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Structure Evolution in Odd-Even Eu- 155 Nucleus within IBFM-2

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Abstract: In this work, We investigate the energy levels and the electromagnetic transition probabilities B(E2) and B(M1) for Eu-155 within framework of IBFM-2. The results are in reasonably agreement with the available experimental values. The result of an IBFM-2 multilevel calculation with the $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, and $1g_{7/2}$ single particle orbits is reported for the positive parity states of the odd-mass Eu-155 isotopes. Also, an IBM-2 calculation is presented for the low-lying states in the even-even Sm-154 core nucleus.

Keywords: Interacting Boson Model, Interacting Boson-Fermion Model, Energy Levels, B(E2), B(M1) and Mixing Ratios

1. Introduction

The interacting boson model represents a significant step forward in our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques, which have also found recent application to problems in atomic, molecular, and high-energy physics [1]. The application of this model to deformed nuclei is currently a subject of considerable interest and controversy.

The interacting boson model-1 (IBM-1) [2] and its extension to the odd-A nuclei, the interacting boson-fermion model (IBFM-1) [3], have proved to be able to give a successful description of widely varying classes of nuclei situated away from closed shell configurations.

In heavy nuclei, the neutron excess prevents the formation of correlated proton-neutron pairs and one thus is led to consider only proton-proton and neutron-neutron pairs. The corresponding model is the interacting boson model-2 (IBM-2) [4, 5]. The introduction of fermions in this models leads to the interacting boson-fermion model-2 (IBFM-2). In addition to a more direct connection with the spherical shell model, the interacting boson-fermion model-2 (IBFM-2) has features that cannot be obtained in the interacting boson-fermion model-1 (IBFM-1). Here, we apply the IBFM-2 model to account for ^{155}Eu isotope.

Detailed work has been done on the structure of europium nuclei in recent years; Bhattacharya *et al.*, [6] studied on level structure, single-nucleon-transfer spectroscopic factors, electromagnetic transition strengths, and relative gamma-ray branching. Guchhait *et al.* .[7] determined the level energies, spectroscopic factor, and E2 transition strengths. Prokofjev *et al,* .[8] studied on the γ-ray and conversion electron spectra of ^{155}Eu from the (n, γ) reaction. Lo Bianco *et al,* .[9] studied gamma-ray transitions in ^{147}Eu and analyzed in terms of the interacting boson-fermion model. There are also theoretical studies of particular isotopes with different models. Yazar *et al*., [10] explored the energy levels and the electric quadrupole transition probabilities B(E2; $J_i \rightarrow J_f$) and γ-ray E2/M1 mixing ratios for selected transitions of some isotopes. Akaya *et al,* . [11] studied on the gamma-gamma angular correlation and $e_K - \gamma(\theta)$ directional correlation methods and γ-ray E2/M1 mixing ratios of ^{154}Eu were investigated. Yazar et al., [12] studied some electromagnetic transition properties of $^{153\text{-}155}$Eu Isotopes within IBFM-2. The aim of the present work is to do a systematic study of the ^{155}Eu isotope within the IBFM-2 model.

It is generally believed that such positive parity spin states can be explained in particle-core coupled type models. ^{155}Eu has 63 protons and 92 neutrons, it is thus appropriate to describe ^{155}Eu in the IBFM-2 by the coupling of a single fermion to the ^{154}Sm even-even core. Over the major shell N

= 50, there are four available positive parity single-particle levels, the $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, and $3s_{1/2}$. For the boson core, the IBM-2 basis states are used. To describe the positive-parity states, however, it is necessary to consider the inclusion of all four negative-parity single particle levels. The inclusion of multilevel possibilities into the IBFM has been analyzed by Scholten [13], who developed a formalism based on the BCS equations. The single particle energies were calculated using the relations given by [14]. Here, we apply the IBFM model to account for the ^{155}Eu isotope. The results of the IBFM-2 multilevel calculations for ^{155}Eu are presented for the energy levels and the transitions probabilities, which are compared with the corresponding experimental data.

2. The Interacting Boson Model and Even-Even Core

The interacting boson model [15] provides a unified description of collective nuclear states in terms of a system of interacting bosons. The ^{155}Eu isotopes have 63 protons 92 neutrons, which fill the orbits above major shell closure at N = 50, characterized by 13 particle-like proton states. It is thus appropriate to describe ^{155}Eu in the IBFM-2 model by coupling of a single fermion (proton) to the ^{154}Sm even-even nuclear core**.**

The interacting boson model (IBM) has become widely accepted as a tractable theoretical scheme of correlating, describing, and predicting low-energy collective properties of complex nuclei. In this model it was assumed that low-lying collective states of even-even nuclei could be described as states of a given (fixed) number N of bosons. Each boson could occupy two levels one with angular momentum J = 0 (s-boson) and another with J = 2 (d-boson). In the original form of the model known as IBM-1, proton, and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure, associated with it. In the IBM-2 model the neutrons and protons degrees of freedom are taken into account explicitly. Thus the Hamiltonian [16, 17] can be written as,

$$H = H_\pi + H_\nu + V_{\pi\nu} \tag{1}$$

$$H = \varepsilon_\pi d_\pi^+ d_\pi^\sim + \varepsilon_\nu d_\nu^+ d_\nu^+ + V_{\pi\pi} + V_{\nu\nu} + \kappa Q_\pi . Q_\nu + M_{\pi\nu} \tag{2}$$

Here ε is the *d*-boson energy, κ is the strength of the quadrupole interaction between neutron and proton bosons.

In the IBM-2 model, the quadrupole moment operator is given by:

$$Q_{\rho\rho} = \left(s^+ d^\sim + d^+ s^\sim\right)_\rho^{(2)} + \chi_\rho \left(d^+ d^\sim\right)_\rho^{(2)} \tag{3}$$

where $\rho = \pi$ or ν, $Q_{\rho\rho}$ is the quadrupole deformation parameter for neutrons $(\rho = \nu)$ and protons $(\rho = \pi)$. Where the terms $V_{\nu\nu}$ and $V_{\pi\pi}$ are the neutron-neutron and proton-proton d-boson interactions only and given by:

$$V_{\rho\rho} = \sum_{J=0.2.4} \frac{1}{2} C_{L\rho} \left(2J+1\right)^{1/2} \left[\left(d^+ d^+\right)_\rho^{(2)} \left(\tilde{d}\,\tilde{d}\right)_\rho^{(2)} \right]^{(0)} \tag{4}$$

The last term $M_{\pi\nu}$ is the Majorana interaction, shits the states with mixed proton-neutron symmetry with respect to the totally symmetric ones. Since little experimental information is known about such states with mixed symmetry, which has the form:

$$M_{\pi\nu} = -\sum_{k=1.3} 2\xi_k \left(d_\pi^+ d_\pi^+\right)^{(k)} \left(d_\pi^\sim d_\pi^\sim\right)^{(k)} + \xi_2 \left(d_\pi^+ s_\nu^+ - s_\pi^\sim d_\nu^\sim\right)^{(2)} (d_\pi^\sim s_\nu^\sim - s_\pi^\sim d_\nu^\sim)^{(2)} \tag{5}$$

The general one-body E2 transition operator in the IBM-2 is

$$T(l) = T_\pi(l) + T_\nu(l) \tag{6}$$

$$T(E2) = e_\pi \left[\left(s^+ d^\sim + d^+ s^\sim\right)_\pi^{(2)} + \chi_\pi \left(d^+ d^\sim\right)_\pi^{(2)} \right]^{(2)} + e_\nu \left[\left(s^+ d^\sim + d^+ s^\sim\right)_\nu^{(2)} + \chi_\nu \left(d^+ d^\sim \right)_\nu^{(2)} \right]^{(2)}$$

$$T\left(E2\right) = e_\pi Q_\pi + e_\nu Q_\nu \tag{7}$$

Where Q_ρ is in the form of Eq.(3). For simplicity, the χ_ρ has the same value as in the Hamiltonian. This is also suggested by the single *j*-shell microscopy. In general, the *E*2 transition results are not sensitive to the choice of e_ν and e_π, whether $e_\pi = e_\nu$ or not. Thus, the reduced electric quadrupole transition rates between $J_i \to J_f$ states are given by:

$$B(E2; J_i^+ \to J_f^+) = \frac{1}{2J_i+1} \left| < J_f^+ \left\| T(E2) \right\| J_i^+ > \right|^2 \tag{8}$$

The electric quadrupole moment in IBM-2 is given:

$$Q_l = \left[\frac{16\pi}{5}\right]^{1/2} \begin{bmatrix} J & 2 & J \\ -J & 0 & J \end{bmatrix} < J \left\| T\left(E2\right) \right\| J > \quad \ldots\ldots.(9)$$

In the IBM-2, the M1 transition operator up to the one-body term (l =1) is

$$T\left(M1\right) = \left[3/4\pi\right]^{1/2} \left(g_\pi L_\pi^{(1)} + g_\nu L_\pi^{(2)}\right) \tag{10}$$

Where $L_\rho^{(1)} = \sqrt{10}(d^+ \tilde{d})_\rho$ and $L^{(1)} = L_\pi^{(1)} + L_\nu^{(1)}$. The g_π

and g_v are the boson g-factors (gyromagnatic factors) in unit μ_n that depends on the nuclear configuration. They should be different for different nuclei.

$$T(M1)=\left[\frac{3}{4\pi}\right]^{1/2}\left[\frac{1}{2}(g_\pi+g_v)\left(L_\pi^{(1)}+L_v^{(1)}\right)+\frac{1}{2}(g_\pi-g_v)\left(L_\pi^{(1)}+L_v^{(1)}\right)\right] \quad (11)$$

The magnetic dipole moment operator is given by:

$$T(M1)=0.77\left[\left(d^+d^\sim\right)_\pi-\left(d^+d^\sim\right)_v\right]^{(1)}(g_\pi-g_v) \quad (12)$$

The reduced magnetic dipole transition rates between $I_i \to I_f$ states are given by:

$$B\left(M1,J_i^+\to J_f^+\right)=\frac{1}{2I_i+1}|<J_i^+\|T(M1)\|J_f^+>|^2 \quad (13)$$

3. The Interacting Boson-Fermion Model

In the IBFM, odd-A nuclei are described by the coupling of the odd fermionic quasi-particle to a collective boson core. The total Hamiltonian can be written as the sum of three parts:

$$\mathrm{H} = \mathrm{H_B} + \mathrm{H_F} + \mathrm{V_{BF}} \quad (14)$$

where H_B is the usual IBM-2 Hamiltonian [16-17] for the even-even core, H_F is the fermion Hamiltonian containing only one-body terms and V_{BF} is the boson-fermion interaction that describes the interaction between the odd quasi-nucleon and the even-even core nucleus. H_F is the fermion Hamiltonian containing only one-body terms and V_{BF} is the boson-fermion interaction that describes the interaction between the odd quasi-nucleon and the even-even core nucleus. V_{BF} is dominated by three terms: a monopole interaction characterized by the parameter A_0 which plays a minor role in actual calculations; the most important arise from the quadrupole interaction [18] characterized by Γ_0 and the exchange of the quasi-particle with one of the two fermions forming a boson [19] characterized by Λ_0. H_F is the fermion Hamiltonian containing only one-body terms and

$$H_F=\sum_{jm}\varepsilon_j a_{jm}^+ a_{jm} \quad (15)$$

where the ε_j are the quasiparticle energies and $a_{jm}^+ a_{jm}$ is the creation (annihilation) operator for the quasiparticle in the eigen state $|jm\rangle$. The boson-fermion interaction V_{BF} that describes the interaction between the odd quasi-nucleon and the even-even core nucleus contains, in general, many different terms and is rather complicated, but has been shown to be dominated by the following three terms:

$$V_{BF}=\sum_j A_j\left[\left(d^+\times d^\sim\right)^{(0)}\times\left(a_j^+\times a_j^\sim\right)^{(0)}\right]+\sum_{jj'}\Gamma_{jj'}\left[Q^{(2)}\times\left(a_j^+\times a_{j'}^\sim\right)^{(2)}\right]_0^{(0)}+\sum_{jj'j''}\Lambda_{jj'}^{j''}:\left[\left(d^+\times a_j^\sim\right)^{(j')}\times\left(a_{j'}^+\times d_j^\sim\right)^{(j'')}\right]_0^{(0)} \quad (16)$$

Where the core boson quadrupole operator is given by the equation (3), and χ is a parameter shown by microscopic theory to lie between $\sqrt{7}/2$ and $-\sqrt{7}/2$. V_{BF} is dominated by three terms: a monopole interaction characterized by the parameter A_0 which plays a minor role in actual calculations, the most important arise from the quadrupole interaction [20, 21] characterized by Γ_0, and the exchange of the quasiparticle with one of the two fermions forming a boson [22] characterized by A_0.

$A_j=A_0\sqrt{2j+1}$ s,d,s^+,d^+ are boson operators with $\delta_{jm}=(-1)^{j-m}$ and denotes normal ordering whereby contributions that arise from commuting the operators are neglected. The first term in V_{BF} is a monopole interaction which plays a minor role in actual calculations and the dominant term are the second and third, which arise from the quadrupole interaction. The third term represents the exchange of the quasiparticle with one of the two fermions forming a boson; Talmi [19] has shown that this exchange force is a consequence of the Pauli principle for the quadrupole interaction between protons and neutrons. The remaining parameters in Equation (17) can be related to the BCS occupation probabilities u_j , v_j of the single-particle orbits:

$$\Gamma_{jj'}=\sqrt{5}\Gamma_0\left(u_j u_{j'}-v_j v_{j'}\right)Q_{jj'} \quad (17)$$

$$A_{jj'}^{j''}=\sqrt{5}\Lambda_0\left[u_j v_{j''}+v_j u_{j''}\right]Q_{j'j''}\beta_{j'j''}+\left(u_{j'} v_{j''}+v_{j'} u_{j''}\right)Q_{j''j}\beta_{jj''}/\sqrt{2j''+1} \quad (18)$$

Where $Q_{jj'}$ are single particle matrix elements of the quadrupole operator and

$$\beta_{jj'}=\left(u_j v_{j'}+v_j u_{j'}\right)Q_{jj'}/\left(\varepsilon_j+\varepsilon_{j'}-\hbar\omega\right) \quad (19)$$

are the structure coefficients of the d boson deduced from microscopic considerations, with $\hbar\omega$ being the energy of a $|D\rangle$ pair relative to an $\langle S|$ pair [23].

The BCS occupation probability v_j and the quasi-particle energy ε_j of each single particle orbital can be obtained by solving the gap equations:

$$\varepsilon_j = \left[\left(E_j - \lambda\right)^2 + \Delta^2\right]^{1/2} \tag{20}$$

$$\nu_j^2 = \frac{1}{2}\left[1 - \frac{\left(E_j - \lambda\right)}{\varepsilon_j}\right] \tag{21}$$

where E_j is the single particle energy calculated from the relations in [20], λ is the Fermi level energy, and Δ is the pairing gap energy, which was chosen to be $12A^{-1/2}$ MeV [24]. That leaves the strengths A_0, Γ_0, and Λ_0 as free parameters which are varied to give the best fit to the excitation energies.

The total number of bosons and fermions is then: $N_B = N_{B\pi} + N_{B\nu}$, and $N_F = N_{F\pi} + N_{F\nu}$.

4. Results and Discussion

4.1. Interacting Boson Model -2

4.1.1. Energy Levels

The isotopes chosen in this work are A=154 due to the presents of experimental data for the energy levels. We have $N_\pi = 6$, (12 protons outside the closed shell 50), and N_ν = 5 for Sm^{154}, measured from the closed shell at 82. While the parameters $\kappa, \chi_\rho, and\, \varepsilon_\rho$,as well as the Majorana parameters ξ_k, with k =1,2,3, were treated as free parameters and their values were estimated by fitting with the experimental values. The procedure was made by selecting the traditional value of the parameters and allowing one parameter to vary while keeping the others constant until the best fit with the experimental obtained. This was carried out until one overall fit was obtained. The best values for the Hamiltonian parameters are given in table 1.

The IBM-2 Hamiltonian is non-linear in the parameters. To obtain the values of the parameters which give the best fit, we have to calculate for each energy level the difference between its experimental and calculated values. Then we have to sum over the squares of all these differences and to find a local minimum to this summation. The least square fit procedure was used to find the best fit to the three lowest bands of the ^{154}Sm isotopes under consideration.

Values of the interaction parameters for the ^{154}Sm isotopes in the IBM–2 Hamiltonian (for ^{154}Sm, in terms of code NPBOS notation are given in the table (1)).

Table 1. *IBM-2 Hamiltonian parameters , all parameters in MeV units except χ_π and χ_ν are dimensionless.*

Isotope	ε	K	χ_ν	χ_π	$\xi_1=\xi_2$	ξ_3	$C_{L\nu}$(L=0,2,4)	$C_{L\pi}$(L=0,2,4)
Sm^{154}	0.34	-0.039	-0.8	-1.2	0.12	0.1	-0.5, 0.4, -0.8	0.5, 0.4, -0.8

Concentration was made on the 2_1^+ to make a reasonable fit to experimental data. A sample of experimental and theoretical values of energy levels are taken in Fig.1. As one can see an overall a good agreement was obtained for the gamma and beta bands for ^{154}Sm. Figure (1), show a comparison between experimental and theoretical energy levels of the ground band in ^{154}Sm isotope, the agreement is very good for the 2_1 and 4_1 states.

The ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ for ^{154}Sm equal 3.256 for experimental date [25] and 3.191 for IBM-2. These values for the ration put the nucleus in transitional region from gamma soft to rotational shape.

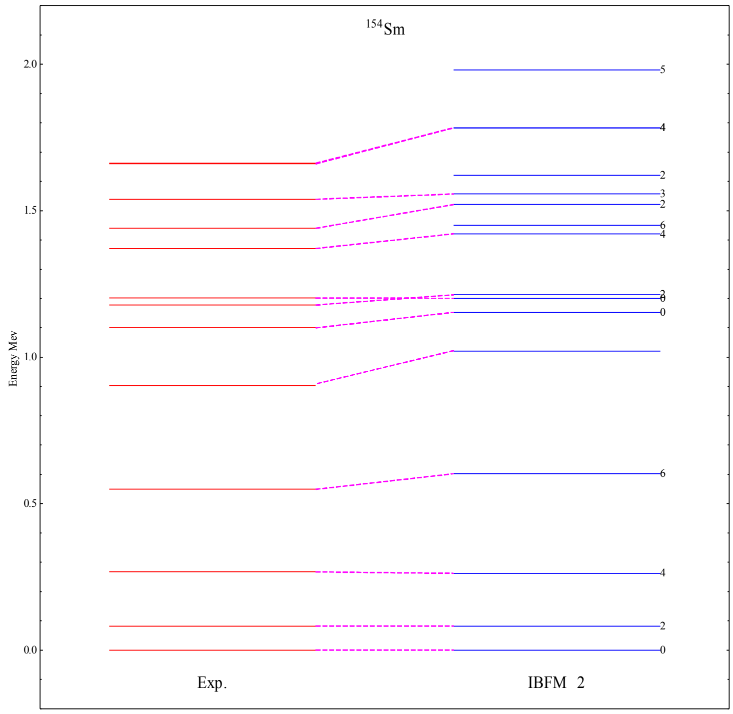

Figure 1. *Comparison of calculated IBFM-2 energy levels for positive parity with experimental data of ^{154}Sm [25].*

4.1.2. Electric Transition Probability

In IBM-2, the E2, transition operator is given by the equation (10), e_π and e_ν are boson effective charges depending on the boson number N_ρ ($\rho = \pi$ or ν) and they can take any value to fit the experimental results $(B(E2;2_1^+ \rightarrow 0_1^+))$. The method explained in reference [26]. The effective charges calculated by this method for ^{154}Sm isotopes were $e_\nu = 0.10.0\,eb$ and $e_\pi = 0.130\,eb$ .Table 3 given the electric transition probability.

The $B(E2;2_1^+ \rightarrow 0_1^+)$ and $B(E2;4_1^+ \rightarrow 2_1^+)$ values increased as neutron number increases toward the middle of the shell as the value of $B(E2;2_2^+ \rightarrow 2_1^+)$ has small value because contain mixtures of M1. The value of $B(E2;2_2^+ \rightarrow 0_1^+)$ is small because this transition is forbidden (from quasibeta band to ground state band). The values of IBM-2 in a good agreement with available experimental data [25].

The quadrupole moment is given in equation (9) for first excited state in ^{154}Sm isotopes are very well described. As mentioned above, the calculated values of $Q(2_1^+)$ ($Q(2_1^+)$ = -1.765 e.b) indicated this nucleus has prolate shape in first excited states.

Table 2. *Electric Transition probability B(E2) for ^{154}Sm isotopes in e^2b^2 units.*

$2_1 \rightarrow 0_1$		$4_1 \rightarrow 2_1$		$6_1 \rightarrow 4_1$		$2_2 \rightarrow 2_1$	
Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
0.922(40)	0.913	1.186(39)	1.231	1.374(47)	1.393	0.012	0.014

4.1.3. Magnetic Transition Probability

After calculated the E2 matrix elements we lock after the M1 matrix elements as in equation (13) .The direct measurement of B(M1) matrix elements is difficult normally, so the M1 strength of gamma transition may be expressed in terms of the multipole mixing ratio which can be written as [27]

$$\delta\left(E2/M1\right) = 0.835\,E_\gamma(MeV).\frac{< I_f \left|T(E2)\right| I_i >}{< I_f \left|T(M1\right| I_i >} \tag{22}$$

Having fitted E2 matrix elements, one can then use them with to obtain M1 matrix elements and then the mixing ratio $\delta(E2/M1)$, compare them with the prediction of the model using the operator (eq.9). The g_π *and* g_ν have to be estimated, if they are not been measured in the case of ^{154}Sm isotope. The g factors may be estimated from experimental magnetic (μ) moment of the 2^+_1 state (μ=2g). In phenomenological studies g_π and g_ν are treated as parameter and kept constant for a whole isotope chain. The total g factor defined by Sambataro *et. al.*, [28] as:

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \tag{23}$$

Many relations could be obtained for a certain mass region and then the average g_π *and* g_π values for this region could be calculated. One of the experimental B(M1) and the relation above been used to find that $g_\pi - g_\nu = 0.53\mu N$ .The estimated values of the parameter are $g_\pi = 0.84\mu_N$ *and* $g_\nu = 0.31\mu_N$, these were used to calculate the mixing ratio $\delta(E2/M1)$. The ratios were calculated for some selected transitions and listed with the available experimental data in table 3. A good agreement between theoretical results (IBM-2 and experimental data in sign and magnitude.

Table 3. *Mixing Ratios for ^{154}Sm (Exp. data are taken from Ref. [27]).*

$2_1 \rightarrow 2_1$		$4_2 \rightarrow 4_1$		$2_3 \rightarrow 2_1$		$3_1 \rightarrow 2_1$	
Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
56^{+130}_{-25}	34	-1.1	0.055	0.8^{+15}_{-6}	0. 20	-7.5	-5.22

The magnetic dipole transition probability is given in table 4, there is no experimental data to compare the theoretical results. The $B(M1;2_3^+ \rightarrow 2_1^+)$ value is small that is, implying some collective effects, The large B(M1) values in IBM-2 are due to the F-spin vector character of 2_3^+ state in ^{154}Sm. The $B(M1;0_1^+ \rightarrow 1_1^+)$ is still sizable in ^{154}Sm (increased with increased neutron number) because the transition from ground state to mixed symmetry state 1_1^+ in IBM-2 .

Table 4. *Mixing Ratios for ^{154}Sm (Exp. data are taken from Ref. [15,21]).*

$2_1 \rightarrow 2_1$	$2_3 \rightarrow 2_1$	$0_1 \rightarrow 1_1$	$3_1 \rightarrow 2_1$
0.007	0.020	1.460	0.262

4.2. Interacting Boson Fermion Model -2

4.2.1. Energy Levels

The Hamiltonian of IBFM-2 eq. (5) was diagonal's by means of the computer program ODDA [29] in which the IBFM-2 parameters are identified as: A_0 Γ_0 and Λ_0 . The parameters for the ^{154}Sm core are derived in the present work and given in table 1, while the quasi-particle energies and occupation probabilities used in this work are given in Table 6. the boson-fermion monopole interaction was omitted (A_0 = 0.0), there are only two ($\Gamma\rho$ and $\Lambda\rho$) free varying boson-fermion interaction parameters for the even-odd ^{155}Eu isotope.

Table 5. *Boson- Fermion interaction in ^{155}Eu isotopes for positive- parity state.*

nucleon	symbol	$\Gamma\rho$	$\Lambda\rho$	$A\rho$
Proton	$\rho=\pi$	0.170	0.210	0.0
neutron	$\rho=\upsilon$	0.172	0.220	0.0

The BCS parameters for the multilevel calculations of ^{155}Eu are given in table 6.

Table 6. *BCS parameters for the multilevel calculations of ^{155}Eu.*

$\varepsilon_j(MeV)$	$2d_{5/2}$	$1g_{7/2}$	$3s_{1/2}$	$2d_{3/2}$
ε_j	1.279	0.955	2.198	2.099
v_j^2	0.810	0.438	0.050	0.052

The IBFM-2 energy levels calculation is used to fit experimental energy levels with the boson-fermion parameters which is given in table (1) for ^{155}Eu nucleus. The monopole interaction parameter A_0 is set to zero. The dependence of V_{BF} on the specificity of each nucleus is counted for in the occupation probabilities appearing in the exchange term Λ_0 and in the quadrupole term Γ_0. The best agreement with experiment for the level calculations of ^{155}Eu nucleus is found by slightly varying the occupation probability to 2j to allow a better fit with the experiment (see fig. 2). The present choice of parameters gives also a good agreement with experimental data.

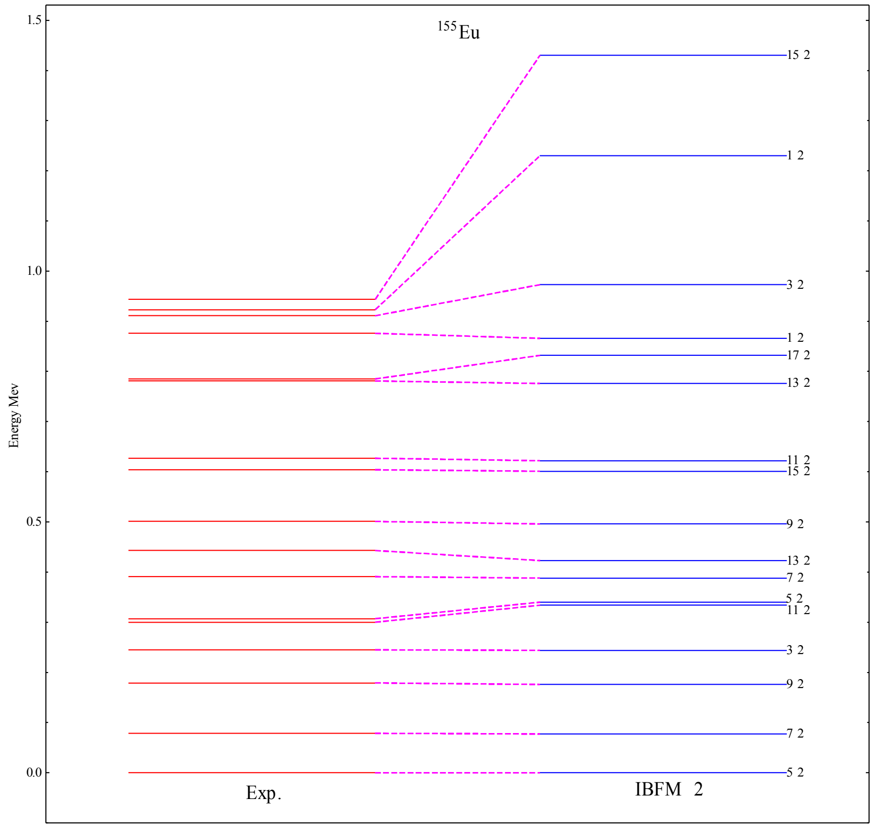

Figure 2. *Comparison of calculated IBFM-2 energy levels for positive parity with experimental data of ^{155}Eu [25].*

4.2.2. Electric Transition Probability

The calculation of electromagnetic transitions gives a good test of the nuclear model wave functions. In this section we discuss the calculation of the E2 transition strengths and results with the available experimental data. In general, the electromagnetic transition operators can be written as a sum of two terms, the first of which acts only on the boson part of the wave function and second only on the fermion part.

Transition operators can be written in the same way as in eq. (3). There are now four terms describing proton and neutron bosons and fermions,

$$T_{\mu}^{(L)}=T_{\pi B,\mu}^{(L)}+T_{\nu B,\mu}^{(L)}+T_{\pi F,\mu}^{(L)}+T_{\nu F,\mu}^{(L)} \tag{24}$$

The boson terms are given in equation (3). The fermion terms can, to the lowest order, be written as:

$$T_{\pi F,\mu}^{(L)}=f_{\pi,0}^{(0)}\delta_{L0}+\sum_{j_\pi j'_\pi} f_{j_\pi j'_\pi}^{(L)}\left[a_{j_\pi}^{+}\times\tilde{a}_{j'_\pi}\right]_{\mu}^{(L)}$$

$$T_{\nu F,\mu}^{(L)}=f_{\nu,0}^{(0)}\delta_{L0}+\sum_{j_\nu j'_\nu} f_{j_\nu j'_\nu}^{(L)}\left[a_{j_\nu}^{+}\times\tilde{a}_{j'_\nu}\right]_{\mu}^{(L)} \tag{25}$$

Particularly important in odd-even nuclei are the transition operators which induce E2 and *M*l transitions. It is customary in the operators to separate the dependence on the angular momenta j_π and j_ν from the coefficients that determine the strengths of the transitions. This is done by introducing effective charges and moments. For *E*2 transitions, one has:

$$f^{(2)}_{j_\rho j'_\rho} = -e^F_\rho \left\langle n_\rho, l_\rho \left| r^2 \right| n'_\rho, l'_\rho \right\rangle \left\langle l_\rho, \tfrac{1}{2}, j_\rho \left\| Y^{(2)} \right\| l'_\rho, \tfrac{1}{2}, j'_\rho \right\rangle / \sqrt{5}\,, \rho = \pi, \nu \quad (26)$$

where now the single particle indices $n, l, s = 1/2, j$ are written explicitly. The quantities e^F_π and e^F_ν are the fermion effective charges. The free values of these charges are 1 and 0 respectively, in units of the electron charge. Shell model calculations indicate that $e^F_\pi \approx 1.5e$ and $e^F_\nu \approx 0.5e$. Following, the boson part is written as

$$T^{(L)}_{\rho B,\mu} = e^B_\rho \hat{Q}^\chi_{\rho,\mu}\,, \rho = \pi, \nu \quad (27)$$

A superscript B has been added to e_ρ in order to distinguish it from the fermion charges. The units of e^B_ρ are different from those of e^F_ρ since the radial integral is already included in eq. (18). The boson effective charges e^B_ρ have the same units as the product

$$e'^F_\rho = e^F_\rho \left\langle n_\rho, l_\rho \left| r^2 \right| n'_\rho, l'_\rho \right\rangle, \rho = \pi, \nu \quad (28)$$

That is the units are $e\,fm^2$.

In the table 7 the values of the E2 transitions for ^{155}Eu with the experimental data, The transition $3^+/2_1 \rightarrow 5^+/2_1$ is a good agreement with experimental data. The main discrepancies occur in the case of the B(E2) involving the depopulation of excited states of ^{155}Eu at about 0.213 MeV. This apparent breakdown of the present model has two probable cases:

1- The configuration space used in the present calculation is not large enough. It may be better to include protons and neutrons as active nucleons.

2- A satisfactory comparison with the experiments is quite difficult due to the large errors on the experimental values, moreover the theoretical B(E2) values for that the transition seem to be systematically too high. This can be explained by the fact that many small components of the initial and final wave functions contribute coherently to the value of this reduced E2 transition probability.

In general, the calculated values agree with the experimental data reasonably well. The B(E2) values depend quite sensitively on the wave functions, which suggest that the wave functions obtained in this work are reliable. The model may be applied to many other nuclei and its many other nuclear properties.

Table 7. *Electric Transition Probability B(E2) in $e^2.b^2$ (Exp. data are taken from Ref. [30]).*

$3/2_1 \rightarrow 5/2_1$		**$3/2_2 \rightarrow 7/2_1$**		**$5/2_2 \rightarrow 5/2_1$**		**$5/2_2 \rightarrow 7/2_1$**	
Exp.	IBFM-2	Exp.	IBFM-2	Exp.	IBFM-2	Exp.	IBFM-2
0.0036	0.0053	0.0023	0.0033	-	0.0732	-	0.00346

4.2.3. Magnetic Transition Probability

The δ -mixing ratios for some selected transitions in the ^{155}Eu nucleus is calculated from the useful eq. (10) of mixing ratio as above and with the help of B (E2) and B (MI) values, which are obtained from ODDA program [29] ; the results are given in table (8). In general, the calculated mixing ratio of ^{155}Eu nucleus.

Table 8. *Mixing ratio of ^{155}Eu.*

$3/2_1 \rightarrow 5/2_1$	**$3/2_2 \rightarrow 7/2_1$**	**$5/2_2 \rightarrow 5/2_1$**	**$5/2_2 \rightarrow 7/2_1$**
0.02	0.0043	0.006	2.0

5. Conclusions

We can summarize the main results and conclusions of this study as follows. Energy level for even-even ^{154}Sm nucleus for ground, beta gamma bands are reproduced well. The energy spectra of the odd-even ^{155}Eu nucleus can be reproduced quite well with the help of only two (Λ_0 and Γ_0) freely varying boson-fermion interaction parameters. The monopole interaction (A_0) plays a minor role in the actual calculations. The most important effects arise from the quadrupole interaction (Γ_0) and the exchange of the quasiparticle with one of the two fermions forming a boson interaction (Λ_0).

A satisfactory comparison with the experiments is quite difficult due to the errors in the experimental values; moreover the theoretical B (E2) values for the transition seem to be systematically too small. This can be explained by the fact that many small components of the initial and final wave functions contribute coherently to the value of this reduced E2 transition probability. In general, the calculated electromagnetic properties of the ^{154}Sm nucleus do not differ significantly from those calculated in experimental work. The calculated values in this study show that the transitions connect the levels with the same parity and the E2 transitions are predominant. The later includes transitions originating from the beta and gamma bands, which supports the idea that the beta and bands may be quadrupole excitations of the perturbed ground state, but the existence of M1 indicates that the beta and gamma bands cannot be pure quadrupole excitations of the ground state band.

We have also examined the mixing ratio δ (E2/M1) of transitions linking the ground state bands. We find that the transitions which link low-spin states and which were obtained in the present work are largely consistent with this requirement, although some may be considered to show irregularities.

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