

Higher-Order Nonlinear Schrödinger Equation Family in Optical Fiber and Solitary Wave Solutions

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Abstract: In this paper, we modify with an appropriate analytical technique, the characteristics of the optical fiber through the modification of the coefficients of the highly nonlinear partial differential equation, which initially governs the dynamics of the propagation in such a wave guide. The procedure consists to assign arbitrary coefficients to the various terms of the established nonlinear partial differential equation, such as the one that embodies the propagation dynamics in a strongly nonlinear optical fiber and subsequently establishing the constraint equations linking these coefficients and thus the analysis makes it possible to enumerate the criteria for which obtaining the desired solutions is possible. These coefficients are like indicators which characterize the various modifications made in this medium of transmission. The nonlinear evolution equation that served as mathematical model for this study is the higher-order nonlinear Schrödinger equation which better describes the propagation of an ultrafast pulse in an optical fiber. The use of the Bogning-Djeumen Tchaho-Kofané method enabled not only to establish the constraint relations, but also the solitary wave solutions and plane wave solutions. We want through the results obtained in this article to give the specialists of the manufacture of transmission media such as optical fiber, to consider the modification of the properties of this wave guide during manufacture, depending on the type of signal that one wants to propagate in this case notably the solitary wave.

Keywords: Schrödinger Equation, Higher-Order Nonlinear Effects, Solitary Wave Solution, Periodic Travelling Wave Solutions, Bogning-Djeumen-Kofané Method

1. Introduction

Nonlinear Schrödinger (NLS) equation is one of the most interesting nonlinear evolution equation (NEE) used to model phenomena in many area of physics in which nonlinear optics, plasma physics, condensed matter physics, nonlinear quantum field theory, bio-physics and hydrodynamic [1-8]. Particularly in nonlinear optics, the propagation of picoseconds pulse in monomode fibers is governed by the well-known NLS equation. This equation is known to be integrable and admits bright and dark soliton solutions in anomalous dispersion region and normal dispersion regime respectively [9-10]. For femtosecond pulse propagation, the standard NLS equation is inappropriate. Thin width pulses will induce higher-order

effects like third order dispersion (TOD) and nonlinearity such as self-steepening (SS) and stimulated Raman scattering (SRS) [11-12]. The governing equation of the femtosecond pulse propagation in monomode fibers is the Higher-order Nonlinear Schrödinger (HNLS) equation [13]

$$E_z - i\alpha_1 E_{tt} - i\alpha_2 |E|^2 E - \alpha_3 E_{ttt} - \alpha_4 \left(|E|^2 E \right)_t - \alpha_5 \left(|E|^2 \right)_t E = 0 \quad (1)$$

Where E is the slowly varying envelope of the electric field, α_1 and α_3 the dispersion coefficients, α_2 , α_4 and α_5 the nonlinear coefficients.

In nonlinear sciences, one thing is to model phenomena

using NEEs, another thing is to construct their exact solutions. The survey of these exact solutions of nonlinear evolution equations has a great importance in the study of nonlinear phenomena. In optical fiber for example, they permit to understand better, phenomena like soliton propagation and soliton interactions, and their Modulational instabilities [11, 14-17]. Several methods have been proposed to solve NLS equation of higher-order, we can quote the inverse scattering transform, Galilean transformation, Hirota's bilinear method,

Liouville integrability and Gauge equivalence, the unified transform method, the Sub-ODE method [18-22]. Thus many type of solutions have also been found, such as dark and bright solitary wave solutions, Jacobi elliptic function solutions, and rotational solutions [23-24].

The goal in this study on the one hand is to construct optical solitary wave solutions and periodic travelling wave solutions (PTWS) of the generalized higher-order nonlinear Schrödinger (GHNLS) equation

$$n_0 E_z + in_1 E_{tt} + in_2 |E|^2 E + n_3 E_{ttt} + n_4 (|E|^2 E)_t + n_5 (|E|^2)_t E = 0 \quad (2)$$

Where n_0 represents coefficient related to the variation of the electricfield with respect to z , n_1 and n_3 the group velocity dispersion coefficient and the TOD coefficient, n_2 , n_4 and n_5 respectively the coefficient related to the self-phase modulation, SS and SRS.

To reduce the density of work in this study, we set

$$E(z, t) = \bar{E}(z, t) e^{-i(kz - \omega t)} \quad (3)$$

Substituting Eq.(3) into Eq.(2), this last equation is written asfollows

$$-i(kn_0 + n_1 \omega^2 + n_3 \omega^3) \bar{E} + i(n_1 + 3n_3 \omega) \bar{E}_{tt} + i(n_2 + n_4 \omega) \bar{E}^3 + (-2n_1 \omega - 3n_3 \omega^2) \bar{E}_t + n_0 \bar{E}_z + n_3 \bar{E}_{ttt} + (3n_4 + 2n_5) |\bar{E}|^2 \bar{E}_t = 0 \quad (4)$$

The mathematical method employed to obtain the results is the Bogning-Djeumen Tchaho-Kofané method (BDKm) [25-32]. The BDKm is a method proposed by J. R. Bogning, C. T. Djeumenand T. C. Kofané in 2012 to construct exact solutions of certain NEEs of the form

$$a_i \sum_i \left(\frac{\partial q}{\partial x_i} \right)_i + b_i \sum_i \left(\frac{\partial^2 q}{\partial x_i^2} \right) + \dots + c_i \sum_i \left(\frac{\partial^l q}{\partial x_i^l} \right) + d_i \sum_{m,n} \left(\frac{\partial^n q \partial^m q}{\partial x_i^n \partial x_i^m} \right) + f(q, |q|^2) = 0 \quad (5)$$

Where a_i , b_i , c_i and d_i are constants that characterize partial differential equations, i , l , m and n positive natural integers, f an arbitrary linearity function of q and $|q|^2$, and q the unknown function to determine and $|q|$ its magnitude.

A brief presentation of the novel BDKm is also important for the calculations made.

2. Description of the Method

To research exact solutions of Eq.(5), the main steps are the following:

Step 1: Setting the change of variable

$$q(z, t) = q(\xi), \xi = \alpha(z - \beta t) \quad (6)$$

Where β is a real number and α a real or pure imaginary constant which will be consider in a first time as real, Eq.(5) is

$$\sum_{i,j} F(a_i, \alpha, \beta, \omega, k) \cosh^j(\xi) + \sum_{i,k} H(a_i, \alpha, \beta, \omega, k) \operatorname{sech}^k(\xi) + \sum_{i,s} G(a_i, \alpha, \beta, \omega, k) \sinh(\xi) \sec h^s(\xi) + \sum_{i,t} T(a_i, \alpha, \beta, \omega, k) \sinh(\xi) \cos h^t(\xi) + \sum_i V(a_i, \alpha, \beta, \omega, k) = 0 \quad (9)$$

Step 3: If the functions $F(a_i, \alpha, \beta, \omega, k)$, $H(a_i, \alpha, \beta, \omega, k)$, $G(a_i, \alpha, \beta, \omega, k)$, $T(a_i, \alpha, \beta, \omega, k)$, $V(a_i, \alpha, \beta, \omega, k)$, can be null for some values of the coefficients a_i , α , β , ω or k (nontrivialvalues), the ansatz given in Eq. (8) can be supported

transformed into and ordinary differential equation

$$P(q, q', q'', \dots, |q|^2 q', q(|q|^2)') = 0 \quad (7)$$

where $q' = \frac{\partial q}{\partial \xi}$

Step 2: Assuming that solution of Eq. (7) can be expressed under the form:

$$q(\xi) = a_1 \sinh^m(\xi) \sec h^n(\xi) + \dots + a_l \sinh^q(\xi) \sec h^r(\xi) \quad (8)$$

Where m , n , q and r are numbers to determine, Eq. (8) into Eq. (7), and taking into account the transformations related to the BDKm [30], gives an equation of the form

by Eq. (5) for any values of m, n, \dots, q, r and can be considered as solution of Eq. (5).

Step4: For $m = n = 0$, and $q = r$, we look for the values of r which can make some terms of Eq.(9) merge. The values of

r ($r_{\min} \leq r \leq r_{\max}$) obtained permit to select the reals m, n, q that can give solutions of Eq. (5).

Step5: For each values of (m, n, \dots, q, r) selected between the previous values, we obtain from Eq. (9) a range of equations in $a_i, \alpha, \beta, \omega$ and k to determine. Finally, the solitary wave solutions and other types of exact solutions (traveling wave solutions, hyperbolic function solutions...) are obtained.

Lemma: when the values $(m, n, q, r = l)$ give a trivial solution of Eq. (5), values $m, n, q, (m, n, q, r = l + 1)$ (if $l > 0$) and $(m, n, q, r = l - 1)$ (if $l < 0$) cannot give a nontrivial solution.

Step6: After getting the solutions when α is a real number (hyperbolic function solutions and solitary wave solutions), we consider α as a pure imaginary number by setting $\alpha = i\mu$ where μ is areal. The insertion of the new values of α in Eq. (9), permits to write all PTWS from hyperbolic function solutions

through the transformations

$$\begin{aligned}\sinh(i\xi) &= i \sin(\xi), \cosh(i\xi) = \cos(\xi) \\ \operatorname{sech}(i\xi) &= \sec(\xi), \tanh(i\xi) = i \tan(\xi)\end{aligned}\quad (10)$$

3. Determination of the Coefficient Range Equations

Setting the travelling wave transformation

$$\bar{E}(z, t) = \bar{E}(\xi), \xi = \alpha(t - \beta z) \quad (11)$$

Eq. (2) becomes

$$\begin{aligned}-\alpha(\beta n_0 + 2n_1\omega + 3n_3\omega^2)\bar{E}_\xi + n_3\alpha^3\bar{E}_{\xi\xi\xi} + \alpha(n_4 + n_5)\left(\left|\bar{E}\right|^2\right)_\xi \bar{E} + \alpha n_4\left|\bar{E}\right|^2 \bar{E}_\xi \\ + i[-(kn_0 + n_1\omega^2 + n_3\omega^3)\bar{E} + \alpha^2(n_1 + 3n_3\omega)\bar{E}_{\xi\xi} + (n_2 + n_4\omega)\bar{E}^2 E] = 0\end{aligned}\quad (12)$$

To look for the solution of Eq. (11), let consider the solitary ansatz

$$\bar{E}(t) = a \sinh^m \alpha(t - \beta z) \sec h^n \alpha(t - \beta z) + b \sinh^q \alpha(t - \beta z) \sec h^r \alpha(t - \beta z) \quad (13)$$

Where a and b are real constants, α the pulse width and β the shift of the inverse velocity group. Taking into account the fact that \bar{E} is a real physical size, Eq. (12) can be separate in real part and imaginary part as follows

$$\alpha(\beta n_0 - 2n_1\omega - 3n_3\omega^2)\bar{E}_\xi + n_3\alpha^3\bar{E}_{\xi\xi\xi} + \alpha(3n_4 + 2n_5)\bar{E}^2 \bar{E}_\xi + i[-(kn_0 + n_1\omega^2 + n_3\omega^3)\bar{E} + \alpha^2(n_1 + 3n_3\omega)\bar{E}_{\xi\xi} + (n_2 + n_4\omega)\bar{E}^2 E] = 0 \quad (14)$$

Eq. (14) can also be written

$$\alpha\kappa_1 \bar{E}_\xi + \alpha\kappa_2 \bar{E}^2 \bar{E}_\xi + \alpha^3 \kappa_3 \bar{E}_{\xi\xi\xi} + i\kappa_4 \bar{E} + i\kappa_5 \bar{E}^3 + i\alpha^2 \kappa_6 \bar{E}_{\xi\xi} = 0 \quad (15)$$

where,

$$\kappa_1 = -\beta n_0 - 2n_1\omega - 3n_3\omega^2 \quad (16)$$

$$\kappa_2 = 3n_4 + 2n_5, \kappa_3 = n_3 \quad (17)$$

$$\kappa_4 = -kn_0 - n_1\omega^2 - n_3\omega^3 \quad (18)$$

and

$$\kappa_5 = n_2 + n_4\omega, \kappa_6 = n_1 + 3n_3\omega \quad (19)$$

When α is a pure complex number i.e. $\alpha = i\mu$, Eq. (15) becomes

$$i[\mu\kappa_1 \bar{E}_\xi + \mu\kappa_2 \bar{E}^2 \bar{E}_\xi + (-\mu^3)\kappa_3 \bar{E}_{\xi\xi\xi} + \kappa_4 \bar{E} + \kappa_5 \bar{E}^3 + (-\mu^2)\kappa_6 \bar{E}_{\xi\xi}] = 0 \quad (20)$$

\bar{E} been a real number, it is easy to see that a solution of Eq. (17) is also a solution of Eq. (15) when the following transformations are considered:

$$\alpha \leftarrow \mu, \alpha^2 \leftarrow -\mu^2, \alpha^3 \leftarrow -\mu^3 \quad (21)$$

Thus, from Eq. (15), leading to the hyperbolic function solutions of the GHNLS, we can deduce their corresponding PTWS without solve it again.

Inserting Eq. (13) into Eq.(15), and separating in real part and imaginary part, we obtain the main equations which will be in the center of all analysis,

$$\begin{aligned}
 & \alpha\kappa_2 a^3 \left(m \sinh^{3m-1} \xi \sec h^{3n-1} \xi - n \sinh^{3m+1} \xi \sec h^{3n+1} \xi \right) + \alpha\kappa_2 q b^3 \left(q \sinh^{3q-1} \xi \sec h^{3r-1} \xi - r \sinh^{3q+1} \xi \sec h^{3r+1} \xi \right) \\
 & + ab^2 \alpha\kappa_2 [(m+2q) \sinh^{m+2q-1} \xi \sec h^{n+2r-1} \xi - (n+2r) \sinh^{m+2q+1} \xi \sec h^{n+2r+1} \xi] + a^2 b \alpha\kappa_2 [(q+2m) \sinh^{2m+q-1} \xi \sec h^{2n+r-1} \xi \dots \\
 & - (r+2n) \sinh^{2m+q+1} \xi \sec h^{2n+r+1} \xi] - a \alpha m \left(\frac{2\kappa_3 \alpha^2 + 3\alpha^2 m n \kappa_3}{-3\alpha^2 \kappa_3 m - \kappa_1} \right) \sinh^{\xi m-1} \sec h^{\xi n-1} + a \alpha n \left(\frac{3\alpha^2 \kappa_3 n + 3\alpha^2 m n \kappa_3}{+2\kappa_3 \alpha^2 - \kappa_1} \right) \sinh^{\xi m+1} \sec h^{\xi n+1} \\
 & - abq \left(\frac{2\kappa_3 \alpha^2 + 3\alpha^2 q r \kappa_3}{-3\alpha^2 \kappa_3 q - \kappa_1} \right) \sinh^{\xi q-1} \sec h^{\xi r-1} + \alpha r (2\kappa_3 \alpha^2 + 3\alpha^2 q r \kappa_3 + 3\alpha^2 \kappa_3 r - \kappa_1) \sinh^{\xi q+1} \sec h^{\xi r+1} \\
 & + \alpha^3 \kappa_3 a [m(m-1)(m-2) \sinh^{m-3} \xi \sec h^{n-3} \xi - n(n-1)(n-2) \sinh^{m+3} \xi \sec h^{n+3} \xi] \\
 & + \alpha^3 \kappa_3 b [q(q-1)(q-2) \sinh^{q-3} \xi \sec h^{r-3} \xi - r(r-1)(r-2) \sinh^{q+3} \xi \sec h^{r+3} \xi] = 0
 \end{aligned} \tag{22}$$

and

$$\begin{aligned}
 & 3\kappa_5 a^2 b \sinh^{2m+q} \xi \sec h^{2n+r} \xi + 3\kappa_5 a b^2 \sinh^{2q+m} \xi \sec h^{2r+m} \xi + \kappa_5 a^3 \sinh^{3m} \xi \sec h^{3n} \xi + \kappa_5 b^3 \sinh^{3q} \xi \sec h^{3r} \xi \\
 & - a \left(\frac{-\kappa_4 + \alpha^2 \kappa_6 n}{+2\alpha^2 \kappa_6 m n - \alpha^2 \kappa_6 m} \right) \sinh^m \xi \sec h^n \xi - b \left(\frac{-\kappa_4 + \alpha^2 \kappa_6 r}{+2\alpha^2 \kappa_6 q r - \alpha^2 \kappa_6 q} \right) \sinh^q \xi \sec h^r \xi \\
 & + \alpha^2 \kappa_6 a \left[\frac{m(m-1) \sinh^{m-2} \xi \sec h^{n-2} \xi}{+n(n+1) \sinh^{m+2} \xi \sec h^{n+2} \xi} \right] + \alpha^2 \kappa_6 b \left[\frac{q(q-1) \sinh^{q-2} \xi \sec h^{r-2} \xi}{+r(r+1) \sinh^{q+2} \xi \sec h^{r+2} \xi} \right] = 0
 \end{aligned} \tag{23}$$

Equations (19) are the range equations of the coefficients from which we study all the possibilities of obtaining solutions.

a) Form $n = q = r = 0$, Eqs. (22) and (23) reduce to

$$(a+b) \left[\kappa_5 (a+b)^2 + \kappa_4 \right] = 0 \tag{24}$$

4. Analysis of the Range Equations (22) and (23) and Solutions

The search for the terms of the equations (13) and (14) that are grouped allows to obtain the values of m, n, q and r for which it is possible to obtain solutions. So, for $m = n = 0$ and $q = r$, some terms of Eqs. (22) and (23) merge for $q = r = \{0, \pm 1, \pm 2, \pm 3, \pm 4\}$.

and the constant solution of Eq. (2) is

$$E(x, t) = \pm \sqrt{\frac{-\kappa_4}{\kappa_5}} \pm \sqrt{\frac{\kappa n_0 + n_1 \omega^2 + n_3 \omega^3}{n_2 + n_4 \omega}} = cst \tag{25}$$

b) For $m=n=0$ and $q=r=\pm 1$ Eqs. (19) become

$$\alpha b (4\alpha^2 \kappa_3 + \kappa_2 b^2 + \kappa_2 a^2 + \kappa_1) \sec h^2 \xi - \alpha b (\kappa_2 b^2 + 6\alpha^2 \kappa_3) \sec h^4 \xi + 2\alpha \kappa_2 a b^2 \sinh \xi \sec h^3 \xi = 0 \tag{26}$$

and

$$a(\kappa_5 a^2 + 3\kappa_5 b^2 + \kappa_4) - 3\kappa_5 a b^2 \sec h^2 \xi + b(\kappa_4 + 3\kappa_5 a^2 + \kappa_5 b^2) \sinh \xi \sec h \xi - b(\kappa_5 b^2 + 2\alpha^2 \kappa_6) \sinh \xi \sec h^3 \xi = 0 \tag{27}$$

The following coefficient equations are obtained from Eqs. (26) and (27):

$$4\alpha^2 \kappa_3 + \kappa_2 b^2 + \kappa_2 a^2 + \kappa_1 = 0 \tag{28}$$

$$\kappa_2 b^2 + 6\alpha^2 \kappa_3 = 0 \tag{29}$$

$$a(\kappa_5 a^2 + 3\kappa_5 b^2 + \kappa_4) = 0 \tag{30}$$

$$2\alpha \kappa_2 a b^2 = 0 \tag{31}$$

$$\kappa_5 a b^2 = 0 \tag{32}$$

$$b(\kappa_4 + 3\kappa_5 a^2 + \kappa_5 b^2) = 0 \tag{33}$$

$$b(\kappa_5 b^2 + 2\alpha^2 \kappa_6) = 0 \tag{34}$$

The resolution of this algebraic system, taking into account Eqs. (16),..., (19) permits to obtain the parameters

$$a = 0, b = \pm \sqrt{\frac{-6n_3}{3n_4 + 2n_5}} \alpha \tag{35}$$

with $-n_3(3n_4 + 2n_5) > 0$,

$$\omega = \frac{3n_2 n_3 - n_1(3n_4 + 2n_5)}{6n_3(n_4 + n_5)} \tag{36}$$

with $n_3(n_4 + n_5) \neq 0$,

$$k = \frac{\begin{pmatrix} -2\alpha^2 n_1 - 6\alpha^2 n_3 \omega \\ -\omega^2 n_1 - n_3 \omega^3 \end{pmatrix}}{n_0}, n_0 \neq 0 \quad (37)$$

$$\beta = \frac{\begin{pmatrix} -2\alpha^2 n_3 - 2n_1 \omega \\ -3n_3 \omega^2 \end{pmatrix}}{n_0}, n_0 \neq 0 \quad (38)$$

$$k = \frac{\begin{bmatrix} 216\alpha^2 n_1 n_3^2 n_4 (n_4 - n_5)^2 \\ -648\alpha^2 n_2 n_3^2 (n_4 + n_5)^2 - 27(n_2 n_3 + n_1 n_4) \times \\ (n_1 n_4 - n_2 n_3)^2 - 72 n_1^2 n_4 n_5 (n_1 n_4 - n_2 n_3) \\ -4 n_1^2 n_5^2 (4 n_1 n_5 - 9 n_2 n_3 + 15 n_1 n_4) \end{bmatrix}}{216 n_3^2 (n_4 + n_5)^3} \quad (39)$$

$$\beta = \frac{\begin{bmatrix} -24\alpha^2 n_3^2 (n_4 + n_5)^2 + 4 n_1^2 n_5 (n_5 + 2 n_4) \\ +3(n_1 n_4 + 3 n_2 n_3)(n_1 n_4 - n_2 n_3) \end{bmatrix}}{12 n_0 n_3 (n_4 + n_5)^2} \quad (40)$$

Inserting the values of ω in Eqs. (37) and (38), k and β can be rewritten as

The optical dark soliton solution and the travelling wave solution of the GHNLS equation in this condition read

$$E(z, t) = \pm \sqrt{\frac{-6n_3}{3n_4 + 2n_5}} \alpha \tanh \alpha \left(t - \frac{\begin{pmatrix} -2\alpha^2 n_3 - 2n_1 \omega \\ -3n_3 \omega^2 \end{pmatrix}}{n_0} z \right) \times \exp - i \left(\frac{\begin{pmatrix} -2\alpha^2 n_1 - 6\alpha^2 n_3 \omega - \omega^2 n_1 - n_3 \omega^3 \end{pmatrix}}{n_0} z - \frac{3n_2 n_3 - n_1(3n_4 + 2n_5)}{6n_3(n_4 + n_5)} t \right) \quad (41)$$

and

$$E(z, t) = \pm \sqrt{\frac{-6n_3}{3n_4 + 2n_5}} \alpha \cot \alpha \left(t - \frac{\begin{pmatrix} -2\alpha^2 n_3 - 2n_1 \omega \\ -3n_3 \omega^2 \end{pmatrix}}{n_0} z \right) \times \exp - i \left(\frac{\begin{pmatrix} -2\alpha^2 n_1 - 6\alpha^2 n_3 \omega - \omega^2 n_1 - n_3 \omega^3 \end{pmatrix}}{n_0} z - \frac{3n_2 n_3 - n_1(3n_4 + 2n_5)}{6n_3(n_4 + n_5)} t \right) \quad (42)$$

The corresponding TPWS are

$$E(z, t) = \pm i \sqrt{\frac{-6n_3}{3n_4 + 2n_5}} \mu \tan \mu \left(t - \frac{2\mu^2 n_3 - 2n_1 \omega - 3n_3 \omega^2}{n_0} z \right) \times \exp - i \left(\frac{2\mu^2 n_1 + 6\mu^2 n_3 \omega - \omega^2 n_1 - n_3 \omega^3}{n_0} z - \frac{3n_2 n_3 - n_1(3n_4 + 2n_5)}{6n_3(n_4 + n_5)} t \right) \quad (43)$$

and

$$E(z, t) = \pm i \sqrt{\frac{-6n_3}{3n_4 + 2n_5}} \mu \cot \mu \left(t - \frac{2\mu^2 n_3 + 2n_1 \omega - 3n_3 \omega^2}{n_0} z \right) \times \exp - i \left(\frac{2\mu^2 n_1 + 6\mu^2 n_3 \omega - \omega^2 n_1 - n_3 \omega^3}{n_0} z - \frac{3n_2 n_3 - n_1(3n_4 + 2n_5)}{6n_3(n_4 + n_5)} t \right) \quad (44)$$

No solution is found for $m = n = q = r$ and $|r| \geq 2$.

c) For $m = 0$, $n = 0$, $q = 0$ and $r = 1$, Eqs. (19), become

$$-\alpha b(\kappa_1 + \kappa_2 a^2 + \alpha^2 \kappa_3) \sinh \xi \operatorname{sech}^2 \xi - 2\alpha \kappa_2 a b^2 \sinh \xi \operatorname{sech}^3 \xi + \alpha b(-\kappa_2 b^2 + 6\alpha^2 \kappa_3) \sinh \xi \operatorname{sech}^4 \xi = 0 \quad (45)$$

and

$$i[a(\kappa_4 + \kappa_5 a^2) + (\kappa_4 b + 3\kappa_5 a^2 b + \alpha^2 \kappa_6 b) \operatorname{sech} \xi + 3\kappa_5 a b^2 \operatorname{sech}^2 \xi + (\kappa_5 b^3 - 2\alpha^2 \kappa_6 b) \operatorname{sech}^3 \xi] = 0 \quad (46)$$

The resolution of the algebraic system resulting from this equation gives

$$a = 0, b = \pm \sqrt{\frac{6n_3}{3n_4 + 2n_5}} \alpha \quad (47)$$

$$\omega = \frac{3n_2n_3 - 3n_1n_4 - 2n_1n_5}{6n_3(n_4 + n_5)} \quad (48)$$

$$k = (\alpha^2 n_1 + 3\alpha^2 n_3 \omega - n_1 \omega^2 - n_3 \omega^3) / n_0 \quad (49)$$

and

$$\beta = (\alpha^2 n_3 - 2n_1 \omega - 3n_3 \omega^2) / n_0 \quad (50)$$

Inserting the value of ω in Eq. (35), k and β are express as follow

$$E(z, t) = \pm \sqrt{\frac{6n_3}{3n_4 + 2n_5}} \alpha \operatorname{sech} \alpha \left(t - \frac{\alpha^2 n_3 - 2n_1 \omega - 3n_3 \omega^2}{n_0} z \right) \times \exp - i \left(\frac{\alpha^2 n_1 + 3\alpha^2 n_3 \omega - n_1 \omega^2 - n_3 \omega^3}{n_0} z - \frac{3n_2 n_3 - 3n_1 n_4 - 2n_1 n_5}{6n_3(n_4 + n_5)} t \right) \quad (53)$$

and

$$E(z, t) = \pm \sqrt{\frac{-6n_3}{3n_4 + 2n_5}} \mu \operatorname{cosec} \mu \left(t - \frac{-\mu^2 n_3 - 2n_1 \omega - 3n_3 \omega^2}{n_0} z \right) \times \exp - i \left(\frac{-\mu^2 n_1 - 3\mu^2 n_3 \omega - n_1 \omega^2 - n_3 \omega^3}{n_0} z - \frac{3n_2 n_3 - 3n_1 n_4 - 2n_1 n_5}{6n_3(n_4 + n_5)} t \right) \quad (54)$$

The new constraint relations susceptible to make propagate dark and bright optical solitons in the nonlinear fiber are $n_3(3n_4 + 2n_5) < 0$ and $n_3(3n_4 + 2n_5) > 0$ respectively. From these conditions, we can see that optical dark and bright

$$k = - \frac{\left[108\alpha^2 n_1 n_3^2 n_4 (n_4 - n_5)^2 - 324\alpha^2 n_2 n_3^2 (n_4 + n_5)^2 + 27(n_2 n_3 + n_1 n_4) \times (n_1 n_4 - n_2 n_3)^2 + 72 n_1^2 n_4 n_5 (n_1 n_4 - n_2 n_3) + 4 n_1^2 n_5^2 (4 n_1 n_5 - 9 n_2 n_3 + 15 n_1 n_4) \right]}{216 n_3^2 (n_4 + n_5)^3} \quad (51)$$

and

$$\beta = \frac{\left[12\alpha^2 n_3^2 (n_4 + n_5)^2 + 4 n_1^2 n_5 (n_5 + 2 n_4) + 3(n_1 n_4 + 3 n_2 n_3)(n_1 n_4 - n_2 n_3) \right]}{12 n_0 n_3 (n_4 + n_5)^2} \quad (52)$$

Thus, the optical bright soliton obtained here which its equivalent PTWS read

solitons can propagate both in anomalous dispersion regime and normal dispersion regime. The changing of these results is due to the presence of higher-nonlinear effects and the TOD.

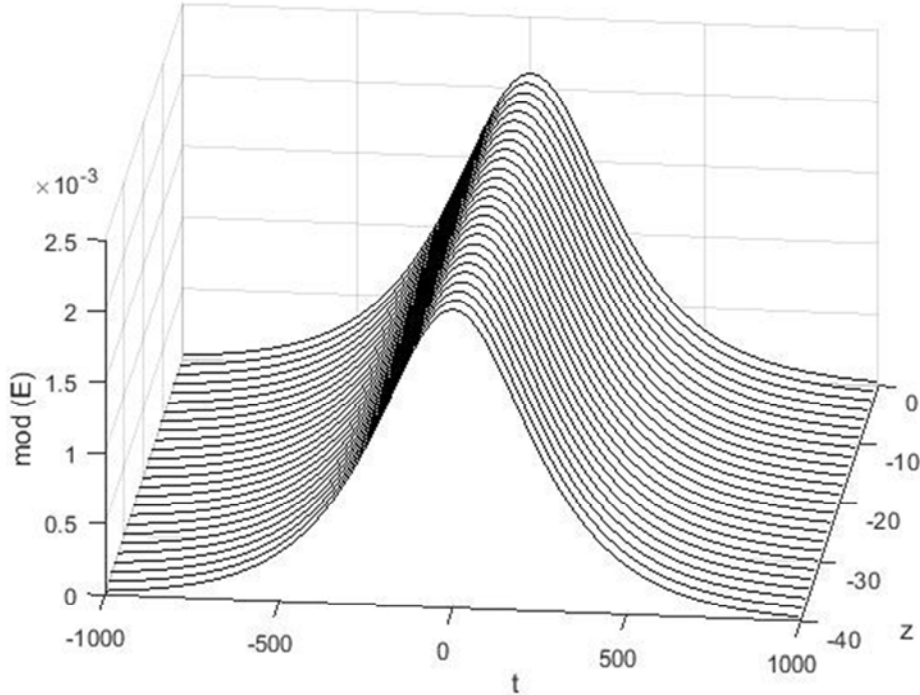


Figure 1. Numerical propagation of the optical bright and dark solitons of the GHNLS respectively for (a): $n_0=1500$, $n_1=0.75$, $n_2=-0.08556$, $n_3=-0.059$, $n_4=0.5$, $n_5=0.25$, $\alpha=0.06$.

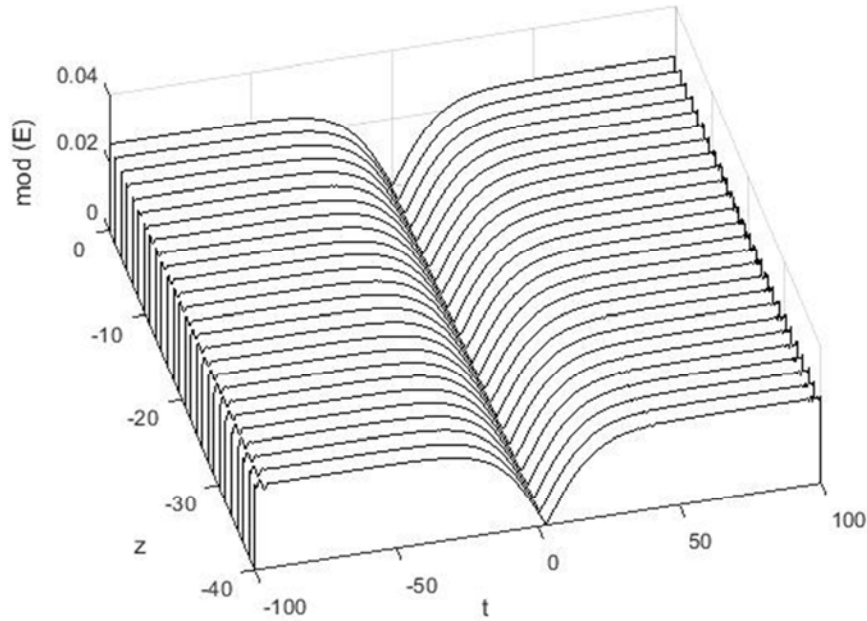


Figure 2. Numerical propagation of the optical bright and dark solitons of the GHNLS respectively for $n_0=3000$, $n_1=0.75$, $n_2=-0.08556$, $n_3=-0.059$, $n_4=0.5$, $n_5=0.25$, $\alpha=0.05$.

Using the one-soliton ansatz, H. Kumar and F. Chand derive optical dark and bright soliton solutions, corresponding to previous solutions for $n_0=1$. They also derive periodic solutions like dn, ns, ds, and cs functions and show that these Jacobi elliptic functions can degenerate into trigonometric functions, i.e., $\text{sn}(\xi) \rightarrow \sin(\xi)$, $\text{cn}(\xi) \rightarrow \cos(\xi)$, $\text{dn}(\xi) \rightarrow 1$, and the periodic traveling wave solutions could become the periodic trigonometric solutions which correspond to weak localization [33]. Figures 1 and 2 show the numerical propagation of these different optical solitons.

The numerical simulation is obtained using the fourth order Runge-Kutta method and integrating factor.

- d) For $m=n=q=0, r=-1$, the computation of the resulting algebraic system gives

$$\omega = -\frac{n_2}{n_4} \quad (55)$$

$$k = \frac{\alpha^2 n_4^2 (n_1 n_4 - 3n_2 n_3) - n_2^2 (n_1 n_4 - n_2 n_3)}{n_0 n_4^3} \quad (56)$$

$$\beta = \frac{\alpha^2 n_3 n_4^2 + 2n_1 n_2 n_4 - 3n_2^2 n_3}{n_0 n_4^2} \quad (57)$$

when $a=0$, with the constraint equation $2n_5 + 3n_4 = 0$.

When $a \neq 0$, the constraint equations are $n_1 n_4 - 3n_2 n_3 = 0$, $2n_5 + 3n_4 = 0$ and the constants

$$\omega = -\frac{n_2}{n_4}, k = -\frac{2n_1^3}{27n_0 n_3^2}, \beta = \frac{3\alpha^2 n_3^2 + n_1^2}{3n_0 n_3} \quad (58)$$

Using the transformations Eq. (21), the obtained PTWS here read

$$E(z, t) = b \cos \mu \left(t - \frac{-\mu^2 n_3 n_4^2 + 2n_1 n_2 n_4 - 3n_2^2 n_3}{n_0 n_4^2} z \right) \times \exp -i \left(\frac{-\mu^2 n_4^2 (n_1 n_4 - 3n_2 n_3) - n_2^2 (n_1 n_4 - n_2 n_3)}{n_0 n_4^3} z + \frac{n_2}{n_4} t \right) \quad (59)$$

and

$$E(z, t) = \left[a + b \cos \mu \left(t - \frac{-3\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \right] \times \exp -i \left(-\frac{2n_1^3}{27n_0 n_3^2} z + \frac{n_2}{n_4} t \right) \quad (60)$$

- e) For $m=n=q=0, r=-2$, we found

$$\omega = -\frac{n_2}{n_4}, k = -\frac{2n_1^3}{27n_0 n_3^2}, \beta = \frac{4\alpha^2 n_3^2 + n_1^2}{3n_0 n_3} \quad (61)$$

with the parametric equations $2n_5 + 3n_4 = 0$, and $n_1 n_4 - 3n_2 n_3 = 0$.

Under these parametric constraints, solution is written

$$E(z,t) = \left[a + b \cos^2 \mu \left(t - \frac{-4\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \right] \times \exp - i \left(-\frac{2n_1^3}{27n_0 n_3^2} z + \frac{n_2}{n_4} t \right) \quad (62)$$

f) For $m = n = 0, q = 1, r = 0$. The constants ω , k , and β are given by

$$\omega = -\frac{n_2}{n_4} \quad (63)$$

$$k = \frac{\left[\begin{array}{c} \alpha^2 n_4^2 (n_1 n_4 - 3n_2 n_3) \\ -n_2^2 (n_1 n_4 - n_2 n_3) \end{array} \right]}{n_0 n_4^3} \quad (64)$$

$$\beta = \frac{\alpha^2 n_3 n_4^2 + 2n_1 n_2 n_4 - 3n_2^2 n_3}{n_0 n_4^2} \quad (65)$$

with parametric constraint $2n_5 + 3n_4 = 0$, when $a=0$.

For $a \neq 0$, constants are rewritten

$$\omega = -\frac{n_2}{n_4}, k = -\frac{2n_1^3}{27n_0 n_3^2}, \beta = \frac{3\alpha^2 n_3^2 + n_1^2}{3n_0 n_3} \quad (66)$$

with the constraint equations $2n_5 + 3n_4 = 0$,

and $n_1 n_4 - 3n_2 n_3 = 0$.

The different solutions of the GHNLS reported here are

$$E(z,t) = ib \sin \mu \left(t - \frac{-\mu^2 n_3 n_4^2 + 2n_1 n_2 n_4}{n_0 n_4^2} z \right) \times \exp - i \left[\frac{-\mu^2 n_4^2 (n_1 n_4 - 3n_2 n_3)}{n_0 n_4^3} z + \frac{n_2}{n_4} t \right] \quad (67)$$

and

$$E(z,t) = \left[a + ib \sin \mu \left(t - \frac{-3\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \right] \times \exp - i \left(-\frac{2n_1^3}{27n_0 n_3^2} z + \frac{n_2}{n_4} t \right) \quad (68)$$

g) $m = n = 0, q = 1, r = -1$ Following the same procedure, we find the constants

$$\omega = -\frac{n_2}{n_4} \quad (69)$$

$$k = \frac{\left(\begin{array}{c} \alpha^2 n_4^2 (n_1 n_4 - 3n_2 n_3) \\ -n_2^2 (n_1 n_4 - n_2 n_3) \end{array} \right)}{n_0 n_4^3} \quad (70)$$

$$\beta = \frac{\alpha^2 n_3 n_4^2 + 2n_1 n_2 n_4 - 3n_2^2 n_3}{n_0 n_4^2} \quad (71)$$

and constraint equation $2n_5 + 3n_4 = 0$, when $a=0$,

and

$$\omega = -\frac{n_2}{n_4}, k = -\frac{2n_1^3}{27n_0n_3^2}, \beta = \frac{4\alpha^2n_3^2 + n_1^2}{3n_0n_3} \quad (72)$$

when the parameters n_i also satisfy the constraints $2n_5 + 3n_4 = 0$ and $n_1n_4 - 3n_2n_3 = 0$ when $a \neq 0$.

The PTWS of the GHNLS read

$$E(z, t) = i \frac{b}{2} \sin 2\mu \left(t - \frac{-\mu^2 n_3 n_4^2 + 2n_1 n_2 n_4 - 3n_2^2 n_3}{n_0 n_4^2} z \right) \times \exp - i \left[\frac{-\mu^2 n_4^2 (n_1 n_4 - 3n_2 n_3) - n_2^2 (n_1 n_4 - n_2 n_3)}{n_0 n_4^3} z + \frac{n_2}{n_4} t \right] \quad (73)$$

and

$$E(z, t) = \left[a + i \frac{b}{2} \sin 2\mu \left(t - \frac{-4\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \right] \times \exp - i \left(-\frac{2n_1^3}{27n_0 n_3^2} z + \frac{n_2}{n_4} t \right) \quad (74)$$

For $m=n=0, q=2, r=0$, the following TPWSs are obtained

$$E(z, t) = -b \sin^2 \mu \left(t - \frac{-4\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \times \exp - i \left(-\frac{2n_1^3}{27n_0 n_3^2} z + \frac{n_2}{n_4} t \right) \quad (75)$$

and

$$E(z, t) = \left\{ a - 2a \sin^2 \mu \left(t - \frac{-4\mu^2 n_3 n_4^2 + 2n_1 n_2 n_4 - 3n_2^2 n_3}{n_0 n_4^2} z \right) \right\} \times \exp - i \left[\frac{-4\mu^2 n_4^2 (n_1 n_4 - 3n_2 n_3) - n_2^2 (n_1 n_4 - n_2 n_3)}{n_0 n_4^3} z + \frac{n_2}{n_4} t \right] \quad (76)$$

For $m=1, n=0, q=0, r=-1$, setting the coefficients of each hyperbolic function of the main equations Eqs. (22) and (23) to zero, and Computing the resulting algebraic system, coefficients allowing to write solution is obtained such as

$$E(z, t) = \left[\begin{array}{l} ia \sin \mu \left(t - \frac{-3\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \\ + b \cos \mu \left(t - \frac{-3\mu^2 n_3^2 + n_1^2}{3n_0 n_3} z \right) \end{array} \right] \times \exp - i \left(-\frac{2n_1^3}{27n_0 n_3^2} z + \frac{n_2}{n_4} t \right) \quad (77)$$

No solution is found for $m=0, n=1, q=1$ and $r=1$.

The different constraints on the parameters of the equation also allow to define Schrodinger equations family with strong nonlinearity, and their exact solutions immediately.

When $n_1 n_4 - 3n_2 n_3 \neq 0$ and $2n_5 + 3n_4 = 0$ for example, the GHNLS equation of eq.(2) lead to the following HNLS equation family

$$n_0 E_z + in_1 E_{tt} + in_2 |E|^2 E + \frac{n_1 n_4}{3n_2} E_{ttt} + n_4 \left(|E|^2 E \right)_t - \frac{3n_4}{2} \left(|E|^2 \right)_t E = 0 \quad (78)$$

From a purely mathematical point of view, it is now possible to propose an infinity of strongly non-linear differential equations of the Schrodinger type, describing the propagation dynamics of solitary waves in optical fibers admitting exact solutions.

The procedure for obtaining these equations consists in assigning the values to the coefficients n_i ($i=0,1,2,3,4,5$) through the different constraint equations that bind them, and

thus deduce from Eq. (78) the corresponding NPDEs as well as their exact solutions given by Eqs. (59, 60, ..., 77).

Considering for example, the values $n_0=1, n_1=3, n_2=2, n_4=2$, we obtain from the constraint equation coefficients ($n_1 n_4 - 3n_2 n_3 = 0, 2n_5 + 3n_4 = 0$) and $n_3=1$ and $n_5=-3$. One of the solutions of Eq.(78) given by Eq.(60) reads

$$E(z, t) = \left\{ a + b \cos \mu [t - (-\mu^2 + 3)z] \right\} \times \exp -i(-2z + t) \quad (79)$$

Other wise, when the coefficients $(n_0 = 1, n_1 = 3, n_2 = 1, n_3 = 2, n_4 = 2, n_5 = 1)$ with $\alpha = 1$ are chosen respecting the constraints $n_3(2n_5 + 3n_4) > 0, n_0 \neq 0, n_3(n_5 + n_4) \neq 0$, Eq.(2) becomes

$$E_z + 3iE_{tt} + i|E|^2 E + 2E_{ttt} + 2(|E|^2 E)_t + (|E|^2)_t E = 0 \quad (80)$$

Based on these values, the pulsed solitary wave solution of Eq. (80) given by Eq. (53), is written

$$E(z, t) = \pm \sqrt{\frac{3}{2}} \operatorname{sech} \left(t - \frac{7}{2}z \right) \times \exp -i \left(-\frac{1}{2}z + \frac{1}{2}t \right) \quad (81)$$

5. Conclusion

We have established in this work the conditions to obtain all partial differential equations of the same family as the Schrodinger nonlinear equation describing the dynamic of the propagation of a soliton wave in a strongly nonlinear optical fiber. In practical terms, the results obtained can be part of a framework in which the properties or components of the optical fiber are modified so as to choose a solitary wave or any type of signal that can propagate in the transmission medium thus constituted, with months of dispersion and dissipation, simply because the coefficients related to these terms have been judiciously selected.

In a purely mathematical sense, this way of investigating strongly nonlinear Schrödinger equations allows not only to determine the exact solutions, but also new partial differential equations of the same family at the same time with their exact solutions.

To bring closer to physical reality in a practical sense, we have used numerical simulations to verify the propagation of some solutions obtained. The propagation of some solutions obtained reassures when to the practical and experimental application of our study and our obtained results.

The depth of this work lies in the possibility for engineers specialized in the construction of transmission media and more specifically optical fibers, to set up new optical fibers whose properties are adapted to the types of signals that we want to propagate, among which solitary waves.

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