Optimal Dynamic Pricing for Assembly Product Supply Chain

Yufang Chen[1], Yong Luo[1, 2]

[1]School of Electrical Engineering, Zhengzhou University, Zhengzhou, China

[2]Dept. Industrial & Manufacturing Eng., University of Wisconsin, Milwaukee, USA

Email address:

luoyong@zzu.edu.cn (Yong Luo)

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Abstract: To study the optimal decisions of suppliers and assemblers in an assembly product supply chain which contains two generations of product. With updated components, a dynamic assembly product supply chain model whose demand is time-varying was built based on product diffusion model. The optimal dynamic pricing decisions and profits of all entities in the supply chain were acquired through theoretical analysis and simulation based on Stackelberg and Nash game. Some insights have been derived: The profits of two assemblers are increased, while two suppliers' profits are relatively reduced if the two assemblers cooperate with each other. The growth rates of suppliers' wholesale prices of two generations of products are opposite, and those of assemblers' retail prices are also opposite whether two assemblers are cooperative or not. With cooperation, the ranges of wholesale prices changing over time are higher, while the ranges of assemblers' retail prices changing over time are lower than those without cooperation.

Keywords: Components Update, Assembly Product Supply Chain, Stackelberg Game, Optimal Pricing

1. Introduction

With the development of market economy, the style of product is becoming diverse, and product assembling is an important reason for product diversification. Assembly products are often completed among a number of enterprises. Taking computer as an example, its mainboard and CPU are respectively produced by different suppliers, and then they are assembled into a complete product. This production mode is the mode of assembling product, which is very common, especially in electronics industry, so it is very useful to study an assembly products supply chain. The typical assembly product supply chain consists of suppliers and assemblers. According to the importance of the product, the components of assembly product are divided into key component and non-critical component. The non-critical component works for auxiliary function of product, and the key component works for main function. Because of the importance of key component, the key component supplier is often in upstream monopoly status in assembly product supply chain. As technology advances, key component is constantly upgrading, and a new assembly product will be formed whose innovation

mainly replies on key components updating. Updated and original products coexist in market and form a series of products (such as Apple iPhone). So it is an urgent and economic significance issue to study the assembly product supply chain considering components updating in order to maximize the benefits of supply chain entities.

Researches on the supply chain of assembly product and updated product became a hot topic in recent years. At present, some researches focus on assembly product supply chain. For example, Leng and Parlar [1] introduced the appropriate buy-back and lost-sales cost-sharing contracts to coordinate the assembly supply chain including a multiple-supplier, single manufacturer in order to find the globally-optimal solution that maximizes the system-wide expected profit. Chen, Ding and Qu [2] studied the impact of supply chain power structure on firms' profitability in an assembly system with one assembler and two suppliers. Fang, Ru and Wang [3] developed an efficient algorithm with a complexity of O(n) to compute the optimal contract to maximize assembler own expected profit in a decentralized assembly supply chain in which each supplier holds private cost information to himself, for which the assembler only has a subjective estimate. Zhang [4] developed a supply chain model in which an original

equipment manufacturer (OEM) procures a key component from a supplier and considered an ingredient branding strategy under which the supplier and the original manufacturer formed a brand alliance. Li [5] researched how an Assemble-to-Order manufacturer matched customers' diverse heterogeneous demand through making reasonable product selection and pricing policy. Another researches focus on renewal product supply chain. For example, Luo [6] built a market share shift model for renewal product on an increment function and a shift function to study the process of product renewal in a supply chain, which is composed of one manufacturer and one retailer. Quan [7, 8] established the sell pricing models when updated and existing products co-existed in the market and present a profit model based on Bass diffusion model, analyzed the relationship of optimal launch time with the param-eters numerically. Ces and Liddo [9] studied an optimal price and advertising strategy of new product joined the subsidy policy by using a Stackelberg differential game. Chanda [10] studied optimal control policies for quality and price when two technology generations was present in a dynamic market and also suggested a policy for the optimal launching time of an advanced generation. The above studies do not involve the assembly products supply chain considering components update.

In order to obtain the dynamic demand of market, product diffusion model is needed. Bass [11] proposed a new product diffusion model in 1969. Norton and Bass [12] developed a model including successive generations which encompassed both diffusion and substitution, building upon the Bass diffusion model. Fisher [13] established a diffusion model specifically for the old and new generations of products. Jiang and Jain [14] developed a generalized Norton–Bass (GNB) model to separate the two different types of substitution. Chen and Carrillo [15] developed a single firm product diffusion model for single-function and fusion products. Christionde [16] introduced an agent-based model that dealt with repeat purchase decisions, and addressed the competitive diffusion of multiple products, and took into consideration both the temporal and the spatial dimension of innovation diffusion. Avagyan Vardan et al [17] studied how companies can simultaneously license their innovations to other firms when launching a new product to speed up the new product diffusion via a differential game. Liang Xiaoying et al [18] extended the classic Bass diffusion model to address the case in which existing adopters could depress the growth of adoption. Hong Jungsik et al [19] proposed a new forecasting method using linearization of the hazard rate formula of the Bass model.

In the studies of product mentioned above, the market demands of product can be divided into two groups: the one is static, and the other is dynamic. The studies which aim at static market demand only contribute for a fixed time's demand, but cannot be suitable for the analysis of a long period in the future. The existing researches on the dynamic demand are usually focused on the product diffusionsimulation and analysis (e.g Christionde [16]), but

they are lack of theoretical analysis on dynamic decisions of each entity in supply chain.

To solve the above problems, and more in-depth research on how dynamic process of product updating affects assembly product supply chain, this paper builds a model of assembly product supply chain considering components update, and a dynamic demand model of assembly product based on Fisher product diffusion model [13], and updated product substitution model [20, 21]. Furthermore, this paper studies the optimal dynamic pricing decisions and profits of all entities in assembly products supply chain which has two generations products based on Fisher model. It is the first time to study the optimal decisions of assembly products supply chain including components update, and it will be as a reference for optimal product decisions of IT and other related industries.

The remained of this paper is organized as follows: Section 2 builds and describes the model of two assembly product supply chains. The original product supply chain consists of an original key components supplier and an assembler, and updated product supply chain consists of a key component supplier and an updated product assembler. Dynamic demand functions of updated and original products are be acquired in this section. In Section 3, the entities of the supply chains play a Stackelberg Game and Nash game to acquire their dynamic optimal prices and optimal profits without cooperation between two assemblers. In Section 4, the entities of supply chains play a Stackelberg Game to get their dynamic optimal prices and optimal profits with cooperation between two assemblers. In Section 5, numerical simulation is shown. Finally, the conclusion of this paper is given.

2. Supply Chain Model

The supply chains consisting of two suppliers and two assemblers are shown in Figure 1. The key component supplier, S_1, sells the key component at wholesale price W_1 to downstream assembler A_1, and the component is assembled into original product by the assembler, who is sells it in a pricing-sensitive market at retail pricing P_1 in the original product supply chain. Because of key component updating, key component supplier, S_2, sells new component at wholesale price W_2 to downstream assembler A_2, and the component is assembled into updated product which is sold at retail pricing P_2 to the pricing-sensitive market in the updated product supply chain. The suppliers manufacture original and updated components at marginal costs C_1, C_2, respectively. Two assemblers sell the two products at the same marginal cost, c, including the cost of buying accessories and selling cost.

According to [21], the statics market demand functions of two differentiated products are $D_1 = m - bP_1 + kP_2$ and $D_2 = m - bP_2 + kP_1$, respectively, where m denotes the saturation value of market demand of two products. b is market price sensitivity, and with b increasing, the sensitivity to price becomes greater and greater. k represents the ability of the two generations of a product to replace each other. With k increasing，the ability to replace strengthens, and $0<k<1$. Two

generations of the product which are assembled by different assemblers using the same non-critical components and different key components coexist in the same market. Due to the different functions of two generations of the product, they

continue to diffuse as time goes on in the market, while the market share of original product will reduce gradually, and the updated product will continue to seize the market of the original product.
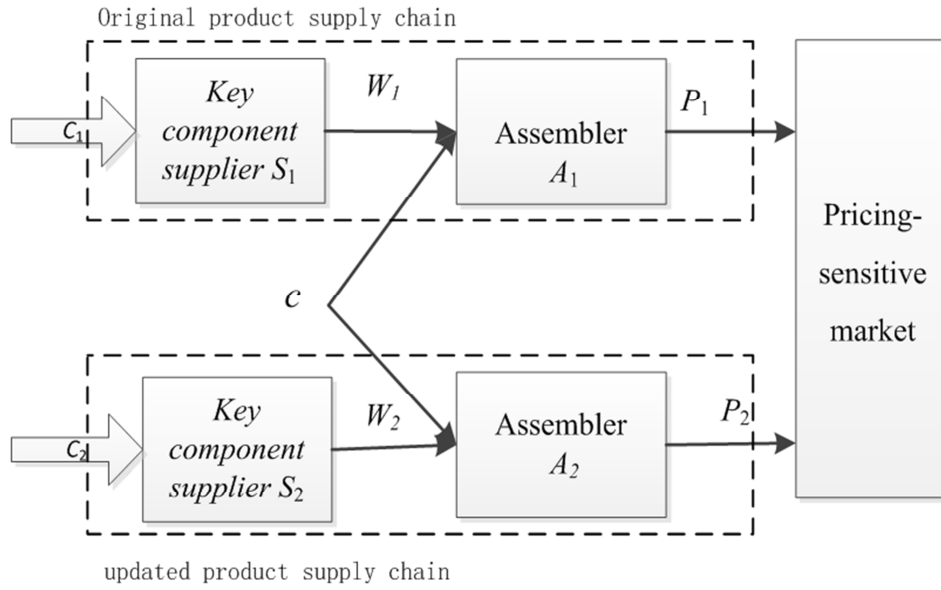

Figure 1. *Assembly product supply chains structure.*

Due to the different functions of two generations of the product, the introduction of updated products provides an opportunity for potential buyers of the old products to switch to the more advanced technology. The market shares of two generations of products are different, in other word, the demand for two generations of products change with time, (such as, Apple iPhone). According to Fisher diffusion model [13], the market share of updated product is *f*, and that of original products is (1-*f*) when two generations of the products coexist in the same dynamic market. According to [21] and Fisher diffusion model [13], the dynamic market demands of two generations of the assembly product considering key components update can be respectively written as:

$$D_1 = m(1-f) - bP_1 + kP_2 \tag{1}$$

$$D_2 = mf - bP_2 + kP_1 \tag{2}$$

Where D_1 denotes the dynamic demand of the original product, and D_2 is the dynamic demand of updated product.

From Fisher diffusion model, the expression of *f* is

$$f = \frac{1}{2}\left[1 + \tanh \alpha(t - t_0)\right] \tag{3}$$

In equation 3, parameter α is the innovation degree of updated product compared to original product. The greater the parameter α is, the higher the innovation degree of updated product is, and $0 < \alpha < 1$.

3. Stackelberg-Nash Game

There are two assembly product supply chains if two

assemblers are not cooperative. The original product supply chain consists of original key component supplier S_1 and one assembler A_1, and the updated product supply chain consists of supplier S_2 and one assembler A_2. There is a Nash game between A_1 and A_2. In each supply chains above, the supplier first decides its wholesale price to maximize its profit, then the assembler sets a retail price to maximize its profit. Hence, the supplier and the assembler play a Stackelberg game, in which the supplier is a leader and the assembler is a follower.

The profit functions of the assemblers, A_1, A_2, can be respectively written as:

$$\pi_{A_1} = (P_1 - W_1 - c)D_1 \tag{4}$$

$$\pi_{A_2} = (P_2 - W_2 - c)D_2 \tag{5}$$

The profit functions of the supplier S_1, S_2 can be respectively written as

$$\pi_{S_1} = (W_1 - C_1)D_1 \tag{6}$$

$$\pi_{S_2} = (W_2 - C_2)D_2 \tag{7}$$

Theorem1. The profit function of assembler A_1 is a concave function of price P_1, and the profit function of assembler A_2 is a concave function of price P_2. The maximum values of π_{A_1}, π_{A_2} are obtained at the only stationary points, respectively.

Proof. If wholesale prices are given, the two assemblers play a Nash game. From (4) and (5), the first-order derivatives and second-order derivatives of the two assemblers' profits to P_1 and P_2 are can be derived as

$$\begin{cases} \dfrac{d\pi_{A_1}}{dP_1} = -2bP_1 + m(1-f) + b(W_1 + c) + kP_2 \\[2mm] \dfrac{d\pi_{A_2}}{dP_2} = -2bP_2 + mf + b(W_2 + c) + kP_1 \end{cases} \quad (8)$$

$$\frac{d^2\pi_{A_1}}{dP_1^2} = -2b, \frac{d^2\pi_{A_2}}{dP_2^2} = -2b \quad (9)$$

According to $b>0$, so $d^2\pi_{A_1}/dP_1^2 = d^2\pi_{A_2}/dP_2^2 = -2b < 0$ can be deduced. In other words, the second derivatives of π_{A_1} and π_{A_2} are less than 0. Therefore, the profit function of the assembler A_1 is a concave function of price P_1, and the profit function of assembler A_2 is a concave function of price P_2.

Let $d\pi_{A_1}/dP_1 = 0$ and $d\pi_{A_2}/dP_2 = 0$, and equation (8) can be reduced to yield

$$\begin{cases} P_1 = \dfrac{2b^2(W_1 + c) + kb(W_2 + c) + m(2b + (k - 2b)f)}{4b^2 - k^2} \\[3mm] P_2 = \dfrac{2b^2(W_2 + c) + kb(W_1 + c) + m(k + (2b - k)f)}{4b^2 - k^2} \end{cases} \quad (10)$$

Obviously, there is only one fixed point for each assembler, which makes the first-order derivative of the assembler' profit function 0, and second-order derivative of the assembler' profit less than 0. Therefore, the assemblers can obtain the optimal profits at the above points.

The proof is completed.

Theorem 2. If $b>k>0$, the profit function of the supplier S_1 is a concave function of the wholesale price W_1, and the profit function of the assembler S_2 is a concave function of the wholesale price of W_2, and the optimal dynamic pricing of the two suppliers and assemblers are respectively:

$$\begin{cases} W_1(t) = -\dfrac{(2b^2 - k^2 - kb)c}{4b^2 - 2k^2 - kb} + \dfrac{2(2b^2 - k^2)^2 C_1}{4k^4 + 16b^4 - 17b^2 k^2} + \dfrac{kb(2b^2 - k^2)C_2}{4k^4 + 16b^4 - 17b^2 k^2} \\[3mm] \quad + \dfrac{(8b^3 - 3bk^2)m}{4k^4 + 16b^4 - 17k^2 b^2} - \dfrac{(2b - k)m[1 + \tanh\alpha(t - t_0)]}{2(4b^2 - 2k^2 + kb)} \\[3mm] W_2(t) = -\dfrac{(2b^2 - k^2 - kb)c}{4b^2 - 2k^2 - kb} + \dfrac{2(2b^2 - k^2)^2 C_2}{4k^4 + 16b^4 - 17b^2 k^2} + \dfrac{kb(2b^2 - k^2)C_1}{4k^4 + 16b^4 - 17b^2 k^2} \\[3mm] \quad + \dfrac{(6b^2 k - 2k^3)}{4k^4 + 16b^4 - 17k^2 b^2}m + \dfrac{(2b - k)m[1 + \tanh\alpha(t - t_0)]}{2(4b^2 - 2k^2 + kb)} \end{cases} \quad (11)$$

$$\begin{cases} P_1(t) = \dfrac{b(2b^2 - k^2)c}{(2b - k)(4b^2 - 2k^2 - kb)} + \dfrac{b(2b^2 - k^2)((8b^3 - 3bk^2)C_1 + (6b^2 k - 2k^3)C_2)}{(4b^2 - k^2)(4k^4 + 16b^4 - 17k^2 b^2)} \\[3mm] \quad + \dfrac{2b(3b^2 - k^2)(8b^2 - 3k^2)}{(4b^2 - k^2)(4k^4 + 16b^4 - 17k^2 b^2)}m - \dfrac{(3b^2 - k^2)[1 + \tanh\alpha(t - t_0)]}{(2b + k)(4b^2 - 2k^2 + kb)} \\[3mm] P_2(t) = \dfrac{b(2b^2 - k^2)c}{(2b - k)(4b^2 - 2k^2 - kb)1} + \dfrac{b(2b^2 - k^2)((8b^3 - 3bk^2)C_2 + (6b^2 k - 2k^3)C_1)}{(4b^2 - k^2)(4k^4 + 16b^4 - 17k^2 b^2)} \\[3mm] \quad + \dfrac{4k(3b^2 - k^2)^2}{(4b^2 - k^2)(4k^4 + 16b^4 - 17k^2 b^2)}m + \dfrac{(3b^2 - k^2)m[1 + \tanh\alpha(t - t_0)]}{(2b + k)(4b^2 - 2k^2 + kb)} \end{cases} \quad (12)$$

Proof: Substitute equation (10) into (6) and (7), respectively, then:

$$\pi_{S_1} = (W_1 - C_1)\begin{cases} m(1 - f) - b \cdot \dfrac{2b^2(W_1 + c) + kb(W_2 + c) + m(2b + (k - 2b)f)}{4b^2 - k^2} \\[3mm] + k\dfrac{2b^2(W_2 + c) + kb(W_1 + c) + m(k + (2b - k)f)}{4b^2 - k^2} \end{cases} \quad (13)$$

$$\pi_{S_2} = (W_2 - C_2) \left\{ \begin{array}{l} mf - b \cdot \dfrac{2b^2(W_2+c)+kb(W_1+c)+m(k+(2b-k)f)}{4b^2-k^2} \\[2ex] +k \dfrac{2b^2(W_1+c)+kb(W_2+c)+m(2b+(k-2b)f)}{4b^2-k^2} \end{array} \right. \tag{14}$$

From (13) and (14), the first-order derivative and the second-order derivative of the suppliers' profits can be derived as:

$$\left\{ \begin{array}{l} \dfrac{d\pi_{S_1}}{dW_1} = (W_1-C_1) \cdot \dfrac{k^2 b - 2b^3}{4b^2-k^2} + m(1-f) - b \cdot \dfrac{2b^2(W_1+c)+kb(W_2+c)+m(2b+(k-2b)f)}{4b^2-k^2} \\[2ex] \qquad + k \dfrac{2b^2(W_2+c)+kb(W_1+c)+m(k+(2b-k)f)}{4b^2-k^2} \\[2ex] \dfrac{d\pi_{S_2}}{dW_2} = (W_2-C_2) \dfrac{k^2 b - 2b^3}{4b^2-k^2} + mf - b \cdot \dfrac{2b^2(W_2+c)+kb(W_1+c)+m(k+(2b-k)f)}{4b^2-k^2} \\[2ex] \qquad + k \dfrac{2b^2(W_1+c)+kb(W_2+c)+m(2b+(k-2b)f)}{4b^2-k^2} \end{array} \right. \tag{15}$$

$$\frac{d^2\pi_{S_1}}{dW_1^2} = -\frac{2(2b^2-k^2)}{4b^2-k^2} \tag{16}$$

$$\frac{d^2\pi_{S2}}{dW_2^2} = -\frac{2(2b^2-k^2)}{4b^2-k^2} \tag{17}$$

If $b>k>0$, $d^2\pi_{S_1}/dW_1^2 = d^2\pi_{S_2}/dW_2^2 = -\dfrac{2(2b^2-k^2)}{4b^2-k^2} < 0$

can be deduced. The second derivatives of π_{S_1} and π_{S_2} are less than 0. Therefore, the profit function of the supplier S_1 is a concave function of the wholesale price W_1, and the profit function of the assembler S_2 is a concave function of the wholesale price W_2. Therefore, the maximum values of the profits can be obtained at the stationary points, and the stationary points are also the optimal values of the wholesale prices. Making $d\pi_{S_1}/dW_1 = 0$ and $d\pi_{S_2}/dW_2 = 0$, the optimal values of the wholesale prices can be obtained as:

$$\left\{ \begin{array}{l} W_1 = -\dfrac{(2b^2-k^2-kb)c}{4b^2-2k^2-kb} + \dfrac{2(2b^2-k^2)^2 C_1}{4k^4+16b^4-17k^2b^2} + \dfrac{kb(2b^2-k^2)C_2}{4k^4+16b^4-17k^2b^2} \\[2ex] + \dfrac{(8b^3-3bk^2)m}{4k^4+16b^4-17k^2b^2} - \dfrac{2b-k}{4b^2-2k^2+kb} mf \\[2ex] W_2 = -\dfrac{(2b^2-k^2-kb)c}{4b^2-2k^2-kb} + \dfrac{2(2b^2-k^2)^2 C_2}{4k^4+16b^4-17k^2b^2} + \dfrac{kb(2b^2-k^2)C_1}{4k^4+16b^4-17k^2b^2} \\[2ex] + \dfrac{(6b^2k-2k^3)}{4k^4+16b^4-17k^2b^2} m + \dfrac{2b-k}{4b^2-2k^2+kb} mf \end{array} \right. \tag{18}$$

From (3), (10) and (18), the optimal dynamic pricing of the two assemblers and the two suppliers can be obtained as:

$$\left\{ \begin{array}{l} P_1(t) = \dfrac{b(2b^2-k^2)c}{(2b-k)(4b^2-2k^2-kb)} + \dfrac{b(2b^2-k^2)\left((8b^3-3bk^2)C_1+(6b^2k-2k^3)C_2\right)}{(4b^2-k^2)(4k^4+16b^4-17k^2b^2)} \\[2ex] + \dfrac{2b(3b^2-k^2)(8b^2-3k^2)}{(4b^2-k^2)(4k^4+16b^4-17k^2b^2)} m - \dfrac{(3b^2-k^2)[1+\tanh\alpha(t-t_0)]}{(2b+k)(4b^2-2k^2+kb)} \\[2ex] P_2(t) = \dfrac{b(2b^2-k^2)c}{(2b-k)(4b^2-2k^2-kb)1} + \dfrac{b(2b^2-k^2)\left((8b^3-3bk^2)C_2+(6b^2k-2k^3)C_1\right)}{(4b^2-k^2)(4k^4+16b^4-17k^2b^2)} \\[2ex] + \dfrac{4k(3b^2-k^2)^2}{(4b^2-k^2)(4k^4+16b^4-17k^2b^2)} m + \dfrac{(3b^2-k^2)m[1+\tanh\alpha(t-t_0)]}{(2b+k)(4b^2-2k^2+kb)} \end{array} \right. \tag{19}$$

$$
\begin{cases}
W_1(t) = -\dfrac{\left(2b^2 - k^2 - kb\right)c}{4b^2 - 2k^2 - kb} + \dfrac{2\left(2b^2 - k^2\right)^2 C_1}{4k^4 + 16b^4 - 17b^2 k^2} + \dfrac{kb\left(2b^2 - k^2\right)C_2}{4k^4 + 16b^4 - 17b^2 k^2} \\[3mm]
\quad + \dfrac{\left(8b^3 - 3bk^2\right)m}{4k^4 + 16b^4 - 17k^2 b^2} - \dfrac{\left(2b - k\right)m\left[1 + \tanh\alpha\left(t - t_0\right)\right]}{2\left(4b^2 - 2k^2 + kb\right)} \\[4mm]
W_2(t) = -\dfrac{\left(2b^2 - k^2 - kb\right)c}{4b^2 - 2k^2 - kb} + \dfrac{2\left(2b^2 - k^2\right)^2 C_2}{4k^4 + 16b^4 - 17b^2 k^2} + \dfrac{kb\left(2b^2 - k^2\right)C_1}{4k^4 + 16b^4 - 17b^2 k^2} \\[3mm]
\quad + \dfrac{\left(6b^2 k - 2k^3\right)}{4k^4 + 16b^4 - 17k^2 b^2}m + \dfrac{\left(2b - k\right)m\left[1 + \tanh\alpha\left(t - t_0\right)\right]}{2\left(4b^2 - 2k^2 + kb\right)}
\end{cases}
\tag{20}
$$

The proof is completed.

Lemma1. If the condition $b>k>0$ in Theorem2 is satisfied, as time goes on, the optimal dynamic wholesale price of original product supplier S_1 is decreasing, while the optimal dynamic wholesale price of updated product supplier S_2 is increasing. The change trends of them over time are opposite. As time goes on, the optimal dynamic retail price of original product assembler A_1 is decreasing, and the optimal dynamic retail price of updated product assembler A_2 is increasing. The change trends of them over time are opposite.

Proof: From formulas (19) and (20), the first-order derivatives of the optimal dynamic wholesale prices and retail prices with respect to t can be respectively written as:

$$
\frac{dW_1(t)}{dt} = -\frac{\left(2b - k\right)m\alpha\sec h^2\alpha\left(t - t_0\right)}{2\left(4b^2 - 2k^2 + kb\right)}
\tag{21}
$$

$$
\frac{dW_2(t)}{dt} = \frac{\left(2b - k\right)m\alpha\sec h^2\alpha\left(t - t_0\right)}{2\left(4b^2 - 2k^2 + kb\right)}
\tag{22}
$$

$$
\frac{dP_1(t)}{dt} = -\frac{\left(3b^2 - k^2\right)m\alpha\sec h^2\left(t - t_0\right)}{\left(2b + k\right)\left(4b^2 - 2k^2 + kb\right)}
\tag{23}
$$

$$
\frac{dP_2(t)}{dt} = \frac{\left(3b^2 - k^2\right)m\alpha\sec h^2\left(t - t_0\right)}{\left(2b + k\right)\left(4b^2 - 2k^2 + kb\right)}
\tag{24}
$$

According to the condition $b>k>0$ in Theorem 2 and formulas (21), (22), $dW_1(t)/dt = -dW_2(t)/dt$, and $dW_1(t)/dt < 0$, $dW_2(t)/dt > 0$ can be deduced.

According to the condition $b>k>0$ in Theorem 2 and formulas (23), (24), $dP_1(t)/dt = -dP_2(t)/dt$, and $dP_1(t)/dt < 0$, $dP_2(t)/dt > 0$ can be deduced.

So W_1 is decreasing, while W_2 is increasing over time. The change trends of them over time are opposite. P_1 is decreasing, while P_2 is increasing over time. The change trends of them over time are opposite.
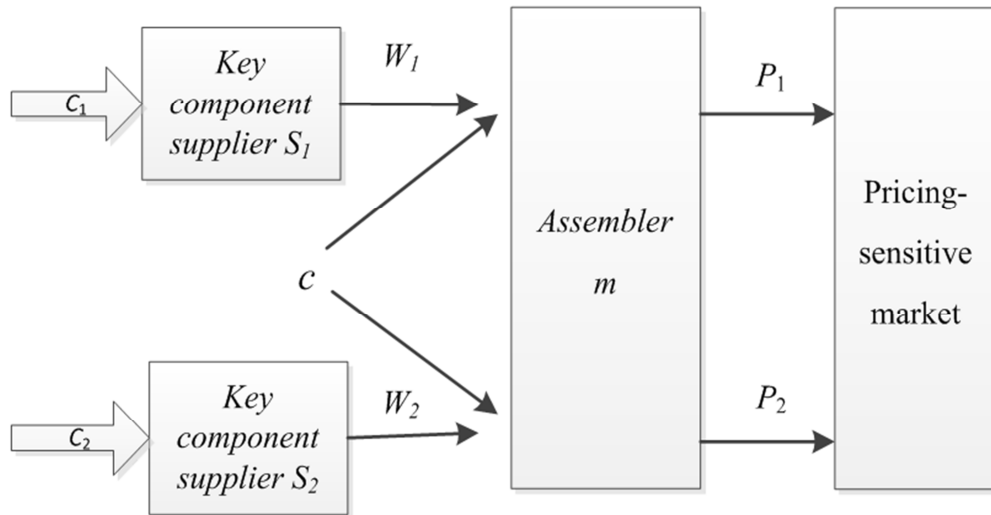
4. Cooperation Model

Figure 2. *Supply chain with cooperation.*

The two assemblers with cooperation should be regarded as a whole assembler in the above supply chain as shown in Figure 2. In the above supply chain, the suppliers are monopolistic because only the two suppliers sell original and updated key components to the assemblers. There is a Stackelberg game, which consists of the two suppliers as

leaders and the whole assembler as follower. The game is divided into three steps, and backward induction is usually used to solve Stackelberg game. Firstly, the supplier S_1 sells original key component to the whole assembler at wholesale price W_{11}, and the supplier S_2 sells updated key component to the whole assembler at wholesale price W_{22}. After accepting the wholesales, the whole assembler sets the retail prices P_{11}, P_{22} of the original product and the updated product to maximize its profit, and the suppliers know the optimal reaction function of wholesale price $P_{11}(W_{11}, W_{22})$, P_{22}

$$\pi_m = \pi_{A11} + \pi_{A22}$$

(25)

$$= (P_{11} - W_{11} - c)(m(1-f) - bP_{11} + kP_{22}) + (P_{22} - W_{22} - c)(mf - bP_{22} + kP_{11})$$

$$\begin{aligned}\pi_{S_{11}} &= (W_{11} - C_1)(m(1-f) - bP_{11} + kP_{22}) \\ \pi_{S_{22}} &= (W_{22} - C_{22})(mf - bP_{22} + kP_{11})\end{aligned}$$

(26)

Theorem 3. If the condition $b>k>0$ in Theorem2 is satisfied, the maximum value of the whole assembler's profit, π_m can be obtained at the only stationary point.

Proof. From (25), the first-order partial derivative of π_m with respect to P_{11}, P_{22} can be written respectively as:

$$\frac{\partial \pi_m}{\partial P_{11}} = m(1-f) - 2bP_{11} + 2kP_{22} + b(W_{11} + c) - k(W_{22} + c) \quad (27)$$

$$\frac{\partial \pi_m}{\partial P_{22}} = mf - 2bP_{22} + 2kP_{11} + b(W_{22} + c) - k(W_{11} + c) \quad (28)$$

From formula (27), (28), the second-order partial derivative of π_m with respect to P_{11}, P_{22} can be written as:

$$\frac{\partial^2 \pi_m}{\partial P_{11}^2} = -2b \quad \frac{\partial^2 \pi_m}{\partial P_{11} \partial P_{22}} = 2k \quad (29)$$

$$\frac{\partial^2 \pi_m}{\partial P_{22}^2} = -2b \quad \frac{\partial^2 \pi_m}{\partial P_{22} \partial P_{11}} = 2k \quad (30)$$

The first-order derivative matrix and second-order partial derivative matrix of π_m can be written as:

$$\nabla \pi_m (P_{11}, P_{22}) = \begin{bmatrix} \dfrac{\partial \pi_m}{\partial P_{11}} \\ \dfrac{\partial \pi_m}{\partial P_{22}} \end{bmatrix} \quad (31)$$

$$\nabla^2 \pi_m = \begin{bmatrix} -2b & 2k \\ 2k & -2b \end{bmatrix} \quad (32)$$

If the condition $b>k>0$ in Theorem 2 is satisfied, the second-order partial derivative matrix of π_m is negative definite. Making $\nabla \pi_m (P_{11}, P_{22}) = 0$, the only stationary point can be obtained:

(W_{11}, W_{22}) in advance. Secondly, substituting $P_{11}(W_{11}, W_{22})$, $P_{22}(W_{11}, W_{22})$ back into the profits functions of the suppliers, and the suppliers set the optimal wholesale prices W_{11}, W_{22} to maximize their profits. The optimal retail prices P_{11}, P_{22} can be obtained based on the optimal wholesale prices W_{11}, W_{22}. At last, the optimal profits of the suppliers and the whole assembler can be deduced.

The profit functions of the whole assembler consisting of the two assemblers and the two suppliers can be respectively written as:

$$\begin{cases} P_{11} = \dfrac{mb - mbf + mkf}{2(b^2 - k^2)} + \dfrac{W_{11} + c}{2} \\ P_{22} = \dfrac{mk + mbf - mkf}{2(b^2 - k^2)} + \dfrac{W_{22} + c}{2} \end{cases} \quad (33)$$

Because the stationary point is only one, π_m has a maximum value at the only stationary point.

So the proof is completed.

Substituting Equation (33) into (26) gives:

$$\begin{aligned}\pi_{S_{11}} &= (W_{11} - C_1)\left(\frac{m(1-f)}{2} - \frac{b(W_{11}+c)}{2} + \frac{k(W_{22}+c)}{2}\right) \\ \pi_{S_{22}} &= (W_{22} - C_2)\left(\frac{mf}{2} - \frac{b(W_{22}+c)}{2} + \frac{k(W_{11}+c)}{2}\right)\end{aligned} \quad (34)$$

From (34), the first-order derivatives and the second-order derivatives of the suppliers' profits can be derived as:

$$\begin{aligned}\frac{d\pi_{S_{11}}}{dW_{11}} &= -\frac{b(2W_{11} + c - C_1)}{2} + \frac{k(W_{22}+c)}{2} + \frac{m(1-f)}{2} \\ \frac{d\pi_{S_{22}}}{dW_{22}} &= -\frac{b(2W_{22} + c - C_2)}{2} + \frac{k(W_{11}+c)}{2} + \frac{mf}{2}\end{aligned} \quad (35)$$

$$\begin{aligned}\frac{d^2\pi_{S_{11}}}{dW_{11}^2} &= -b \\ \frac{d^2\pi_{S_{22}}}{dW_{22}^2} &= -b\end{aligned} \quad (36)$$

According to $b>0$, $d^2\pi_{S_{11}}/dW_{11}^2 = d^2\pi_{S_{22}}/dW_{22}^2 = -b < 0$ can be deduced. The second derivatives of $\pi_{S_{11}}$ and $\pi_{S_{22}}$ are less than 0. Therefore, the profit function of the supplier S_1 is a concave function of the wholesale price W_{11}, and the profit function of the supplier S_2 is a concave function of the wholesale price W_{22}. Hence, the maximum values of the profits can be obtained at the stationary points, and the stationary points are also the optimal values of the wholesale

prices. Making $d\pi_{S_{11}}/dW_{11}=0$ and $d\pi_{S_{22}}/dW_{22}=0$, the   optimal values of the wholesale prices can be obtained as:

$$
\begin{cases}
W_{11}=-\dfrac{(b-k)c}{2b-k}+\dfrac{2b^2C_1+bkC_2}{4b^2-k^2}+\dfrac{2bm}{4b^2-k^2}-\dfrac{m\left(1+\tanh\alpha(t-t_0)\right)}{2(2b+k)} \\[4mm]
W_{22}=-\dfrac{(b-k)c}{2b-k}+\dfrac{2b^2C_2+bkC_1}{4b^2-k^2}+\dfrac{km}{4b^2-k^2}+\dfrac{m\left(1+\tanh\alpha(t-t_0)\right)}{2(2b+k)}
\end{cases}
\tag{37}
$$

Substituting Equation (37) into (33), the optimal values of the retail prices can be obtained as:

$$
\begin{cases}
P_{11}=\dfrac{bc}{2(2b-k)}+\dfrac{2b^2C_1+bkC_2}{2(4b^2-k^2)}+\dfrac{\left(6b^3-3bk^2\right)m}{2\left(b^2-k^2\right)\left(4b^2-k^2\right)}-\dfrac{m(3b+2k)\left(1+\tanh\alpha(t-t_0)\right)}{4(b+k)(2b+k)} \\[4mm]
P_{22}=\dfrac{bc}{2(2b-k)}+\dfrac{2b^2C_2+bkC_1}{2(4b^2-k^2)}+\dfrac{\left(5b^2k-2k^3\right)m}{2\left(b^2-k^2\right)\left(4b^2-k^2\right)}+\dfrac{m(3b+2k)\left(1+\tanh\alpha(t-t_0)\right)}{4(b+k)(2b+k)}
\end{cases}
\tag{38}
$$

Lemma . The optimal dynamic wholesale price of the original key component supplier S_1 is decreasing, while the optimal dynamic wholesale price of the updated key component supplier S_2 is increasing over time. The change trends of them overtime are opposite. The optimal dynamic retail price of the original product is decreasing, while the optimal dynamic retail price of the updated product is increasing over time. The change trends of the mover time are opposite.

Proof: From formulas (37) and (38), the first-order derivatives of the optimal dynamic wholesale prices and retail prices with respect to t can be written as:

$$
\begin{aligned}
\frac{dW_{11}(t)}{dt}&=-\frac{m\alpha\sec h^2\alpha(t-t_0)}{2(2b+k)} \\[3mm]
\frac{dW_{22}(t)}{dt}&=\frac{m\alpha\sec h^2\alpha(t-t_0)}{2(2b+k)}
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
\frac{dP_{11}(t)}{dt}&=-\frac{(3b+2k)m\alpha\sec h^2\alpha(t-t_0)}{4(b+k)(2b+k)} \\[3mm]
\frac{dP_{22}(t)}{dt}&=\frac{(3b+2k)m\alpha\sec h^2\alpha(t-t_0)}{4(b+k)(2b+k)}
\end{aligned}
\tag{40}
$$

According to $b>0$ and $k>0$, and equation (39), $dW_{11}(t)/dt=dW_{22}(t)/dt$ and $dW_{11}(t)/dt<0$, $dW_{22}(t)/dt>0$ can be deduced. From equation (40) $dP_{11}(t)/dt=dP_{22}(t)/dt$ and $dP_{11}(t)/dt<0$, $dP_{22}(t)/dt>0$ can be deduced.

So proof is completed.

Lemma 3. If the condition b>k>0 in Theorem 2 is satisfied, when the two assemblers are cooperative, the ranges of the wholesale prices' changing over time are higher, while the ranges of the retail prices' changing over time are lower than those with no-cooperation between the two assemblers.

Proof. From equations (21-24) and equations (39), (40),

$$
\frac{dW_1(t)}{dt}-\frac{dW_{11}(t)}{dt}=\frac{k(b-k)m\alpha\sec h^2\alpha(t-t_0)}{2(2b+k)\left(4b^2-2k^2+kb\right)}
\tag{41a}
$$

$$
\frac{dW_2(t)}{dt}-\frac{dW_{22}(t)}{dt}=-\frac{k(b-k)m\alpha\sec h^2\alpha(t-t_0)}{2(2b+k)\left(4b^2-2k^2+kb\right)}
\tag{41b}
$$

$$
\frac{dP_1(t)}{dt}-\frac{dP_{11}(t)}{dt}=-\frac{b^2k\left(m\alpha\sec h^2\alpha(t-t_0)\right)}{4(b+k)(2b+k)\left(4b^2-2k^2+kb\right)}
\tag{41c}
$$

$$
\frac{dP_2(t)}{dt}-\frac{dP_{22}(t)}{dt}=\frac{b^2k\left(m\alpha\sec h^2\alpha(t-t_0)\right)}{4(b+k)(2b+k)\left(4b^2-2k^2+kb\right)}
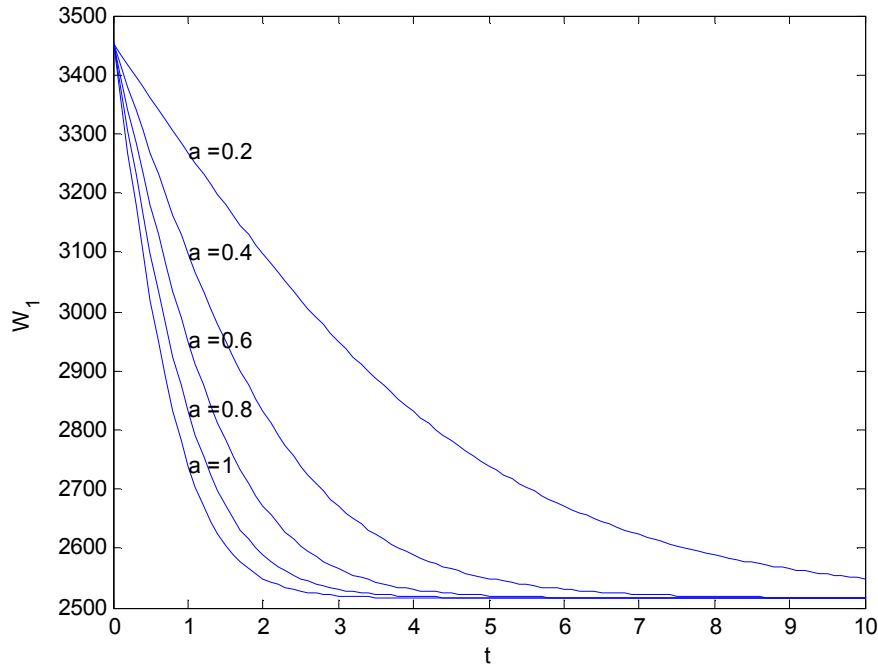\tag{41d}
$$

If the condition $b>k>0$ in Theorem2 is satisfied, from equations (41a), (41b), $dW_{11}(t)/dt<dW_{11}(t)/dt<0$, $dW_{22}(t)/dt>dW_{11}(t)/dt>0$ can be deduced. And from equations (41c), (41d), $dP_1/dt<dP_{11}/dt<0$, $dP_2/dt>dP_{22}/dt>0$ can be deduced. So, the ranges of the wholesale prices changing over time are higher, while the ranges of the retail prices changing over time are lower compared with no-cooperation between the two assemblers.
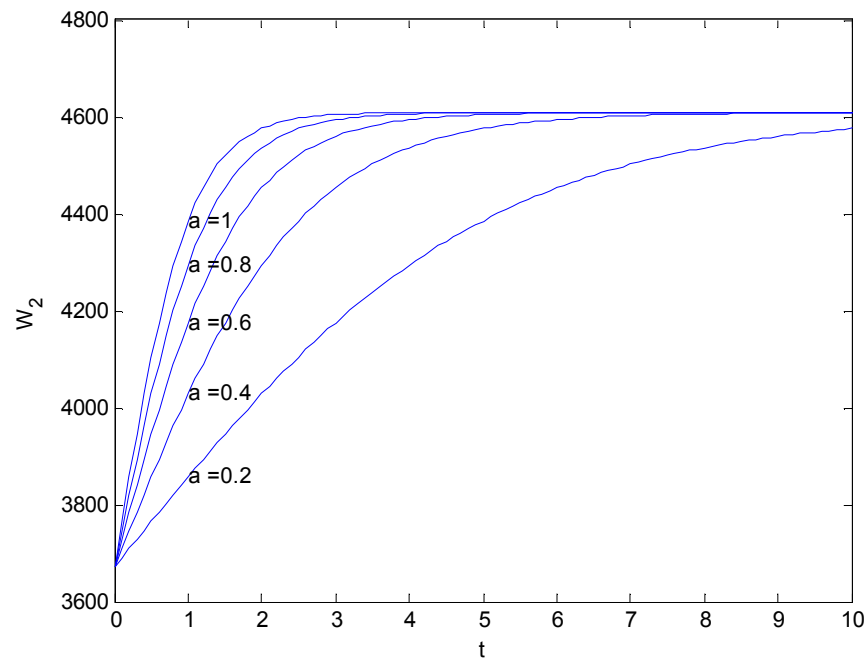
So proof is completed.

5. Numerical Simulation

In the assembly product supply chain above, assume that the two suppliers' unit marginal costs are C_1=2000 and C_2=2500, separately. The two assemblers' marginal cost c is c=300, the saturation value of the market demand m is 1000, and t_0=0.

If there is no cooperation between the two assemblers, with the parameter α changing, time-varying curve of the two suppliers' wholesale price W_1, W_2 are as follows:

Figure 3. *Time-varying curve of W₁ with α changing.*

Figure 4. *Time-varying curve of W₂ withα changing.*

From Figure 3 and 4, some insights can be obtained: (1) As time goes on, the wholesale price curves of all original key component are declining and converge at the point nearby W_1=2500, while the optimal dynamic wholesale price curves of all updated key component are rising, and they converge at a fixed point: W_2=4600 at last. (2) The higher the value of α, the faster the curve of W_1 will decrease, while the faster the curve of W_2 will increase.

With increasing of parameter α, the updated product functions will be more completely, and the demand of the updated product will increase, but the demand of the original

product will decrease. In this case, the two suppliers will raise the wholesale price of the updated key component and reduce the wholesale price of the original key component to maximize their profits. The wholesale price of original key components is decreasing to converge at the point nearby W_1=2500. But no matter how much the wholesale price decreases, to be profitable, it must be higher than its cost.

If there is no cooperation between the two assemblers, with the parameter α varying, time-varying curve of the two suppliers' retail price P_1, P_2 can be shown as follows:
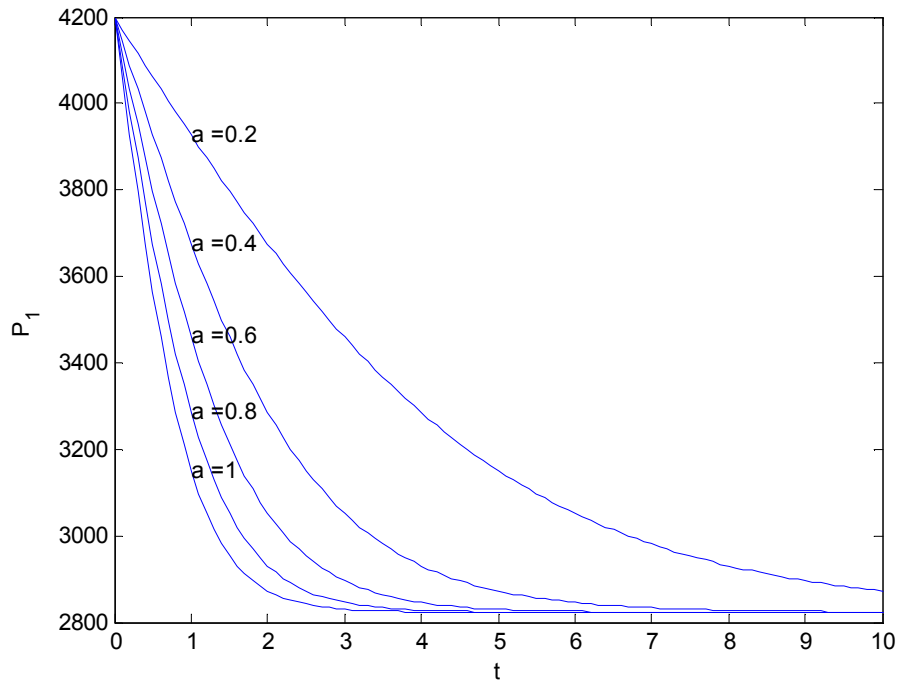
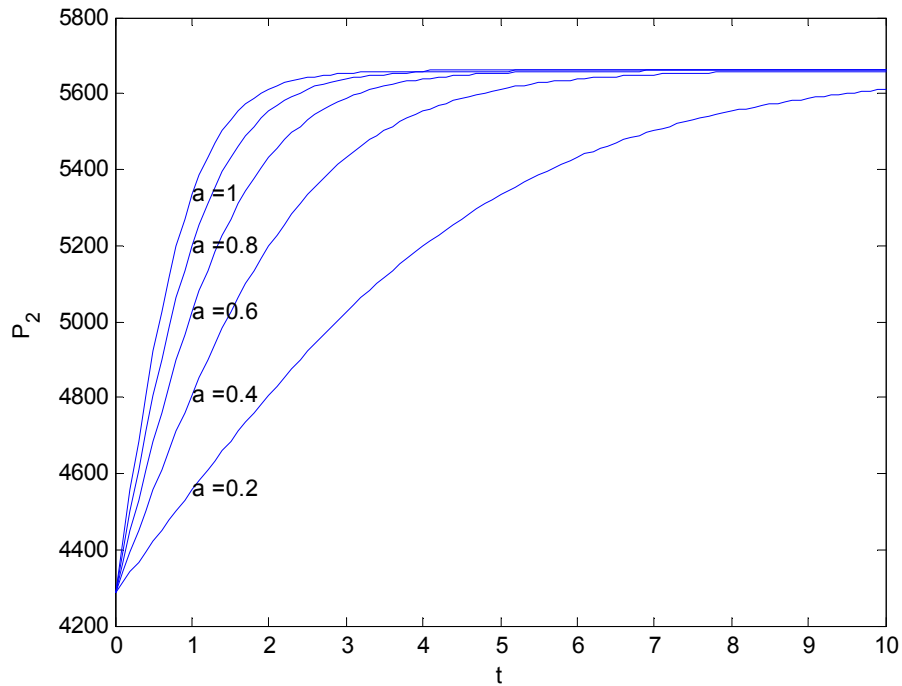*Figure 5.* Time-varying curve of P_1 with different α.

Figure 6. Time-varying curve of P_2 with different α.

From Figure 5 and 6, two conclusions can be obtained: (1) As time goes on, the retail price of the original product is decreasing and reaches a fixed point: P_1=2800, while the optimal dynamic retail price of the updated product is increasing and reaches a fixed point: P_2=5700 at last. (2) The higher the value of α, the faster the curve of P_1 will decrease, while the faster the curve of P_2 will increase.

The reason of these conclusions is similar to that of the wholesale price of key component. The greater the innovation degree is, the greater the attraction of the updated products is. So the demand of the updated product will be greatly increased, while the demand of the old product will be reduced. In order to attract customers and maximize their own profits, the assemblers will reduce the retail price of the original product and increase the retail price of the updated product. At least, the original product's

retail price is higher than the sum of its key component wholesale price and its other cost in order to be profitable, which results in retail price eventually converging at a point: $P_1=2500$ in Figure 5.

Based on the above conclusions, the dynamic strategy of the supply chain entities can be further analyzed. Assuming parameter α is 0.5 and other parameters are constant, time-varying curves of the two suppliers' wholesale prices and the assemblers' retail prices can be shown as Figure. 7 and Figure. 8 without cooperation, and Figure. 9 and Figure. 10 with cooperation.

Figure 7. Time-varying curve of W_1, W_2.

Figure 8. Time-varying curve of P_1, P_2.

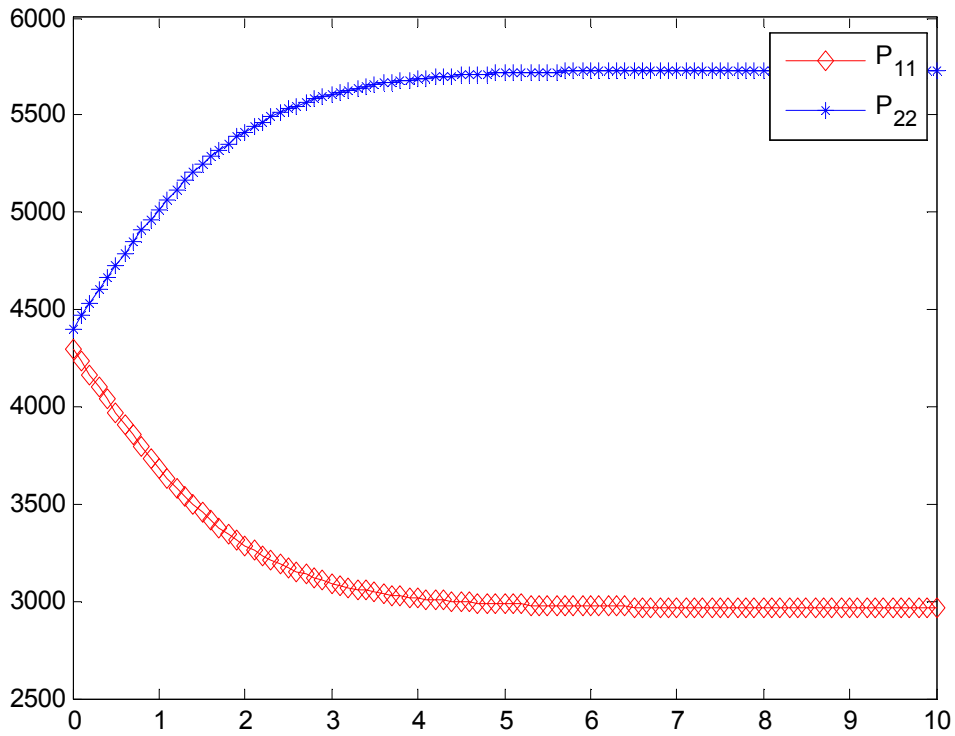**Figure 9.** Time-varying curve of W_{11}, W_{22}.

Figure 10. Time-varying curve of P_{11}, P_{22}.

In Figure. 7 andFigure. 8, the curves denote time-varying curve of the two suppliers' wholesale prices and the two assemblers' retail prices without cooperation between the two assemblers, and in the Figure. 9 andFigure. 10, the curves denote time-varying curve of the two suppliers' wholesale prices and the two assemblers' retail prices with cooperation.

Analyzing Figure 7 and Figure 9, the optimal dynamic wholesale price of the original key component supplier S_1 is decreasing, while the optimal dynamic wholesale price of the updated key component supplier S_2 is increasing and eventually converges at a point over time. The change trends of them over time are opposite with and without

cooperation. From Figure. 8 andFigure. 10, it can be concluded that the optimal retail price of the original product assembler A_1 is decreasing, while the optimal retail price of the updated product assembler A_2 is increasing over time. No matter the two assemblers cooperate or not, the change trends of the retail prices of the two products over time are opposite.
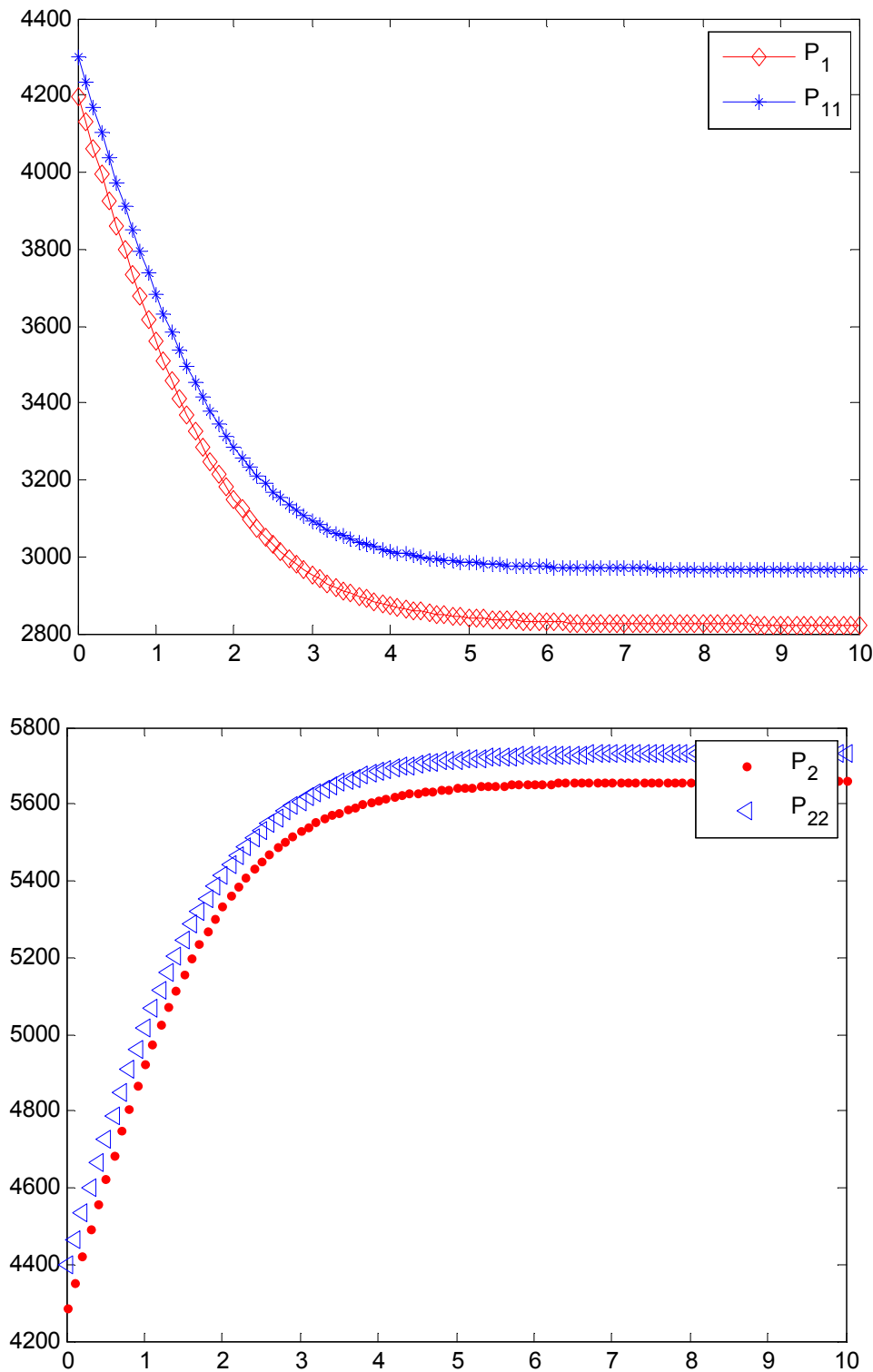

Figure 11. *Time-varying curve of the retail price of A₁, A₂ with and without cooperation.*
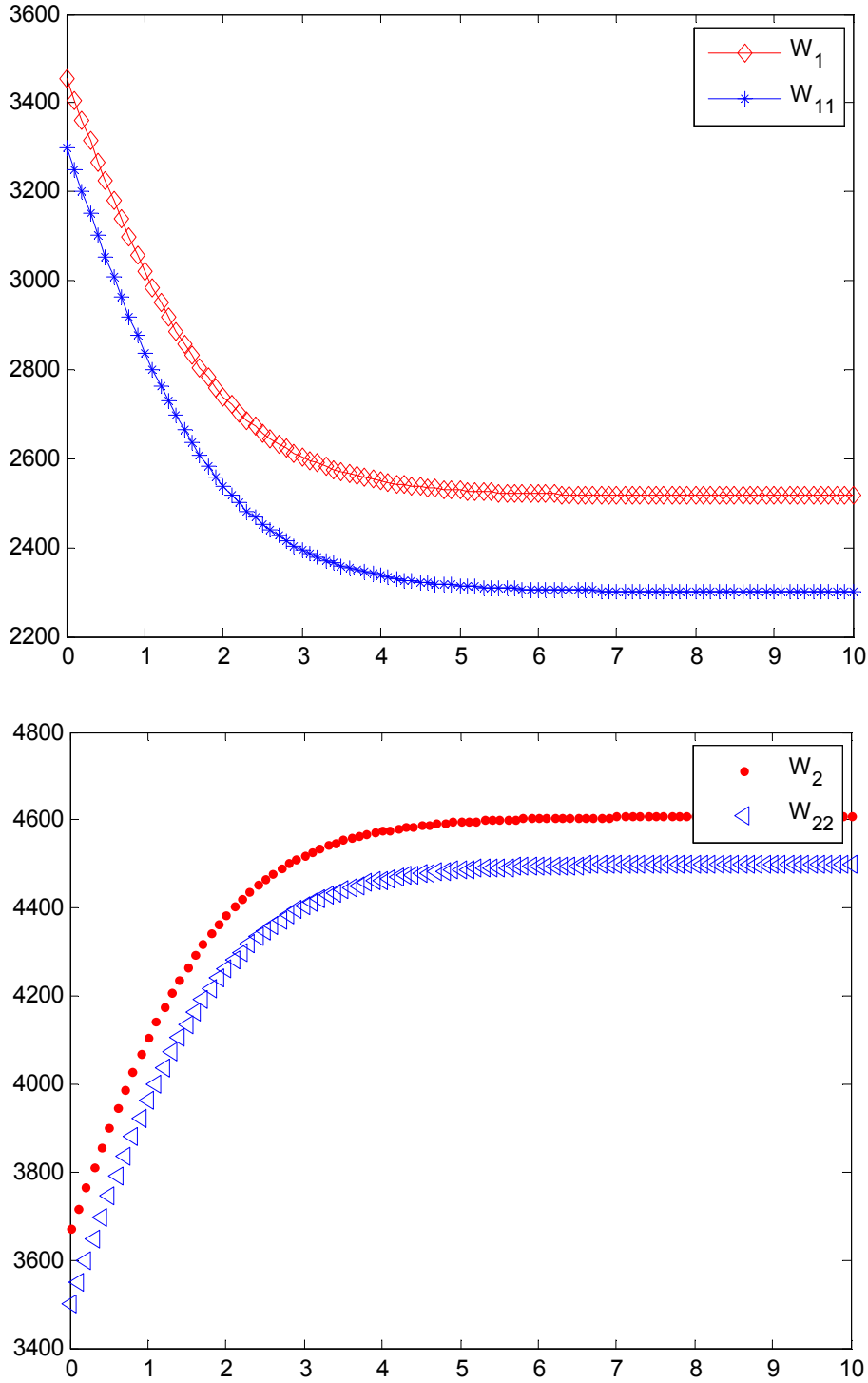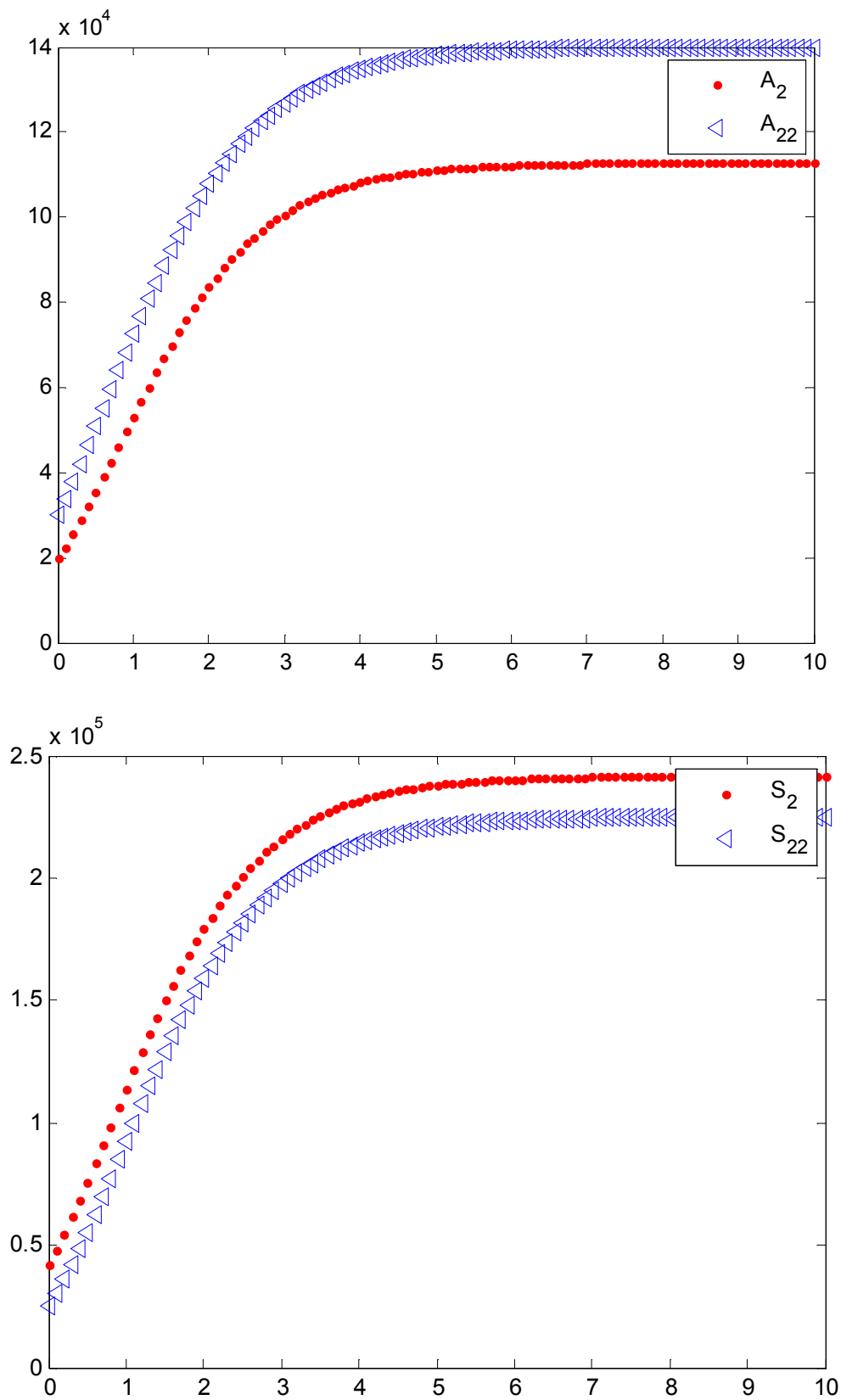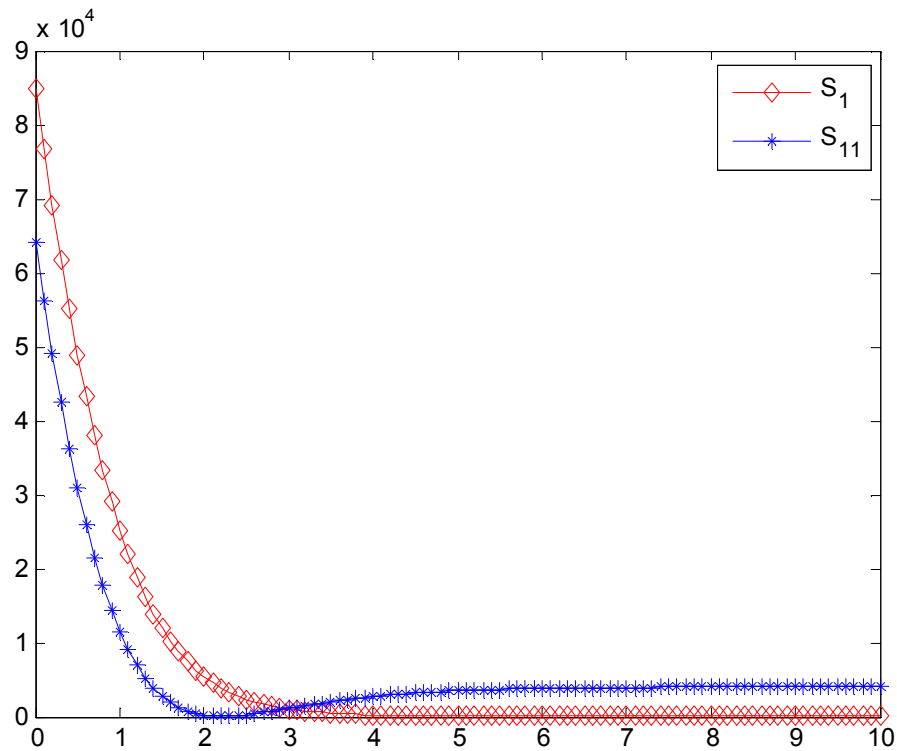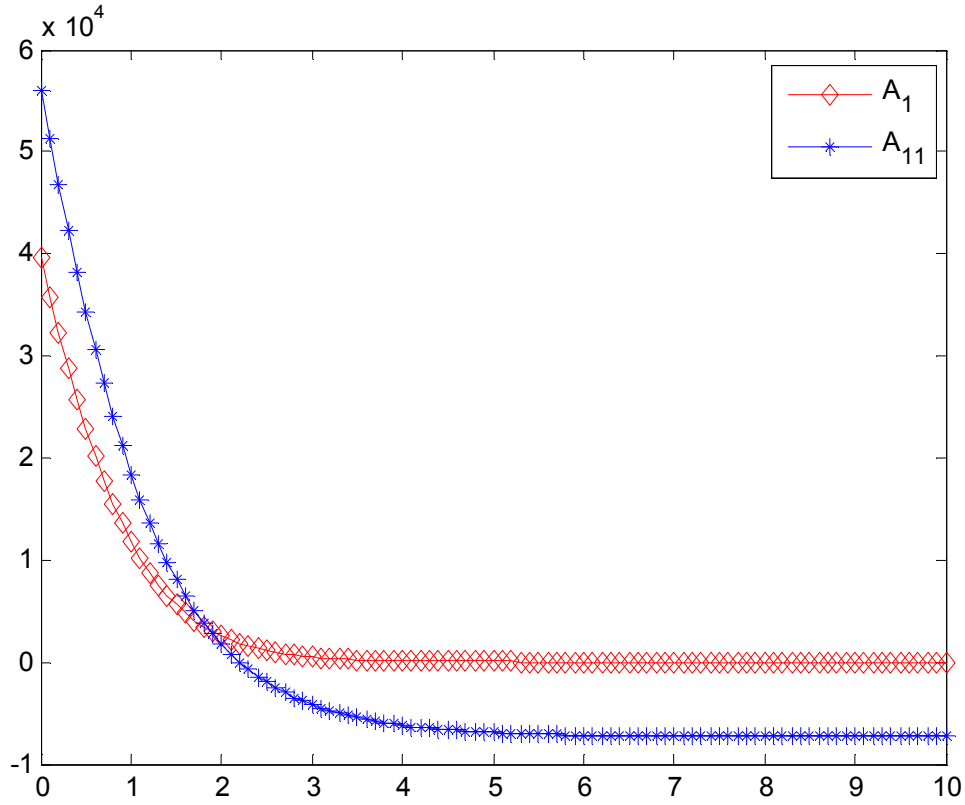
Figure 12. *Time-varying curve of the wholesale price of S₁, S₂ with and without cooperation.*

In Figure. 11, P_1 and P_2 (red line) denote the retail prices of A_1 and A_2 if the two assemblers are not cooperative, and P_{11} and P_{22} (blue line) denote the retail prices of A_1 and A_2 if the two assemblers are cooperative. In Figure. 12, W_1 and W_2 (red line) denote the wholesale prices of S_1 and S_2 if the two assemblers are not cooperative, and W_{11} and W_{22} (blue lines) denote the wholesale prices of S_1 and S_2 if the two assemblers are cooperative. As illustrated in Figure. 11 and 12, it can be concluded that the wholesale prices change faster over time than those without cooperation, while with cooperation the retail prices change lower(less) than those without cooperation. This is because the two assemblers share the same information, and they can be regarded as one whole entity playing game with the two suppliers. Sharing the same information will slow down the change trends over time of the two assemblers' retail prices, while quicken up the change trends over time of the two suppliers' wholesale prices.

Figure 13. *Assembler and supplier profits curve of the updated product with and without cooperation.*

Figure 14. *Assembler and supplier profits curve of the original product with and without cooperation.*

In Figure. 13, the curve A_2 (red line) denotes the profit of the updated product assembler, and the curve S_2 (red line) denotes the profit of the updated component supplier without cooperation between the two assemblers, and the curve A_{22} (blue line) denotes the profit of the updated product assembler, and the curve S_{22} (blue line) denotes the profit of the updated component supplier with cooperation between the two assemblers. In Figure. 14, the curve A_1 (red line) denotes the

profit of the original product assembler and the curve S_1 (red line) denotes the profit of the original component supplier without cooperation between the two assemblers, the curve A_{11} (blue line) denotes the profit of the original product assembler and the curve S_{11} (blue line) denotes the profit of the original component supplier with cooperation between the two assemblers.

As shown in Figure. 13 and Figure. 14, some conclusions can be obtained: (1) the profits of the updated product supplier and assembler are increasing while the profits of the original product supplier and assembler are decreasing over time no matter the two assemblers cooperate or not. (2) The profits of the two assemblers and suppliers with cooperation are bigger than those without cooperation if their profits are bigger than zero.

The reason is that the two assemblers share the same information when they are cooperative. In this case, they as a whole will play game with the suppliers. It is certain that their own revenues are more than those when they complete each other. The suppliers are more likely to obtain more profits from the game where the two assemblers complete each other. So the profits of the two assemblers and the suppliers with cooperation are more than those without cooperation.

In Figure. 14, the profits curves of the two assemblers and the two suppliers will converge at zero at last. The reason is that as time goes on, the original product will no longer be needed, which will be withdrawn from the market, and there will be only the updated product in the market, the profits of the updated product supplier and assembler will be redistributed between them.

6. Conclusions

The assembly product supply chain including two assemblers and two suppliers which sell two generations of a product with component updating is studied in this paper. First, a dynamic supply chain model whose demand is time-changing is built based on Fisher diffusion model and different product model. Then, by using Stackelberg game, in which the suppliers as a leader, assemblers as follower, and Nash game, the optimal dynamic pricing strategy and profits are obtained. At last, some important conclusions are obtained by simulation: (1) The profits of the two assemblers increased, while the two suppliers' profits relatively reduced if the assemblers cooperate with each other; (2) The optimal dynamic wholesale price of the original key component supplier is decreasing over time, while the optimal dynamic wholesale price of the updated key component supplier is increasing over time. The optimal dynamic retail price of the original product assembler is decreasing, while the optimal dynamic retail price of the updated product supplier is increasing over time; (3) The growth rates of the suppliers' wholesale prices of the two generations of a product are opposite, and the growth rates of the assemblers' retail prices are also oppositeno matter the two assemblers are cooperative or not. The study results of this paper have reference value in some extent for supply chain pricing strategies of the assembly product supply chain. The market sales dynamic model for two generations assembly products has established based on Fisher diffusion model. The optimal pricing of suppliers and assemblers has been obtained. It is the first time to study the optimal decisions of assembly products supply chain including components update, and it will be as a reference for optimal product decisions of IT and other related industries.

This paper focuses on dynamic demand and uses the Stackelberg game to get the optimal pricing. Based on these conclusions, the manger can map out the optimal pricing strategy to get the maximum benefits, and predict the old product's exiting. There are some guiding significances for the decisions of assembly product supply chain, but there are still some deficiencies: In the view of mathematical model, this model is simplified, which can lead to distortion of the conclusions. For example, the model assumes that the costs of buying auxiliary parts and assembly cost for the two suppliers are the same, but in reality there is a strong possibility that they are different. On the other hand, the conclusion of this paper is obtained by simulation analysis; and there is no actual enterprise data to verify it. The above aspects are the main objectives to be improved in the future.

Acknowledgement

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