recovered group refers to those that have recovered from infection and cannot infect others unless re-infected [16 - 18]. The four compartments make up the entire population of the

country, Nigeria.

The dynamics of the state variables in the system have been formulated as non-linear ordinary differential equations in form of Initial Value Problem (IVP) with seven unknown parameters. The following is the representation of the parameters. a_1: The rate at which individuals contract the disease (transmission rate). a_2: The rate at which the exposed individuals show symptoms of the disease (progression rate). a_3: The rate at which infectious persons recover from the disease (recovery rate). a_5: The rate at which recovered persons become susceptible again (immunity rate). a_6: The birth rate of persons included in the data set. a_7: The rate at which persons die due to natural factors (natural death rate). The total number of fixed people in the population at time t is given by $N = S(t) + E(t) + I(t) + R(t)$. The initial values of $S(t), E(t), I(t)$ and $R(t)$ are denoted by S_0, E_0, I_0 and R_0 respectively. The dead population as a function of time is $D(t) = N(0) - N(t)$. The basic assumptions of this study are given below.

2.2. SEIR Model Assumptions

1) We assume that all the state variables and parameters are positive.
2) It is assumed that every person in our population is susceptible to COVID-19 attack.
3) The infected and susceptible are assumed to mix homogeneously.
4) Some recovered individuals could go back to the susceptible class.
5) No pre-existing immunity (everyone in Nigeria is susceptible to the virus).
6) The total population N remains constant as we assume that birth rate and natural death rate are equal.
7) All persons in the population have the same probability to contract the disease irrespective of age or health status.

3. Mathematical Formulation of the Model

In this model the total population size, N, is considered closed as birth and death (death induced by the virus) rates are assumed equal. The total population is divided into four classes as shown below with Susceptible (S), Exposed (E), Infected (I) and Recovered (R) compartments. The model diagram is shown below.

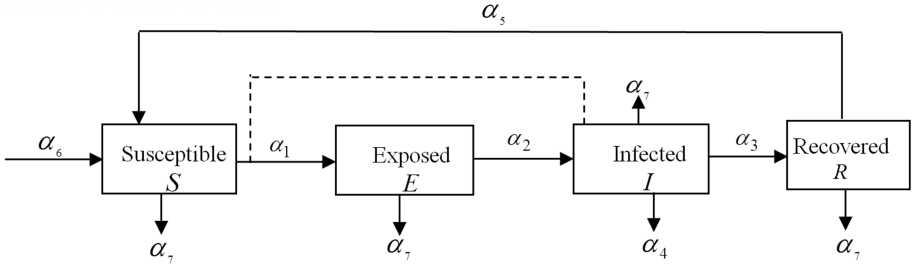

Figure 2. *Model diagram for the four compartments.*

From figure 2, the susceptible people (S) will move to the exposed compartment (E) updating the number of exposed person to a_1SI. Out of this exposed ones, a_2E individuals will move from E compartment to the infectious compartment (I). From the infectious compartment, a_3I persons move to the recovery group.

COVID-19 SEIR Model without Control parameter

Using the model diagram, we derived the following system of ordinary differential equations.

$$\frac{dS}{dt} = \alpha_6 N - \alpha_1 \frac{SI}{N} + \alpha_5 R - \alpha_7 S, \quad S(0) = S_0 > 0,$$

$$\frac{dE}{dt} = \alpha_1 \frac{SI}{N} - (\alpha_2 + \alpha_7)E, \quad E(0) = E_0 > 0,$$

$$\frac{dI}{dt} = \alpha_2 E - (\alpha_3 + \alpha_4 + \alpha_7)I, \quad I(0) = I_0 > 0 \text{ and}$$

$$\frac{dR}{dt} = \alpha_3 I - (\alpha_5 + \alpha_7)R, \quad R(0) > 0.$$

Considering the varying population, $N(t)$, and the proportions of each compartment of individuals in the population namely $s = S/N, e = E/N, i = I/N$ and $r = R/N$, we obtain the state variables s, e, i and r. These variables satisfy the following system of differential equations.

$$\frac{ds}{dt} = \alpha_6 - \alpha_1 si + \alpha_5 r - \alpha_7 s, \quad s(0) = s_0 > 0, \tag{1}$$

$$\frac{de}{dt} = \alpha_1 si - (\alpha_2 + \alpha_7)e, \quad e(0) = e_0 > 0, \tag{2}$$

$$\frac{di}{dt} = \alpha_2 e - (\alpha_3 + \alpha_4 + \alpha_7)i, \quad i(0) = i_0 > 0, \tag{3}$$

$$\frac{dr}{dt} = \alpha_3 i - (\alpha_5 + \alpha_7)r, \; r(0) = r_0 > 0 \qquad (4)$$

Here, $N(t) = s(t) + e(t) + i(t) + r(t) = 1$ for all $t \in [0, T]$ and T is the total time of investigation.

Existence and uniqueness of solution

The conditions under which the system (1 – 4) has solution is outlined in the following theorem.

Theorem 3.0 (Existence and uniqueness)

Assume that the functions $\alpha_6 - \alpha_1 si + \alpha_5 r - \alpha_7 s$, $\alpha_1 si - (\alpha_2 + \alpha_7)e$, $\alpha_2 e - (\alpha_3 + \alpha_4 + \alpha_7)i$ and $\alpha_3 i - (\alpha_5 + \alpha_7)r$ and their partial derivatives with respect to s, e, i, r are continuous in a rectangle $az < t < bz, \; cz < s, e, i, r < dz.$ Then for any $t_0 \in (az, bz)$ and $s_0, e_0, i_0, r_0 \in (cz, dz),$ the system 1 to 4 has a unique solution valid on some open interval (az, bz) containing t_0.

Proof. Let $y(t) = [s(t), e(t), i(t), r(t)], t \in (az, bz)$ and $s(t), e(t), i(t), r(t) \in (cz, dz)$. Then $\frac{dy}{dt} = (s_0, e_0, i_0, r_0).$

Define a function h as

$h(y(t)) =$ ($\alpha_6 - \alpha_1 si + \alpha_5 r - \alpha_7 s$, $\alpha_1 si - (\alpha_2 + \alpha_7)e$, $\alpha_2 e - (\alpha_3 + \alpha_4 + \alpha_7)i$, $\alpha_3 i - (\alpha_5 + \alpha_7)r$). Let $t_0 \in (az, bz)$ and $y(t_0) = (s(t_0), e(t_0), i(t_0), r(t_0)) = (s_0, e_0, i_0, r_0) = y_0 \in (cz, dz)$. By the assumption, we know that $h(y(t))$ and its partial derivatives $h_y(y(t))$ are continuous in the rectangle $az < t < bz, d < y < dz.$ [20], the initial value problem $\frac{dy}{dt} = h(y(t)), y(t_0) = y_0$ has a unique solution valid on some open interval containing t_0.

Linearization of the SEIR Model

Set

$$x_1 = s - s^*, \; x_2 = e - e^*, x_3 = i - i^*, x_4 = r - r^*.$$

$f_1 = \dot{s}, f_2 = \dot{e}, f_3 = \dot{i}, f_4 = \dot{r},$ At $\left(\dot{s}, \dot{e}, \dot{i}, \dot{r}\right),$ we have

$$\dot{x}_1 = x_1 \frac{\partial f_1}{\partial s} + x_2 \frac{\partial f_1}{\partial e} + x_3 \frac{\partial f_1}{\partial i} + x_4 \frac{\partial f_1}{\partial r}$$

$$\dot{x}_2 = x_1 \frac{\partial f_2}{\partial s} + x_2 \frac{\partial f_2}{\partial e} + x_3 \frac{\partial f_2}{\partial i} + x_4 \frac{\partial f_2}{\partial r}$$

$$\dot{x}_3 = x_1 \frac{\partial f_3}{\partial s} + x_2 \frac{\partial f_3}{\partial e} + x_3 \frac{\partial f_3}{\partial i} + x_4 \frac{\partial f_3}{\partial r}$$

$$\dot{x}_4 = x_1 \frac{\partial f_4}{\partial s} + x_2 \frac{\partial f_4}{\partial e} + x_3 \frac{\partial f_4}{\partial i} + x_4 \frac{\partial f_4}{\partial r}$$

$$\dot{x}_1 = -\alpha_1 i^* x_1 - \alpha_7 x_1 - \alpha_1 s^* x_3 + \alpha_5 x_4$$

$$\dot{x}_2 = \alpha_1 i^* x_1 - (a_2 + \alpha_7)x_2 - \alpha_1 s^* x_3$$

$$\dot{x}_3 = \alpha_2 x_2 - (a_3 + a_4 + \alpha_7)x_3$$

$$\dot{x}_4 = \alpha_3 x_3 - (a_5 + \alpha_7)x_4$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} -\alpha_1 i^* - \alpha_7 & 0 & -\alpha_1 s^* & \alpha_5 \\ \alpha_1 i^* & -(\alpha_2 + \alpha_7) & \alpha_1 s^* & 0 \\ 0 & \alpha_2 & -(\alpha_3 + a_4 + \alpha_7) & 0 \\ 0 & 0 & \alpha_3 & -(\alpha_5 + \alpha_7) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

4. Determination of the Basic Reproduction Number

The basic reproduction number, R_0, of the model is determined by employing the results of the next-generation matrix [21] and the first four differential equations. Let $x = (s, e, i, r), \; f(x)$ be the rate of appearance of new infection and $v(x)$ be the rate of transfer of individuals from all other sources into the compartment and transfer of individuals out of the compartment, then we have model 1 to 4, written as

$$f_i = \begin{pmatrix} 0 \\ \alpha_1 si \\ 0 \\ 0 \end{pmatrix}; \; v_i = \begin{pmatrix} \alpha_1 si + \alpha_7 s - \alpha_5 r - \alpha_6 \\ (a_2 + \alpha_7)e \\ (a_3 + \alpha_4 + \alpha_7)i - \alpha_2 e \\ (a_5 + \alpha_7)r - \alpha_3 i \end{pmatrix}$$

As infected compartments are only e and i then F and V are the Jacobian matrices of order 2×2 as defined as defined in Mathematical Tools for understanding Disease Dynamics [7] and the values of F and V for the new infection terms and the transmission terms are given respectively as

$$F = \begin{pmatrix} 0 & \alpha_1 \\ 0 & 0 \end{pmatrix} \text{and} \, V = \begin{pmatrix} \alpha_2 + \alpha_7 & 0 \\ -\alpha_2 & \alpha_3 + \alpha_4 + \alpha_7 \end{pmatrix}.$$

$$|V| = (\alpha_2 + \alpha_7)(\alpha_3 + \alpha_4 + \alpha_7)$$

$$\bar{V} = \begin{pmatrix} \alpha_3 + \alpha_4 + \alpha_7 & 0 \\ \alpha_2 & \alpha_2 + \alpha_7 \end{pmatrix}$$

$$V^{-1} = \frac{\bar{V}}{|V|} = \begin{pmatrix} \frac{1}{\alpha_2 + \alpha_7} & 0 \\ \frac{\alpha_2}{(\alpha_2 + \alpha_7)(\alpha_3 + \alpha_4 + \alpha_7)} & \frac{1}{\alpha_3 + \alpha_4 + \alpha_7} \end{pmatrix}$$

$$K = FV^{-1} = \begin{pmatrix} \frac{\alpha_1 \alpha_2}{(\alpha_2 + \alpha_7)(\alpha_3 + \alpha_4 + \alpha_7)} & \frac{\alpha_1}{\alpha_7(\alpha_3)} \\ 0 & 0 \end{pmatrix}$$

Hence, the basic reproduction number R_0 for the

COVID-19 model (3a – 3d) is obtained by calculating the spectral radius of the matrix FV^{-1} as:

$$R_0 = \frac{\alpha_1\alpha_2}{(\alpha_2+\alpha_7)(\alpha_3+\alpha_4+\alpha_7)}. \quad (5)$$

The following are the estimate of the unknown parameters.

$\alpha_1 = 0.70746202,\ \alpha_2 = 0.1876435,\ \alpha_3 = 0.31817251,$
$\alpha_4 = 0.002942,\ \alpha_5 = 0.00000049234,\ \alpha_6 = \alpha_7 = 0.0001,$

$$R_0 = \frac{0.70746202\times 0.1876435}{(0.1876435+0.0001)(0.31817251+0.002942+0.0001)} = 2.2012866.$$

$\alpha_1\alpha_2 = 0.70746202\times 0.1876435 = 0.132748$ represents the product of the disease transmission and disease progression rates.

$$\frac{1}{(\alpha_2+\alpha_7)(\alpha_3+\alpha_4+\alpha_7)} = \frac{1}{(0.1876435+0.0001)(0.31817251+0.002942+0.0001)} = 0.06030594$$

shows how long people are sick.

$R_0 = 2.2012866$ shows there is high rate of contagiousness of the infectious agent in the population, i.e one infected person is expected to infect, on the average, over two new persons.

4.1. Fitting Data to the Model

In this paper, we apply the method of non-linear least square which depends on nonlinearity of the residual. The non-linear problems are generally used in the iterative method of refinement in which the given data is fit to the model using the following procedure. Consider the data set $G_T = \{y_0, y_1, \ldots, y_T\}$ where $T = 62$ and $y_t = (s_t, e_t, i_t, r_t, d_t)'$ is the vector of the observed values at time t for the variables s, e, i, r and additional variable d. where $(.)'$ denotes transpose. Given the unknown parameters $\vartheta = (\alpha_1, \alpha_2, \alpha_3, \alpha_4\alpha_5, \alpha_7)'$, the prediction model, $\hat{y}_t(\vartheta)$, becomes

$$\hat{y}_t(\vartheta) = \begin{pmatrix} s_{t-1} + \alpha_7 + \alpha_5 r_{t-1} - \alpha_1 s_{t-1} i_{t-1} - \alpha_7 s_{t-1} \\ e_{t-1} + \alpha_1 s_{t-1} i_{t-1} - (\alpha_2+\alpha_7) e_{t-1} \\ t_{t-1} + \alpha_2 e_{t-1} - (\alpha_3+\alpha_4+\alpha_7) i_{t-1} \\ r_{t-1} - \alpha_3 i_{t-1} - (\alpha_5+\alpha_7) r_{t-1} \\ d_{t-1} + \alpha_4 i_{t-1} \end{pmatrix}$$

Where $\hat{y}_0(\vartheta)$ is a zero vector. The quadratic cost is given by $Q_T(\vartheta) = 0.5\sum_{t=0}^{T}\left\| y_t - \hat{y}_t(\vartheta)\right\|^2$ with $\|.\|$ as the Euclidean norm. The least-squares estimator is defined by the minimum of Q_T : $\vartheta_L = \arg\min_{\vartheta\in\Re^6} Q_T(\vartheta)$

Where $\vartheta_L = (\alpha_1, \alpha_2, \ldots, \alpha_7)$ is the estimated parameters. A curve fitting module, lmfit, implements this process numerically in python programming language. The first 62 daily data points of infectious people was selected for the fitting process prior to the peak of the infectious period. This data is given in the form $\{(t_0, Y_0), (t_1, Y_1), \ldots (t_{62}, Y_{62})\}$ and the SEIR model results appeared in the form $\{(t_0, I_0), (t_1, I_1), \ldots (t_{62}, I_{62})\}$.

4.2. Fitting Error Residue

The fit error or residual is denoted by res_i for $i = 0, 1, 2, \ldots 62$. Given n = 62 as total observations, we found the sum of squares error (SSE), mean square error (MSE) and root mean square error (RMSE) using $SSE = \sum_{i=0}^{i=63} res_i^2$, $MSE = SSE/n$ and $RMSE = \sqrt{MSE}$ respectively. Thereafter, we computed the R-squared and Adjusted R-squared of the fitted parameters.

5. Optimal Control

Here, we modify the SEIR model by introducing a control $u(t)$ which stands for all the control measures, especially, the social distancing order in Nigeria, to the system (1 - 4) as given below.

COVID-19 SEIR Model with Control parameter

$$\frac{ds}{dt} = a_6 - a_1 si(1-u) + a_5 r - a_7 s, \quad s(0) = s_0 > 0, \quad (6)$$

$$\frac{de}{dt} = a_1 si(1-u) - (a_2+a_7)e, \quad e(0) = e_0 > 0, \quad (7)$$

$$\frac{di}{dt} = a_2 e - (a_3+a_4+a_7)i, \quad i(0) = i_0 > 0, \quad (8)$$

$$\frac{dr}{dt} = a_3 i - (a_5+a_7)r, \quad r(0) = r_0 > 0 \quad (9)$$

Where the control $u(t)$ is between 0 and $U_{\max} < 1$ or $0 \le u \le U_{\max}$.

The essence is to reduce the contact rate among the susceptible population and the infectious population. Thus, the term $a_1 si(1-u)$ was introduced.

It follows from the existence and uniqueness of solution (1 - 4) that (6 -9) has a solution too.

5.1. Optimal Control Problem

The optimal control problem is to find the control level which minimizes the spread of COVID-19 in Nigeria and its controlling costs. Thus, we seek the optimal value $u*$ of the control u along time t such that the associated state

trajectories s^*, e^*, i^* and r^* are solutions of the above model equations and $u^*(.)$ maximizes the objective functional defined by

$$J(u(.)) = \int_0^T \left(s(t) - i(t) - u^2(t)\right)dt. \quad (10)$$

Subject to (6 - 9)

The integrand contains the fraction of susceptible individuals (s) and infectious people along with the severity of the side effects, u^2, of the control measures. The control u belongs to the bounded control set

$$U = \{u : 0 \le u < 1, t \in [0, T]\}. \quad (11)$$

Where $0 \le u \le u_{\max}$ for some $u_{\max} < 1$.

5.2. Characterization of the Optimal Control

To derive the necessary conditions for the optimal control, we apply Pontryagin's maximum principle [22, 23] to the Hamiltonian (H) where

$$H = s - i - u \wedge 2 + \lambda_1(\alpha_6 - \alpha_1 si(1-u) + \alpha_5 r - \alpha_7 s) + \lambda_2(\alpha_1 si(1-u) - (\alpha_2 + \alpha_7)e) + \lambda_3(\alpha_2 e - (\alpha_3 + \alpha_4 + \alpha_7)i) + \lambda_4(\alpha_3 i - (\alpha_5 + \alpha_7)r).$$

Where λ_j for $j = 1, 2, \ldots, 4$ denote the adjoint variables associated to the state variables s, e, i and r.

Theorem 4. Given an optimal control u^* and solutions s^*, e^*, i^*, r^* of the control system (6 – 9) that maximizes $J(u^*)$ over U, there existsadjoint variables λ_j satisfying

$$\frac{\partial \lambda_j}{\partial t} = -\frac{\partial H}{\partial t}$$

With the transversality conditions

$$\lambda_j(T) = 0, \; j = s, e, i, r.$$

The optimality condition is given by

$$H_u = \frac{\partial H}{\partial u} = 0.$$

Furthermore, we have the optimal control

$$u^* = \min\left(u_{\max}, \max\left(0, 0.5\left(\alpha_1 si(\lambda_1 - \lambda_2)\right)\right)\right).$$

Proof:

Using the Hamiltonian, we obtain the adjoint variables $\lambda_s, \lambda_e, \lambda_i$ and λ_r by solving the system

$$\dot{\lambda}_s = -\frac{\partial H}{\partial s}, \dot{\lambda}_e = -\frac{\partial H}{\partial e}, \dot{\lambda}_i = -\frac{\partial H}{\partial i} \text{ and } \dot{\lambda}_r = -\frac{\partial H}{\partial r}$$

Where

$$\dot{\lambda} = \frac{d\lambda}{dt}.$$

Thus,

$$\dot{\lambda}_s = -1 + \lambda_s\{\alpha_1 i(1-u) + \alpha_7\} - \lambda_e\{\alpha_1 i(1-u)\}, \lambda_s(T) = 0 \quad (12)$$

$$\dot{\lambda}_e = \lambda_s(\alpha_2 + \alpha_7) - \lambda_i\alpha_2, \; \lambda_e(T) = 0 \quad (13)$$

$$\dot{\lambda}_i = 1 + \lambda_s\{\alpha_1 s(1-u)\} - \lambda_e\{\alpha_1 s(1-u)\} + \lambda_i(\alpha_3 + \alpha_7) - \lambda_r\alpha_3, \; \lambda_i(T) = 0 \quad (14)$$

$$\dot{\lambda}_r = -\lambda_s\alpha_5 + \lambda_r(\alpha_5 + \alpha_7), \; \lambda_r(T) = 0 \quad (15)$$

The following optimal control u^* is derived from the stationary condition $H_u = 0$, using the properties from the control space.

$$u^* = \begin{cases} 0, & if \; H_u < 0. \\ 0.5\alpha_1 si(\lambda_1 - \lambda_2), & if \; H_u = 0. \\ U_{\max}, & if \; H_u > 0 \end{cases} \quad (16)$$

Thus, the optimal control of the optimization problem (6 – 9) and (10) can be characterized as

$$u^* = \min\left(u_{\max}, \max\left(0, 0.5\left(\alpha_1 si(\lambda_1 - \lambda_2)\right)\right)\right) \text{ in compact form.} \quad (17)$$

To provide numerical simulations of the state and adjoint equations, we employ a numerical approach known as Forward-Backward Sweep method. We wrote a MATLAB script for the implementation of the method through Runge-Kutta fourth order method. The basic algorithm is stated below.

5.3. Forward-Backward Sweep Method

Step 1. State the estimated initial value problem parameters. State the time range from $t_0 = 0$ to final time T. Choose the number of discretization points, the maximum control value ($u_{\max}$) and step size h.

Step 2. Choose initial state values using the data initial values. Set the stopping criteria.

Step 3. Solve the state equations forward in time and the adjoint equations backward in time simultaneously using Runge-Kutta 4th Order method.

Step 4. Update the variables in each iteration until the desired solution prescription is obtained.

Step 5. Check for convergence if the stopping criterion is met. If the solution is not optimal, go to step 2 and repeat the process. Otherwise choose the obtained solutions.

Step 6 Simulate the solution and compare with the real data.

5.4. Runge-Kutta 4th Order Method

Consider a step size h and the general state

$\frac{dy}{dt} = g(t, y(t))$. An approximation $y(t+h)$ is defined as $y(t+h) \approx y(t) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where $k_1 = g(t, y(t)),$ $k_2 = g(t + \frac{h}{2}, y(t) + \frac{h}{2}k_1),$ $k_3 = g(t + \frac{h}{2}, y(t) + \frac{h}{2}k_2),$ and $k_4 = g(t+h, y(t) + hk_3)$. The initial conditions are the same as those used for parameter fitting. We considered the effect of $u_{\max} = \{0.1, 0.2, 0.3\}$ on the state trajectories.

6. Results

Table 1. *Summary of estimated Parameter values.*

Parameters	Value	Reference
α_1	0.70746202	Data fitting
α_2	0.18764358	Data fitting
α_3	0.31817251	Data fitting
α_4	0.002942	Data fitting
α_5	0.00000049243	Data fitting
α_6	0.0001	Data fitting
α_7	0.0001	Data fitting

Using the estimated parameters, initial state values, $t_0 = 0, T = 365,$ $u_{\max} = \{0.1, 0.2, 0.3\}$, and population total N = 195,874,740 we got the following results.

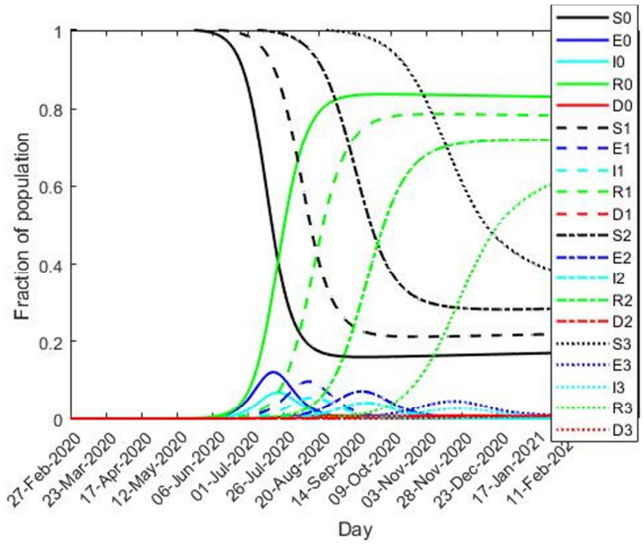

Figure 3. *Optimal states at various level of the control.*

The number of infected people decreases as control level increases. I_0, I_1, I_2 and I_3 denotes number of infected persons at control levels of $u_{\max} = 0, 0.1, 0.2$ and 0.3 respectively. Same notation goes for other state variables. The control, if adhered to will reduce the infectious rate as shown above. More control yields better result.

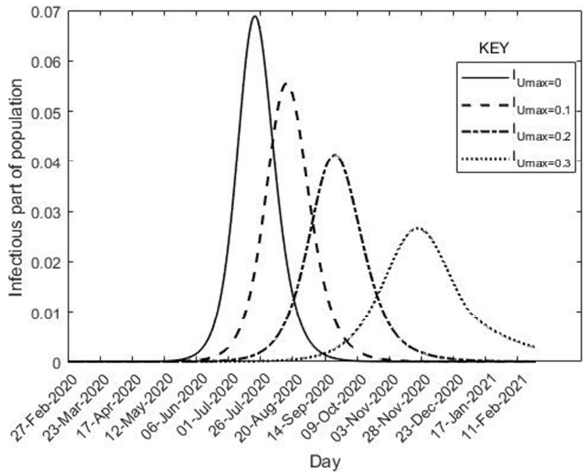

Figure 4. *Number of infected persons at different levels of control.*

Figure 4 highlights the decreasing trend of the infected individuals due to control levels. $I_{U\max=0.1}$ denotes infected individuals when $u_{\max} = 0.1$.

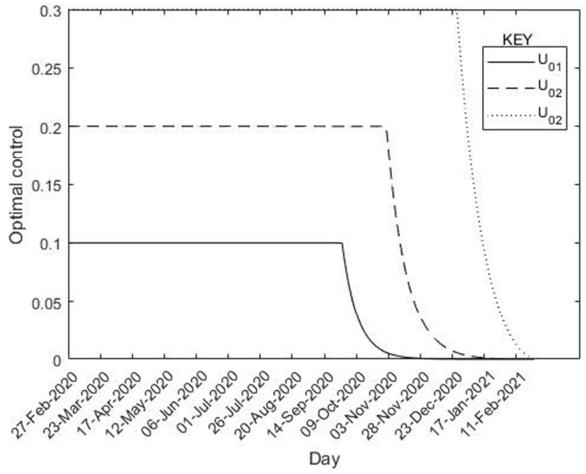

Figure 5. *Three control levels that maximized the objective function.*

U_{01}, U_{02} and U_{03} denote the control trajectories when $u_{\max} = 0.1,$ $u_{\max} = 0.2$ and $u_{\max} = 0.3$ respectively.

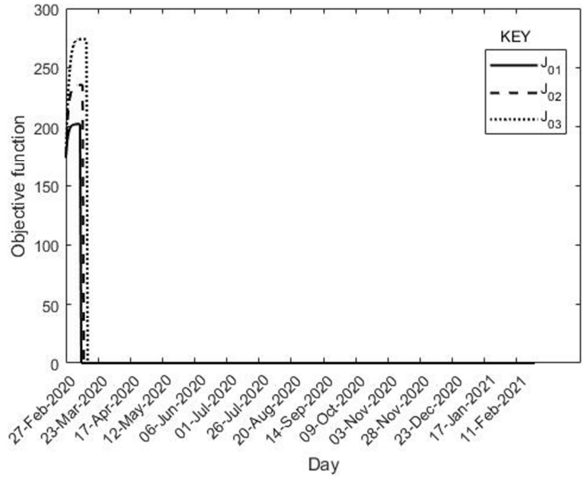

Figure 6. *Three objective function values at the given control levels.*

J_{01}, J_{02} and J_{03} denote the objective function trajectories when $u_{\max} = 0.1$, $u_{\max} = 0.2$ and $u_{\max} = 0.3$ respectively. Fitting a polynomial to the obtained control values when $u_{\max} = 0.1$, with R-squared=0.999459, gives

$$\begin{aligned} &u(t) = 0.1;\ H_u > 0,\ 0 \le t \le 223 \text{ and} \\ &u(t) = 27.3378 - 0.00621019t^2 + 0.0000678564t^3 \\ &\quad - 3.32837\times10^{-7}t^4 + 8.68489\times10^{-10}t^5 \\ &\quad - 1.17721\times10^{-12}t^6 + 6.54737\times10^{-16}t^7; \\ &\quad H_u = 0,\ 223 < t \le 365. \end{aligned}$$

7. Conclusion

This paper provides a study on numerical methods to explain how modeling and optimal control of COVID-19 dynamics could be very informative to researchers and Government's health policy makers. The SEIR results are close to what is obtained in real life cases. The SEIR results have shown that an increase in control measure against the disease will produce a decrease in the number of infected individuals and ultimately a reduction in the number of deaths induced by Covid-19 infection and it equally follows that administration covid-19 is more effective in combating the Covid-19 epidemic in Nigeria. The overall result shows the effectiveness of optimal control theory in medicine. We believe that this paper will be very useful to National Centre of Disease Control (NCDC) and other health policy makers.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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