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Adaptive Impulsive Synchronization for a Class of Delay Fractional-Order Chaotic System

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Abstract: This paper is concerned with the adaptive impulsive synchronization for a class of delay fractional-order chaotic system. Firstly, according to the impulsive differential equations theory and the adaptive control theory, the adaptive impulsive controller and the parametric update law are designed, respectively. Secondly, by constructing the suitable response system, the original fractional-order error system can be converted into the integral-order one. Finally, based on the Lyapunov stability theory and the generalized Barbalat's lemma, some new sufficient conditions are derived to guarantee the asymptotic stability of synchronization error system.

Keywords: Delay, Fractional-Order, Chaotic System, Impulsive, Synchronization

1. Introduction

Fractional-order calculus is a 300-year-old mathematical notion, as a generalization of integer order differentiation and integration to arbitrary non-integer order. The major advantage of the fractional-order derivatives is that they provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. It is recognized that the fractional derivative is more suitable for describing system characteristics of numerous real world fields than integer order [1-4]. For this reason, some specialists and scholars have incorporated fractional calculus into chaotic system and investigated the rich dynamics of fractional chaotic system, meanwhile, abundant literatures have been reported in succession [5–8]. Particularly, the chaotic synchronization of fractional-order chaotic system is becoming the hot topic of in the fields of nonlinear science [9–11], based on its wide applications in control processing and secure communication. Because impulsive control allows the stabilization and synchronization of chaotic systems to use only small control impulsive, it has been widely used to stabilize and synchronize chaotic systems [12–14].

In 2012, a novel impulsive control method [15] based on comparison system was proposed to realize complete synchronization of a class of fractional-order chaotic systems. Liu [16] obtained a new synchronization criterion of fractional-order chaotic systems by using the stability theory of impulsive fractional-order system. In 2008, Zhang et al. [17] investigated the adaptive impulsive synchronization for a class of non-autonomous integral-order chaotic systems with an unknown Lipschitz constant according to the generalized Barbalat's lemma and the Lyapunov stability theory. In 2012, Li et al. [18] discussed the issue of adaptive impulsive synchronization and parameter identification for a class of integral-order chaotic systems, and derived some sufficient conditions to synchronize the systems with different impulse distances. Xi et al. [19] investigated the adaptive impulsive synchronization for a class of fractional-order chaotic and hyper-chaotic systems with an unknown Lipschitz constant, and derived a new sufficient criterion to guarantee the asymptotical stability of synchronization error system by the Lyapunov stability theory and the generalized Barbalat's lemma. On the other hand, delayed differential equations have been abundantly studied by many researchers [20, 21].

Furthermore, time delays are also introduced to fractional differential chaotic systems [22, 23]. In 2007, Deng et al. [24] studied the stability of n-dimensional linear fractional differential equation with time delays, and also considered the synchronization between the coupled Duffing oscillators with time delays by using the linear feedback control method and their theorem. Gu et al. [25] investigated the global synchronization for fractional-order multiple time-delayed memristor-based neural networks with the parameter uncertainty, and derived the synchronization conditions of fractional-order multiple time-delayed memristor-based neural networks with the parameter uncertainty. More Zhang et al. [26] studied the drive-response synchronization fractional-order memristive neural networks with switching jumps mismatch, and obtained some lag quasi-synchronization conditions by the Laplace transform and linear feedback control.

However, to the best of the authors' knowledge, to this day, still less scholars consider the adaptive impulsive synchronization of delay fractional-order chaotic systems. Motivated by the above works, the adaptive impulsive synchronization for a class of fractional-order chaotic systems with an unknown Lipschitz constant and time delay is discussed. The rest of this paper is organized as follows: In Section 2, some preliminaries of fractional derivative are briefly introduced. A new adaptive impulsive synchronization method of delay fractional-order chaotic systems is proposed in Section 3, based on the theory of Lyapunov stability and impulsive differential equations. Finally, conclusions are addressed in Section 4.

2. Preliminaries of Fractional Derivative

The theory of the fractional order calculus is the arbitrary order calculus theory, where the order can be integer or fractional, or even complex. At present, there are several definitions of fractional-order differential operator, such as Grünwald-Letnikov (GL) definition, Riemann- Liouville (RL) definition, Caputo definition, and Jumarie definition. Among them, the method defined by GL is the most direct numerical one to solve the fraction-order system. For continuous function $f(t)$ in the integral interval $[t_0, t]$, the GL definition [27] is defined as

$$D_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[(t-t_0)/h]} (-1)^j \binom{\alpha}{j} f(t-jh), \quad (1)$$

where t_0 is the lower limit of integral, h is the calculation step, $[\bullet]$ rounds down to the nearest integer, α is the order of fractional derivative, and

$$\binom{\alpha}{j} = \frac{\alpha(\alpha-1)\cdots(\alpha-j+1)}{j!}$$

Eq. (1) can be reduced to

$$D_t^{\alpha} y(m) = h^{-\alpha} \sum_{j=0}^{m} \omega_j^{(\alpha)} y(m-j), \quad (2)$$

where

$$\omega_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}, \; j = 0, 1, 2, \cdots,$$

is the binomial coefficients and the numerical values can be recursively approximated as follows:

$$\omega_0^{(\alpha)} = 1, \; \omega_j^{(\alpha)} = \left(1 - \frac{\alpha+1}{j}\right)\omega_{j-1}^{(\alpha)}, \; j = 1, 2, \cdots.$$

3. Description of Adaptive Impulsive Synchronization

Consider the following fraction-order system with delay

$$D_t^{\alpha} \boldsymbol{x}(t) = A\boldsymbol{x}(t) + Bf(\boldsymbol{x}(t)) + Cf(\boldsymbol{x}(t-\tau)), \quad (3)$$

where $0 < \alpha < 1$, $A, B, C \in R^{n\times n}$, $f: R^n \to R^n$ is nonlinear vector function, τ is a positive constant representing delay, and $x(t) \in R^n$ represent the state vectors of the system. System (3) is regarded as the drive system.

Remark 3.1: When $\tau = 0$, the system (3) is reduced to the system (3) proposed in the literature [19].

Assumption 3.1 For any $x, y \in \Omega \subseteq R^n$, $\exists L > 0$ such that

$$\|f(\boldsymbol{y}(t)) - f(\boldsymbol{x}(t))\| \le L\|\boldsymbol{y}(t) - \boldsymbol{x}(t)\|. \quad (4)$$

In order to obtain self-synchronization of fractional-order system (3), the following controlled response system is constructed

$$\begin{cases} D_t^{\alpha} \boldsymbol{y}(t) = A\boldsymbol{y}(t) + Bf(\boldsymbol{y}(t)) + Cf(\boldsymbol{y}(t-\tau)) + u(t, \boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)), \\ \qquad t \ne t_k, k = 1, 2, \cdots, \\ \Delta \boldsymbol{y}(t) = \boldsymbol{y}(t_k^+) - \boldsymbol{y}(t_k^-) = \delta_k e(t_k), \quad t = t_k, \end{cases} \quad (5)$$

where $y(t) \in R^n$ is the state vector of system (5), $u(t, x(t-\tau), y(t-\tau), x(t), y(t)) \in R^n$ is the adaptive controller, and $\delta_k = \delta_k^T$ are $n\times n$ gain matrices. The discrete time set t_k satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, and the initial time t_0 satisfies $0 \le t_0 < t_1$. Set $y(t_k^+) = \lim_{t\to t_k^+} y(t)$,

$\boldsymbol{y}(t_k^-) = \lim_{t \to t_k^-} y(t)$, and further assume that the solution of $y(t)$ is left continuous at the points of impulse, that is, $y(t_k^-) = y(t_k)$. Now, let

$$e(t) = \boldsymbol{y}(t) - \boldsymbol{x}(t) = [\boldsymbol{y}_1(t) - \boldsymbol{x}_1(t), \boldsymbol{y}_2(t) - \boldsymbol{x}_2(t), \cdots, \boldsymbol{y}_n(t) - \boldsymbol{x}_n(t)]^T$$

be synchronization error vector. then the impulsive synchronization error system between system (3) and system (5) can be described as

$$\begin{cases} D_t^\alpha e(t) = Ae(t) + Bf(\boldsymbol{x}(t), \boldsymbol{y}(t)) + Cf(\boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)) \\ \qquad\qquad + u(t, \boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)), \quad t \neq t_k, \quad k = 1, 2, \cdots, \\ \Delta e(t) = e(t_k^+) - e(t_k^-) = \delta_k e(t_k), \quad t = t_k, \end{cases} \tag{6}$$

where $f(\boldsymbol{x}(t), \boldsymbol{y}(t)) = f(\boldsymbol{y}(t)) - f(\boldsymbol{x}(t))$, $f(\boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)) = f(\boldsymbol{y}(t-\tau)) - f(\boldsymbol{x}(t-\tau))$

According to Assumption 3.1, it follows that

$$f(\boldsymbol{x}(t), \boldsymbol{y}(t)) \le L\|e(t)\|, \quad f(x(t-\tau), y(t-\tau)) \le L\|e(t-\tau)\|$$

Thus, the synchronization problem is to design the adaptive controller and the parametric update law to achieve the asymptotical synchronization of the drive system (3) and the response system (5), that is, $\lim_{t\to\infty} e(t) = 0$.

To solve this problem, a new controlled response system is constructed as follows

$$\begin{cases} \dot{y}(t) = Ay(t) + Bf(\boldsymbol{y}(t)) + Cf(\boldsymbol{y}(t-\tau)) + u(t, \boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)) \\ \qquad\quad + N(\boldsymbol{x}(t)), \quad t \neq t_k, \quad k = 1, 2, \cdots, \\ \Delta y(t) = y(t_k^+) - y(t_k^-) = \delta_k e(t_k), \quad t = t_k, \end{cases} \tag{7}$$

where $N(x(t)) = \dot{x}(t) - D_t^\alpha x(t)$, $\dot{x}(t)$ and $D_t^\alpha x(t)$ are from system (3). Then the following new synchronization error system can be obtained by systems (3) and (7)

$$\begin{cases} \dot{e}(t) = Ae(t) + Bf(\boldsymbol{x}(t), \boldsymbol{y}(t)) + Cf(\boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)) \\ \qquad\quad + u(t, \boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau)), \quad t \neq t_k, k = 1, 2, \cdots, \\ \Delta e(t) = e(t_k^+) - e(t_k^-) = \delta_k e(t_k), \quad t = t_k, \end{cases} \tag{8}$$

It is obvious that the synchronization of fractional-order chaotic system (3) can be converted into the impulsive control of integral-order synchronization error system (8) by constructing the response system (7).

The adaptive impulsive controller is designed as follows

$$u(t, \boldsymbol{x}(t-\tau), \boldsymbol{y}(t-\tau), \boldsymbol{x}(t), \boldsymbol{y}(t)) = -LCe(t-\tau) + (L - \Gamma(t) - \beta)e(t), \tag{9}$$

where $\beta > 0$ is a constant, the parameter $\Gamma(t)$ is used to approach the unknown parameter L, and its update law is given as

$$\begin{cases} \dot{\Gamma}(t) = \gamma\|e(t)\|^2, \quad t \neq t_k, k = 1, 2, \cdots, \\ \Delta\Gamma(t) = \Gamma(t_k^+) - \Gamma(t_k^-) = 0, \quad t = t_k, \end{cases} \tag{10}$$

where $\gamma > 0$ is the adaptive rate.

Remark 3.2: When $\tau = 0, C = E$ (unit matrix), the controller (9) is reduced to the controller (10) proposed in work [19], but the controller of the literature [19] can't be used in this manuscript.

The following generalized Barbalat's lemma is used to obtain the main result.

Lemma 3.1 (See, [17]). Suppose a sequence t_k satisfies $0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$, $\lambda = \inf_k \{t_k - t_{k-1}\} > 0$. And suppose $f(t)$ is defined on the interval $[t_0, +\infty)$ and differentiable on interval $[t_{k-1}, t_k)$. If $f(t)$ and $\dot{f}(t)$ are uniformly bounded for k on the interval $[t_{k-1}, t_k)$, that is, $\exists M_0, M_1 > 0$, $\forall t \in [t_{k-1}, t_k)$, $k \in N$, one has $|f(t)| \le M_0$, $|\dot{f}(t)| \le M_1$, and the generalized integration $\int_0^{+\infty} f(t)dt$ is convergent, then $\lim_{t\to+\infty} f(t) = 0$.

Next, the main result and its proof are given.

Theorem 3.1 Let λ_{A+A^T} be the largest eigenvalue of

$A+A^T$, ε_0 is a sufficiently small positive number. If the adaptive impulsive controller (9) and the parametric update law (10) are adopted, as well as the following conditions are satisfied

$$(H_1)\ \lambda = \inf_k \{t_k - t_{k-1}\} > 0,$$

$$(H_2)\ \lambda_{\max}((E+\delta_k)^T(E+\delta_k)) \le 1,$$

$$(H_3)\ \lambda_{A+A^T} - \beta + \varepsilon_0 < \lambda_{\min}(-L(B+B^T)),$$

then the synchronization error system (8) is asymptotically stable, that is, the impulsive controlled response system (5) and the drive system (3) asymptotically synchronize.

Proof. Let the Lyapunov candidate function be

When $t=t_k$, from Assumption 3.1, Eq. (8), Eq. (10), and the condition (H_2), it follows that

$$\begin{aligned}\Delta V(t) &= V(t_k^+) - V(t_k^-)\\ &= \frac{1}{2}[(e(t_k^+))^T e(t_k^+) - (e(t_k^-))^T e(t_k^-)]\\ &= \frac{1}{2}\{[(E+\delta_k)e(t_k)]^T[(E+\delta_k)e(t_k)] - (e(t_k))^T e(t_k)\}\\ &= \frac{1}{2}(e(t_k))^T[(E+\delta_k)^T(E+\delta_k) - E]e(t_k) \le 0.\end{aligned}$$

For any $t\in[t_{k-1},t_k)$, $k\in N$, its dini-derivative along the trajectory of the error system (8) is given by

$$V(t) = \frac{1}{2}(e(t))^T e(t) + \frac{1}{2\gamma}(\Gamma(t)-L)^2$$

$$\begin{aligned}D^+V(t) &= \frac{1}{2}(\dot e(t))^T e(t) + \frac{1}{2}(e(t))^T \dot e(t) + \frac{1}{\gamma}(\Gamma(t)-L)\dot\Gamma(t)\\
&= \frac{1}{2}[Ae(t) + Bf(\boldsymbol{x}(t),\boldsymbol{y}(t)) + Cf(\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)) + u(t,\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{x}(t-\tau),\\
&\quad \boldsymbol{y}(t-\tau))]^T e(t) + \frac{1}{2}(e(t))^T[Ae(t) + Bf(\boldsymbol{x}(t),\boldsymbol{y}(t)) + Cf(\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau))\\
&\quad + u(t,\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau))] + \frac{1}{\gamma}(\Gamma(t)-L)\dot\Gamma(t)\\
&= \frac{1}{2}(e(t))^T(A+A^T)e(t) + \frac{1}{2}(f(\boldsymbol{x}(t),\boldsymbol{y}(t)))^T B^T e(t) + \frac{1}{2}(e(t))^T Bf(\boldsymbol{x}(t),\boldsymbol{y}(t))\\
&\quad + \frac{1}{2}(f(\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)))^T C^T e(t) + \frac{1}{2}(e(t))^T Cf(\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau))\\
&\quad + \frac{1}{2}(u(t,\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)))^T e(t) + \frac{1}{2}(e(t))^T u(t,\boldsymbol{x}(t),y(t),\\
&\quad \boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)) + \frac{1}{\gamma}(\Gamma(t)-L)\dot\Gamma(t)\\
&\le \frac{1}{2}\lambda_{A+A^T}(e(t))^T e(t) + \frac{1}{2}L(e(t))^T B^T e(t) + \frac{1}{2}L(e(t))^T Be(t) + \frac{1}{2}L(e(t-\tau))^T C^T e(t)\\
&\quad + \frac{1}{2}L(e(t))^T Ce(t-\tau) + \frac{1}{2}(u(t,\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)))^T e(t)\\
&\quad + \frac{1}{2}(e(t))^T u(t,\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)) + \frac{1}{\gamma}(\Gamma(t)-L)\dot\Gamma(t)\\
&= \frac{1}{2}(e(t))^T(\lambda_{A+A^T}E + LB^T + LB)e(t) + L(e(t-\tau))^T C^T e(t)\\
&\quad + (u(t,\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{x}(t-\tau),\boldsymbol{y}(t-\tau)))^T e(t) + \frac{1}{\gamma}(\Gamma(t)-L)\dot\Gamma(t)\\
&= \frac{1}{2}(e(t))^T(\lambda_{A+A^T}E + LB^T + LB)e(t) + L(e(t-\tau))^T C^T e(t) + [-LCe(t-\tau)\\
&\quad + (L-\Gamma(t)-\beta)e(t)]^T e(t) + (\Gamma(t)-L)(e(t))^T e(t)\\
&= \frac{1}{2}(e(t))^T[(\lambda_{A+A^T}-\beta)E + L(B+B^T)]e(t) < -\frac{\varepsilon_0}{2}e^T(t)e(t) \le 0.\end{aligned}$$

From aboveanalysis, so $V(t)$ is monotonically decreasing on the interval $[0,+\infty)$. Based on the fact $V(t)\ge 0$, therefore limit $\lim_{t\to+\infty} V(t)$ exists. Moreover, by the Cauchy convergence principle, it holds that for any $\varepsilon>0$, exists $M\in N$, such that when $t'' > t' > M$, it holds that $V(t') - V(t'') < \varepsilon$.

Hence, it follows that

$$\begin{aligned}\frac{1}{2}\varepsilon_0\int_{t'}^{t''}(e(t))^T e(t)\,dt &= \frac{1}{2}\varepsilon_0\Big(\int_{t'}^{t_{k_1}}(e(t))^T e(t)\,dt + \int_{t_{k_1}}^{t_{k_2}}(e(t))^T e(t)\,dt + \cdots + \int_{t_{k_s}}^{t''}(e(t))^T e(t)\,dt\Big)\\ &\le -\Big(\int_{t'}^{t_{k_1}} D^+V(t)dt + \int_{t_{k_1}}^{t_{k_2}} D^+V(t)dt + \cdots + \int_{t_{k_s}}^{t''} D^+V(t)dt\Big)\\ &= V(t') - V(t_{k_1}^-) + V(t_{k_1}) - V(t_{k_2}^-) + \cdots + V(t_{k_s}) - V(t'')\\ &= V(t') - V(t'') + V(t_{k_1}) - V(t_{k_1}^-) + \cdots + V(t_{k_s}) - V(t_{k_s}^-)\\ &\le V(t') - V(t'') < \varepsilon.\end{aligned}$$

Thus, by the Cauchy convergence principle, it follows that $\frac{1}{2}\varepsilon_0\int_0^{+\infty}(e(t))^T e(t)dt$ is convergent, that is, $\int_0^{+\infty}(e(t))^T e(t)dt$ is convergent. Since chaotic system is bounded, $e(t)$ and $\dot{e}(t)$ are uniformly bounded for k on the interval $[t_{k-1}, t_k)$, so $(e(t))^T e(t)$ and $(e(t))^T \dot{e}(t)$ are uniformly bounded for k on the interval $[t_{k-1}, t_k)$. According to Lemma 3.1, it holds that $\lim_{t\to+\infty}(e(t))^T e(t) = 0$, that is, $\lim_{t\to\infty} e(t) = 0$.

4. Conclusions

In this paper, a class of delay fractional-order chaotic system is proposed and investigated. On the strength of the theory of control system, the theory of impulsive differential equation and the theory of fractional differential equation, some sufficient conditions to ensure the asymptotic synchronization of the drive system and the response system are obtained by applying the generalized Barbalat's lemma and developing some new analysis methods as well as constructing a suitable Lyapunov function. Some known results are extended and improved. Particularly, it is worth pointing out that the proposed method can also be applied to synchronize other fractional-order chaotic systems with delay or without delay.

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