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On The Solution of Newell-Whitehead-Segel Equation

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Abstract: In this paper, we apply a combined form of the Elzaki transform method with the Adomian decomposition method to obtain the solution of Newell-Whitehead Segel equation. This method is called the Elzaki Adomian decomposition method (EADM). The method can be applied to linear and nonlinear problems. The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear problems.

Keywords: Newell-Whitehead-Segel Equation, Elzaki Transform Method, Adomian Decomposition Method

1. Introduction

The nonlinear equations play an important role in modeling various phenomena arising in applied science. Several systems are modeled by partial differential equations and most of them are nonlinear. Solving nonlinear system is an important task in mathematical analysis and applications [1-3]. One of the most important of amplitude equations is the Newell-Whitehead-Segel equation (NWS) [4-6] which describes the appearance of the stripe pattern in two dimensional systems. Moreover, this equation was applied to a number of problem in a variety systems, e.g., Rayleigh-Benard convection, Faraday instability, nonlinear optics, chemical reactions and biological systems. In recent years, Different methods are utilized to solve NWS equation. P. Pue-on [7] applied Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation. Malomed in [8] proposed dispersive NWS equation for the description of traveling waves patterns in binary fluids. Nourazar et al.[9] used homotopy perturbation method (HPM) and Aasaraai [10] used differential transformed method (DTM) to solve these equations. Ezzati and Shakibi [11] applied ADM and multiquadric quasi-interpolation methods for same purpose. Macas- Daz and Ruiz-Ramrez proposed non-standard symmetry-preserving method to compute bounded solutions of NWS equation in [12]. Recently, Tarig M. Elzaki and Sailh M. Elzaki in [13-19], showed Elzaki transform, was applied to partial differential equations, ordinary differential equations, system of ordinary and partial differential equations and integral equations. Elzaki transform is a powerful tool for solving some differential equations which cannot solve by Sumudu transform. In this paper a reliable Elzaki Adomian decomposition method is applied for solving Newell-Whitehead-Segel equation. The method can be employed to linear and nonlinear problems, and The major advantage of this method is its capability of combining the two powerful method to obtain exact solution for nonlinear equation. Moreover, some examples are illustrative for demonstrating the advantage of the method.

2. Elzaki Adomian Decomposition Method

Let us consider the initial value problem in Newell-Whitehead-Segel equation in the form

$$u_t(x,t) = Ku_{xx}(x,t) + au(x,t) - bu^m(x,t) \quad (1)$$

$$u(x,0) = f(x) \quad (2)$$

where a and b are real numbers and k and m are positive integers.

By applying the Elzaki transform on both sides of the equation (1) and using the linearity of the Elzaki transform gives:

$$E[u_t(x,t)] = KE[u_{xx}] + aE[u(x,t)] - bE[u^m] \quad (3)$$

Using the differential property of Elzaki transform Eq. (3) can be written as:

$$\frac{1}{v}E[u(x,t)] - v\,u(x,0) = KE[u_{xx}] + aE[u(x,t)] - bE[u^m] \quad (4)$$

Using initial condition (2), Eq. (4) can be written as:

$$E[u(x,t)] = \frac{v^2 f(x)}{1-av} + \frac{Kv}{1-av}E[u_{xx}] - \frac{bv}{1-av}E[u^m] \quad (5)$$

The Elzaki Adomian decomposition method represents solution as an infinite series of components given below

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \quad (6)$$

and the nonlinear term $F(u) = u^m, m > 1$ can be presented by an infinite series

$$F(u) = \sum_{n=0}^{\infty} A_n \quad (7)$$

where the components A_n are Adomain polynomials [5] of $u_0, u_1, \dots, u_n$ which can be calculated by formula

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} F\left(\sum_0^{\infty} \lambda^i u_i\right)\right]_{\lambda=0} \quad (8)$$

Specific algorithms were seen in [7, 11] to formulate Adomian polynomials [20-21]. The following algorithm:

$$A_n = F(u_0)$$

$$A_1 = u_1 F'(u_0)$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2}u_1^2 F''(u_0)$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3}u_1^3 F'''(u_0)$$

$$A_4 = u_4 F'(u_0) + \left(u_1 u_3 + \frac{1}{2}u_2^2\right)F''(u_0) + \frac{1}{2}u_1^2 u_2 F'''(u_0) + \frac{1}{24}u_1^4 F^{(iv)}(u_0) \quad (9)$$

…..

can be used to construct Adomian polynomial. By substituting (6) and (7) into (5), one get

$$E\left[\sum_{n=0}^{\infty} u_n(x,t)\right] = \frac{v^2 f(x)}{1-av} + \frac{Kv}{1-av}E\left[\left(\sum_{n=0}^{\infty} u_n(x,t)\right)_{xx}\right] - \frac{bv}{1-av}E\left[\sum_{n=0}^{\infty} A_n\right] \quad (10)$$

Operating with the Elzaki inverse on both sides of equation (10) gives:

$$\sum_{n=0}^{\infty} u_n(x,t) = E^{-1}\left[\frac{v^2 f(x)}{1-av}\right] + E^{-1}\left[\frac{Kv}{1-av}E\left[\left(\sum_{n=0}^{\infty} u_n(x,t)\right)_{xx}\right] - \frac{bv}{1-av}E\left[\sum_{n=0}^{\infty} A_n\right]\right] \quad (11)$$

By comparing the both sides of the Eq. (11), we have the recursive relation is given by

$$u_0 = E^{-1}\left[\frac{v^2 f(x)}{1-av}\right] = e^{at} f(x) \quad (12)$$

$$u_{n+1} = E^{-1}\left[\frac{Kv}{1-av}E[u_n(x,t)_{xx}] - \frac{bv}{1-av}E[A_n]\right] n \geq 0 \quad (13)$$

3. Application

In this section, some initial value problems are presented to show the advantages of the proposed method which can be applied to linear and nonlinear problem.

Example 3.1.

Consider linear Newell-Whitehead-Segel equation

$$u_t = u_{xx} - 3u \quad (14)$$

subject to initial condition

$$u(x,0) = e^{2x} \quad (15)$$

By taking the Elzaki transform on both sides of (14), then using the differentiation property of Elzaki transform one obtains

$$\frac{E[u(x,t)]}{v} - v\,u(x,0) = E[u_{xx}] - 3\,E[u(x,t)] \quad (16)$$

Using initial condition (15), Eq. (16) can be written as:

$$E[u(x,t)] = \frac{v^2 e^{2x}}{1+3v} + \frac{v}{1+3v}E[u_{xx}] \quad (17)$$

The inverse Elzaki transform implies that:

$$u(x,t) = E^{-1}\left[\frac{v^2 e^{2x}}{1+3v}\right] + E^{-1}\left[\frac{v}{1+3v}E[u_{xx}]\right]$$

$$u(x,t) = e^{2x}e^{-3t} + E^{-1}\left[\frac{v}{1+3v}E[u_{xx}]\right] \quad (18)$$

The Elzaki Adomian decomposition defines the solution u(x, t) by the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n \quad (19)$$

So, the term u_{xx} can be defined by an infinite series

$$u_{xx}(x,t) = \sum_{n=0}^{\infty} (u_n)_{xx} \quad (20)$$

Substituting (19) and (20) into both sides of Eq.(18) gives

$$\sum_{n=0}^{\infty} u_n = e^{2x}e^{-3t} + E^{-1}\left[\frac{v}{1+3v}E\left[\sum_0^{\infty}(u_n)_{xx}\right]\right] \quad (21)$$

Thus, the recursive relation is defined by

$$u_0 = e^{2x}e^{-3t} \quad (22)$$

$$u_{n+1} = E^{-1}\left[\frac{v}{1+3v}E\left[\sum_0^{\infty}(u_n)_{xx}\right]\right] n \geq 0 \quad (23)$$

The other components of the solution can easily calculated by using the above recursive relation

$$u_1 = E^{-1}\left[\frac{v}{1+3v}E[(u_0)_{xx}]\right] = 4e^{2x}E^{-1}\left[\frac{v^3}{(1+3v)^2}\right] = 4te^{2x}e^{-3t},$$

$$u_2 = E^{-1}\left[\frac{v}{1+3v}E[(u_1)_{xx}]\right] = 16e^{2x}E^{-1}\left[\frac{v^4}{(1+3v)^3}\right] = 8t^2 e^{2x}e^{-3t},$$

$$u_3 = E^{-1}\left[\frac{v}{1+3v}E[(u_2)_{xx}]\right] = 64e^{2x}E^{-1}\left[\frac{v^5}{(1+3v)^4}\right] = \frac{32}{3}t^3e^{2x}e^{-3t},$$

And so on for other components. Using (19), the series solutions are therefore given by

$$u(x,t) = e^{2x}e^{-3t}\left[1+4t+8t^2+\frac{32}{3}t^3+\cdots\right] = e^{2x}e^{-3t}e^{4t}$$

In series form, we can find the exact solution

$$u(x,t) = e^{2x+t}$$

Example 3.2

Consider nonlinear Newell-Whitehead-Segel equation

$$u_t = 5u_{xx} + 2u + u^2 \quad (24)$$

subject to initial condition

$$u(x,0) = \alpha \quad (25)$$

where α is arbitrary constant. By taking the Elzaki transform on both sides of (25), then using the differentiation property of Elzaki transform one obtains

$$\frac{E[u(x,t)]}{v} - v\,u(x,0) = 5E[u_{xx}] + 2E[u(x,t)] + E[u^2] \quad (26)$$

Using initial condition (25), Eq. (26) can be written as:

$$E[u(x,t)] = \frac{\alpha v^2}{1-2v} + \frac{5v}{1-2v}E[u_{xx}] + \frac{v}{1-2v}E[u^2] \quad (27)$$

The inverse Elzaki transformation is applied to Eq.(27) we get

$$u(x,t) = E^{-1}\left[\frac{\alpha v^2}{1-2v}\right] + E^{-1}\left[\frac{5v}{1-2v}E[u_{xx}]\right] + E^{-1}\left[\frac{v}{1-2v}E[u^2]\right]$$

$$u(x,t) = \alpha e^{2t} + E^{-1}\left[\frac{5v}{1-2v}E[u_{xx}]\right] + E^{-1}\left[\frac{v}{1-2v}E[u^2]\right] \quad (28)$$

As before we defines the solution $u(x,t)$ by the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n \quad (29)$$

and u_{xx} can be defined by an infinite series

$$u_{xx}(x,t) = \sum_{n=0}^{\infty} (u_n)_{xx} \quad (30)$$

The nonlinear term $F(u) = u^2$ is decomposed in term of Adomian polynomials

$$u^2 = \sum_{n=0}^{\infty} A_n \quad (31)$$

Substituting (29), (30) and (31) into both sides of Eq.(28) gives

$$\sum_{n=0}^{\infty} u_n = \alpha e^{2t} + E^{-1}\left[\frac{5v}{1-2v}E\left[\sum_{n=0}^{\infty}(u_n)_{xx}\right]\right] + E^{-1}\left[\frac{v}{1-2v}E\left[\sum_{n=0}^{\infty}A_n\right]\right] \quad (32)$$

Thus, the recursive relation is defined by

$$u_0 = \alpha e^{2t} \quad (33)$$

$$u_{n+1} = E^{-1}\left[\frac{5v}{1-2v}E[(u_n)_{xx}]\right] + E^{-1}\left[\frac{v}{1-2v}E[A_n]\right] \quad (34)$$

The other components of the solution can easily calculated by using the above recursive relation

$$u_1 = E^{-1}\left[\frac{5v}{1-2v}E[(u_0)_{xx}]\right] + E^{-1}\left[\frac{v}{1-2v}E[A_0]\right] = \alpha^2E^{-1}\left[\frac{v^3}{(1-2v)(1-4v)}\right] = \frac{\alpha^2}{2}e^{2t}[e^{2t}-1],$$

$$u_2 = E^{-1}\left[\frac{5v}{1-2v}E[(u_1)_{xx}]\right] + E^{-1}\left[\frac{v}{1-2v}E[A_1]\right] = \alpha^3E^{-1}\left[\frac{v^2}{(1-2v)(1-6v)} - \frac{v^2}{(1-2v)(1-4v)}\right] = \frac{\alpha^3}{4}e^{2t}[e^{2t}-1]^2$$

$$u_3 = E^{-1}\left[\frac{5v}{1-2v}E[(u_2)_{xx}]\right] + E^{-1}\left[\frac{v}{1-2v}E[A_2]\right]$$

$$= \alpha^3E^{-1}\left[\frac{v^2}{(1-2v)(1-8v)} - \frac{v^2}{(1-2v)(1-6v)} - \frac{v^2}{(1-2v)(1-4v)}\right] = \frac{\alpha^4}{8}e^{2t}[e^{2t}-1]^3 \quad (35)$$

And so on for other components. Using (29), the series solutions are therefore given by

$$u(x,t) = e^{2t}\left[\alpha + \frac{\alpha^2}{2}[e^{2t}-1] + \frac{\alpha^3}{4}[e^{2t}-1]^2 + \frac{\alpha^4}{8}[e^{2t}-1]^3 + \cdots\right]$$

In series form, we can find the exact solutions

$$u(x,t) = e^{2t}\left[\frac{\alpha}{1-\frac{\alpha(e^{2t}-1)}{2}}\right] = \frac{2\alpha e^{2t}}{2+\alpha(1-e^{2t})} \quad (36)$$

4. Conclusion

In this work, Elzaki Adomian decomposition method is a powerful device to solve many functional equations. Here we have successfully used the method for solving Newell-Whitehead-Segel equation. The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear problems.

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