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Chimera States in Three Populations of Pendulum-Like Elements with Inertia

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Abstract: The aim of this study is to investigate the chimera states in three populations of pendulum-like elements with inertia in varying network topology. Considering the coupling strength between oscillators within each population is stronger than the inter-population coupling, we search for the chimera states in three populations of pendulum-like elements under the ring and the chain structures by adjusting the inertia and the damping parameter. The numerical evidence is presented showing that chimera states exist in a narrow interval of inertia in ring and chain structures. It is found that chimera states cease to exist with the decreasing of damping parameter. Furthermore, it is revealed that there is a linear relationship between the inertia (m) and damping parameter threshold (ε_{th}) in the two network structures.

Keywords: Chimera States, Inertia, Network Topology

1. Introduction

The phenomenon of chimera states in the network of coupled, identical oscillators has attracted a great deal of theoretical and experimental interest. It corresponds to the spatiotemporal patterns, in which some oscillators exhibit coherent dynamics while the others are incoherent in the system. The existence of this remarkable state where the population of identical oscillators split into two parts: one is synchronized and the other is desynchronized were first discovered in 2002 by Kuramoto and Battogtokh [1], and in 2004 Abrams and Strogatz investigated the phenomenon analytically in a ring of phase oscillators coupled by a cosine kernel and defined it chimera state [2].

Since then, several groups have studied the nonlinear dynamics of chimera states in spatiotemporal systems with theories and experiments [1-26]. In 2008, Abrams and Strogatz found chimera states for identical coupled Kuramoto phase oscillators in network of two populations with strong coupling in each population and weaker coupling between them [3]. In 2010, Martens have found chimera states in the case of triangular network [4, 5]. More recently, the systems of phase oscillators with inertia have been studied theoretically [18-21]. In 2014, Bountis investigated the chimera states in two coupled populations of penludum-like elements and found the states occur within a narrow interval of small values of inertia [18]. In particular, chimera states have been shown to emerge in heterogeneous phase lag in the network of two coupled phase oscillators [22-24]. Furthermore, recent theoretical works have also explored chimera states and discussed the dynamics with different frequency distributions [25, 26].

In this paper, the chimera states in three coupled populations of pendulum-like elements in the network of ring and chain structures are investigated. Considering the oscillators within each population are coupled stronger than the neighboring populations and the oscillators oscillate at the same natural frequency and fixed phase lag, we discuss the variations of chimera states by adjusting the inertia and the damping parameter. This paper is organized as follows. In Section 2 the model system is presented and the initial conditions are given. In Section 3, the results of simulations by adjusting the inertia m in network of ring and chain structures are presented. The effects of damping parameter ε on the two network topologies are considered and the relationship between m and ε are analyzed. Section 4 summarizes the findings.

2. Model

The network of three populations of pendulum-like elements is considered. The dynamics of the network is governed by the following equations:

$$\mathrm{m}\frac{d^2\theta_i^\sigma}{dt^2}+\varepsilon\frac{d\theta_i^\sigma}{dt}=\omega-d_\sigma\sin(\theta_i^\sigma)+\sum_{\sigma'=1}^{3}\frac{K_{\sigma\sigma'}}{N_{\sigma'}}\sum_{j=1}^{N_{\sigma'}}\sin(\theta_j^{\sigma'}-\theta_i^\sigma-\alpha) \quad (1)$$

where i denotes the individual oscillators in each population with superscripts $\sigma\in\{1,2,3\}$. The network has N_σ oscillators in each population and the phases of the oscillator are defined by θ. m is the inertia term introduced to each oscillator, d_σ is the gravitational force and α is a fixed phase lag. Here, we keep the natural frequency $\omega=0$. The coupling kernel $K_{\sigma\sigma'}$ describes the coupling strength between populations σ and σ'. We follow previous studies on chimera states [18] and set the coupling strength between the oscillators within each population $K_{\sigma\sigma'}=0.6$, while neighboring populations couple with weaker strength $K_{\sigma\sigma'}=0.4$, such that the inter-population coupling is weaker than the coupling within each subpopulation. In this system, we keep $N=64$, $d_1=d_2=d_3=0$, and $\alpha=\pi/2-0.05$ constant and adjust ε between 0 and 1. And in following simulations we use random initial conditions for oscillators in each population within $(0,2\pi)$.

3. Numerical Simulations and Discussions

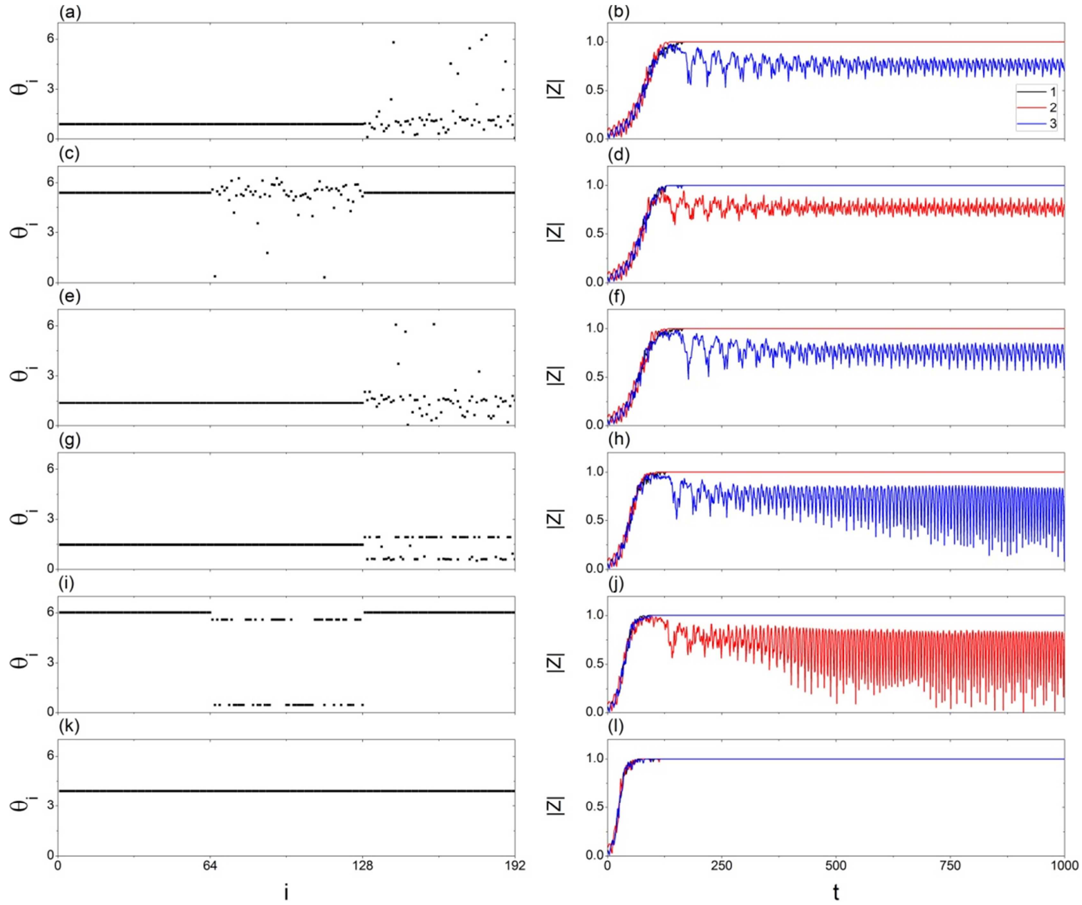

Figure 1. *(Color Online) The left column are snapshots of the variable θ_i for oscillators in all populations, and the right column show order parameters $Z_\sigma(t)$ correspondingly, for different values of inertia m of a ring structure. (a) $m=0$, (c) $m=0.006$, (e) $m=0.012$, (g) $m=0.05$, (i) $m=0.1$, (k) $m=0.2$. The oscillators in population 1, 2 and 3 are numbered 1 to 64, 65 to 128 and 129 to 192, respectively. The black, red and blue curves for $Z_\sigma(t)$ represent population 1, 2 and 3, respectively.*

To start with, the influence of inertia m on chimera states in parameter space is investigated by increasing the value of m continuously while keeping other parameters fixed. Here, we consider the case of ring structure network composed of three populations connect in head to tail, and discuss the effects of inertia on the network behavior. The snapshots of the phase θ_i of oscillators for all three populations at $t=1000$ with different values of inertia m are presented in the left panels of Figure 1. As shown in Figure 1(a), we present the snapshot of the phases θ_i for all the oscillators without inertia ($m=0$) at $t=1000$.

The chimera states can be clearly observed, with the dynamics in one of the three populations shown disordered behavior, and the other two populations are in coherent motion (synchronized). When increasing the value of inertia m up to 0.006 (Figure 1(c)) and 0.012 (Figure 1(e)), we observe that there is always one disordered population appearing in the whole system, and the disordered population can be anyone of the three populations. By keeping increasing the inertia up to $m = 0.05$, we find chimera states cease to exist. It is observed that the oscillators in one of the populations appearing a two layered coherent structure, while the other two populations keep the synchronized dynamics, as shown in Figure 1(g) and (i) for $m = 0.05$ and $m = 0.1$, respectively. Then, as the inertia m further increasing up to 0.2, the whole system reachesfully synchronization, with each population synchronized with one another, as shown in Figure 1(k). By keeping increasing m, there are no sign of chimera states anymore. This is in accordance with previous investigation for the two populations case, where chimera states exist for small mass values $m \ll 1$ [18].

In order to measure the degree of coherence for the oscillators in each population, we introduced the order parameter $Z_\sigma(t) = N^{-1}\sum_{j=1}^{N} e^{i\theta_j^\sigma(t)}$, where $\sigma = 1,2,3$ denotes the population 1, 2, 3, respectively. The modulus of order parameter $|Z| = 1$ denotes the coherent (synchronized) motion, while $|Z| < 1$ quantifies the incoherent (disordered) motion. The right panels show the evolution of $Z_\sigma(t)$ for different values of inertia in correspondence to the left panels. As can be seen in Figure 1(b), for $m = 0$, the modulus of order parameter for the 3-rd population $|Z_3|$ is well below 1, while for the 1-st and 2-nd populations, $|Z_1|$ and $|Z_2|$ approach 1, which means the 3-rd population is disordered and the population 1 and 2 are coherent. Remarkably, it is noticed that in a short period of time from the beginning, $|Z_{1,2,3}|$ increase rapidly from 0, and approach 1. Then $|Z_3|$ starts to derivate from the other two $|Z_{1,2}|$ and oscillate over time below 1. Figure 1(d) and (f) are similar to Figure 1(b). For Figure 1(h), the variations of order parameters $|Z_{1,2}|$ reach to 1 with time and the dynamic of $|Z_3|$ is irregular. We note that the order parameter can not characterize the two layered coherent structure for the case $m = 0.05$ and $m = 0.1$. For $m = 0.2$ in Figure 1(l), the system reaches to fully synchronization, so that all the order parameters reach to 1 with time. Besides, we calculate for higher inertia and find that oscillators escape from the coherent oscillations in each population. This leads to a higher degree of disorder of the system and breaks the chimera state.

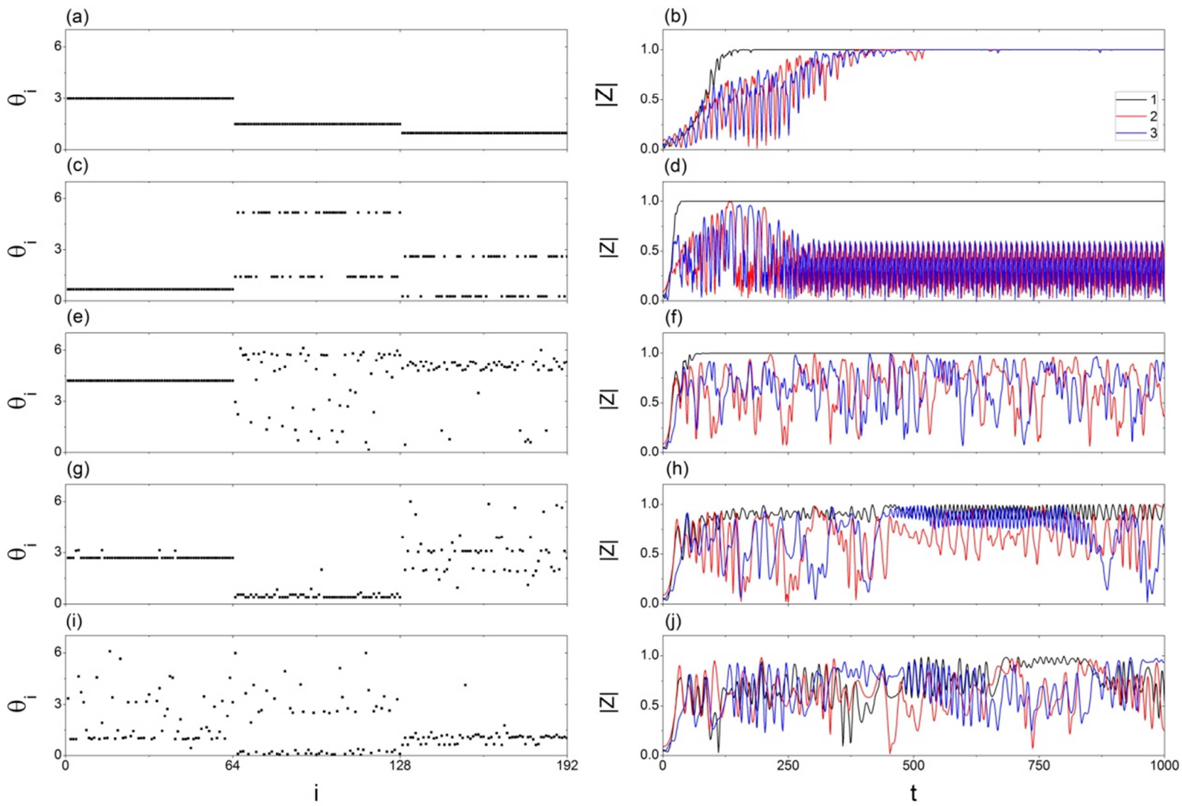

Figure 2. *(Color Online) The left column are snapshots of the variable θ_i for oscillators in all populations for different values of inertia m of a chain structure, and the right column show order parameters $Z_\sigma(t)$ correspondingly. (a) $m = 0$, (c) $m = 0.5$, (e) $m = 4$, (g) $m = 8$, (i) $m = 12$. The oscillators in population 1, 2 and 3 are numbered 1 to 64, 65 to 128 and 129 to 192, respectively. The black, red and blue curves for $Z_\sigma(t)$ represent population 1, 2 and 3, respectively.*

In previous section, the influence of inertia on the emergence of chimera states in a ring structure is discussed. For a network with three nodes, there are two basic types of structure, a ring topology and a chain topology. To investigated the effects of different topology on emergence of chimera state, in the following, we will discuss a chain topology formed by the three populations. The network in chain topology can be created by simply setting the coupling strength in between the 2-nd population and 3-rd population of the above ring topology to zero. For the three populations in a chain topology, we will increase the value of inertia m, while keeping other parameters fixed, to observe the dynamical evolution of the system. The results are shown in Figure 2. In the left panels, we present the snapshots of the phase θ_i of oscillators for all three populations at $t=1000$ with different values of inertia m. As shown in Figure 2(a), we note that the oscillators without inertia in three populations show synchronization in each population, however, the three populations are not synchronized with each other. This is different from the ring structure results[See Figure 1(k)]. It seems to be easier for the three populations in ring structure to reach fully synchronization, but not in chain structure. A simple explanation is that in the ring structure the oscillators are better connected than in the chain structure for them to reach full synchronization. For $m=0.5$, as shown in Figure 2(c), the dynamics of oscillators in population 1 are completely coherent, while the other two populations both show the two layered coherent structure. When the value of inertia is continuously increased up to 4, a clear image of chimera states appears in Figure 2(e), in which population 1 is coherent and the other two populations are disordered. Then, as we increase the value of m continuously, chimera states can still be observed up to $m=8$, as shown in Figure 2(g). However, it is found that there are few oscillators escape from the synchronized population 1. As we continue to increase the inertia, more and more oscillators escape from the synchronized population, and there are no clear sign of chimera states above $m=12$, see Figure 2(i). It is appearant that chimera states exist in different parameter space for the two different network structures. For the chain structure, chimera states will appear with $m>1$. The right panels show the variation of $Z_\sigma(t)$ for the different values of inertia in correspondence to the left panels. As can be seen in Figure 2(b), all three modulus values of order parameter approach 1 with time, which means all the populations are coherent (synchronized). For Figure 2(d), it is observed that $|Z_1|$ increases rapidly from 0 to 1, while the other two $|Z_{2,3}|$ oscillate irregularly between 0 and 1 in the beginning and then oscillate periodically with time below 1. In Figure 2(f), it can be seen that $|Z_1|$ reaches to 1 with time and the other two $|Z_{2,3}|$ oscillate below 1 irregularly. For $m=8$ as shown in Figure 2(h), the variation of order parameter for 1-st population oscillates around 1 over time and the other two $|Z_{2,3}|$ are oscillating below 1 irregularly. As the inertia m further increases, the degree of disorder of the system becomes higher, so the variations of modulus of order parameters for three populations oscillate below 1 for $m=12$ in Figure 2(j). Furthermore, we have tried to simulate with higher inertia and the system has always been disordered.

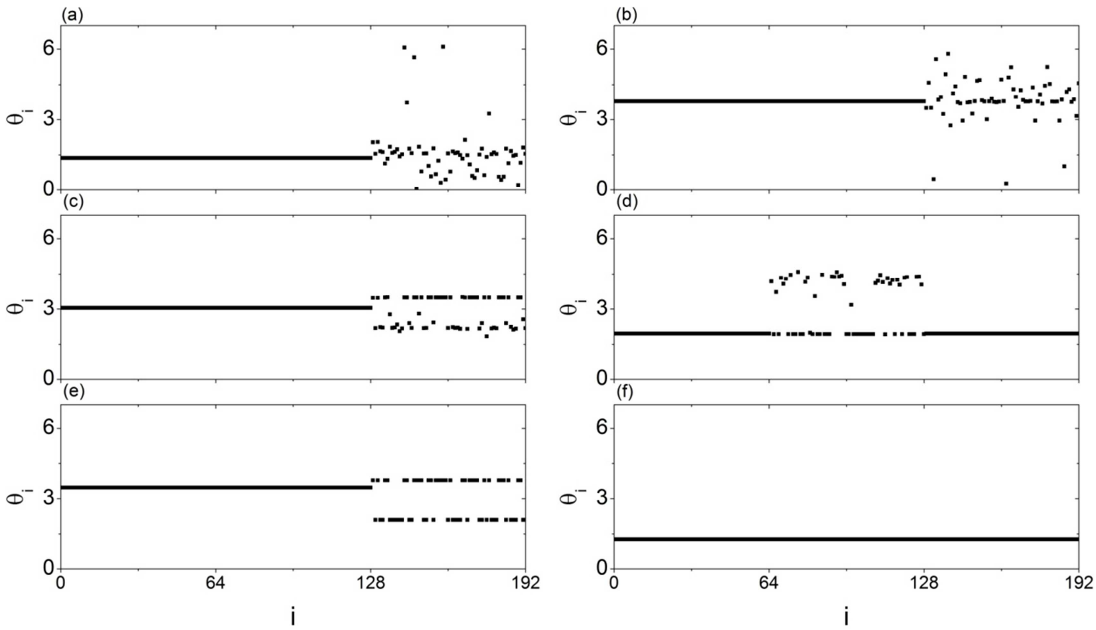

Figure 3. Snapshots of θ_i at $t=1000$ for $m=0.012$ in the ring structure with different damping parameters: (a) $\varepsilon=1$, (b) $\varepsilon=0.9$, (c) $\varepsilon=0.65$, (d) $\varepsilon=0.55$, (e) $\varepsilon=0.5$, (f) $\varepsilon=0.3$. The oscillators in population 1, 2 and 3 are numbered 1 to 64, 65 to 128 and 129 to 192, respectively.

In the above section, we have discovered that chimera states exist in both ring and chain structures, and it will disappear with increasing inertia m. Here, the impact of the damping parameter ε with a fixed inertia in the ring structure is discussed. We decrease the damping parameter ε gradually for the case $m = 0.012$ in Figure 1(e) and present the corresponding snapshots of the phase θ_i of oscillators for all three populations at $t = 1000$. In Figure 3(a) and (b), it is shown that the 1-st and 2-nd populations are coherent, while the 3-rd population is disordered and chimera states can be observed clearly for $\varepsilon = 1$ and $\varepsilon = 0.9$, respectively. Tuning down ε to 0.65, we observe that the 1-st and 2-nd populations keep coherence (synchronized) and the imperfect two layered structure appeared in the 3-rd population. By further decreasing the damping parameter, the two layered coherent state pertains, see results for $\varepsilon = 0.55$ (Figure 3 (d)) and $\varepsilon = 0.5$ (Figure 3 (e)). To decrease the value of ε further, we find that all the populations are fully synchronized in the ring structure, as shown in Figure 3(f) for $\varepsilon = 0.3$. Therefore, below the threshold value of ε, it is found that one of the populations in the system appears the two layered coherent structure while the other two populations are coherent (synchronized) or all of the three populations are fully synchronized, and chimera states no longer exist.

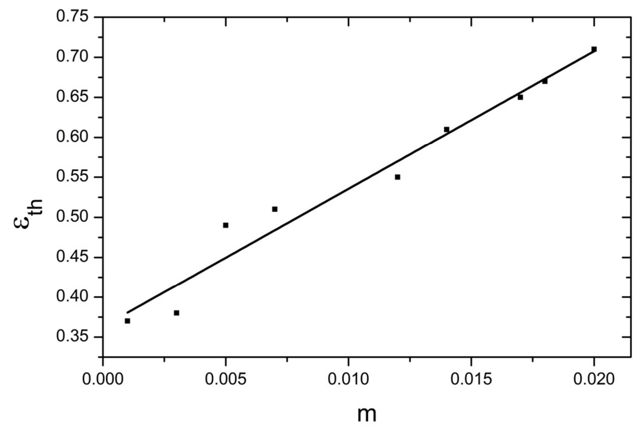

Figure 4. *Threshold values $\varepsilon_{th} = \varepsilon(m)$, below which chimera states cease to exist in the ring structure.*

To better characterize the influence of the damping parameter ε on the dynamics of the ring structure, we have repeated the calculations for different values of m and showed the phase diagram of (m, ε_{th}) in Figure 4. We choose $m = 0.001$, 0.003, 0.005, 0.007, 0.012, 0.014, 0.017, 0.018, 0.02, and obtain the corresponding threshold value $\varepsilon(m) = 0.37$, 0.38, 0.49, 0.51, 0.55, 0.61, 0.65, 0.67, 0.71. When the value of ε is above ε_{th}, chimera states can be observed, otherwise chimera states cease to exist. It is worth noting that ε_{th} varies almost linearly with m.

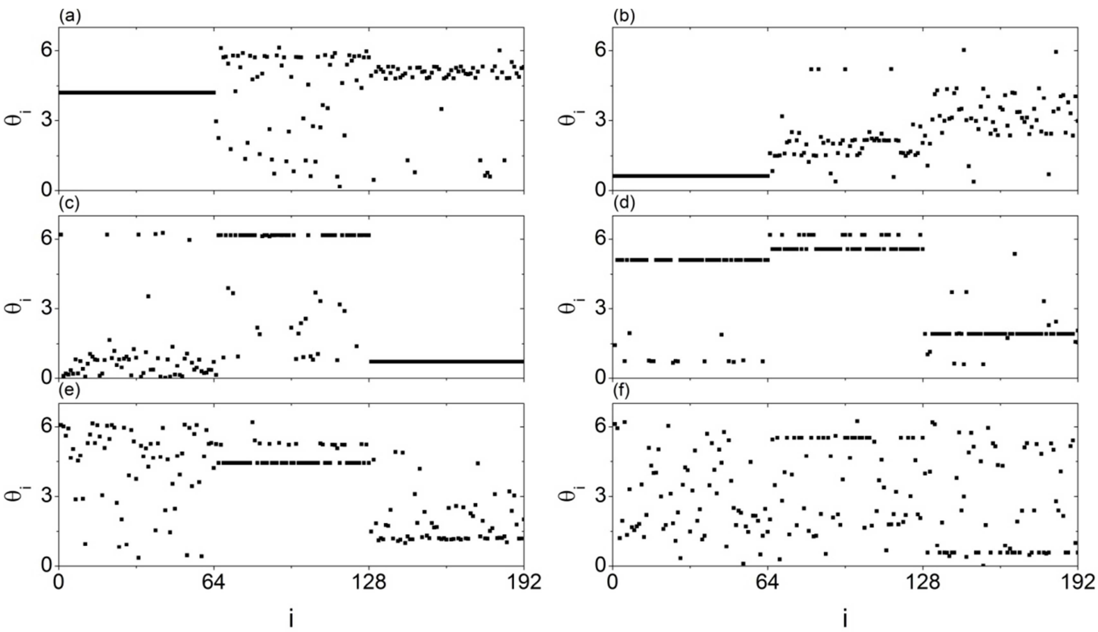

Figure 5. *Snapshots of θ_i at $t = 1000$ for $m = 4$ in the chain structure with different damping parameters: (a) $\varepsilon = 1$, (b) $\varepsilon = 0.8$, (c) $\varepsilon = 0.7$, (d) $\varepsilon = 0.6$, (e) $\varepsilon = 0.4$, (f) $\varepsilon = 0.1$. The oscillators in population 1, 2 and 3 are numbered 1 to 64, 65 to 128 and 129 to 192, respectively.*

In this section, the impact of the damping parameter ε with a fixed inertia in the chain structure is discussed. We consider the case $m = 4$ by decreasing the damping parameter ε continuously and present the corresponding snapshots of the phase θ_i of oscillators for all three populations at $t = 1000$ in Figure 5. As shown in Figure 5(a) and (b), it is observed that the 1-st population is coherent, while the 2-nd and 3-rd populations are disordered. Chimera states can be observed clearly for $\varepsilon = 1$ and $\varepsilon = 0.8$, respectively. By decreasing ε to 0.7, the 3-rd population show coherent dynamics and the other two populations are disordered in Figure 5(c). We can see that some oscillators in the 2-nd population exhibit coherent behavior. By further decreasing the damping parameter, a clear image of the two layered coherent structure occurs in the 2-nd population, meanwhile, a small portion of oscillators escapes from the coherent in population 1-st and 3-rd, see Figure 5(d). Then, by decreasing the value of ε further and it is found that all the populations are disordered in the chain structure, as shown in Figure 5(f) for $\varepsilon = 0.1$. Therefore, it suggests that more and more oscillators escape from the coherent population as ε decreasing continuously below the threshold value, so that chimera states cease to exist.

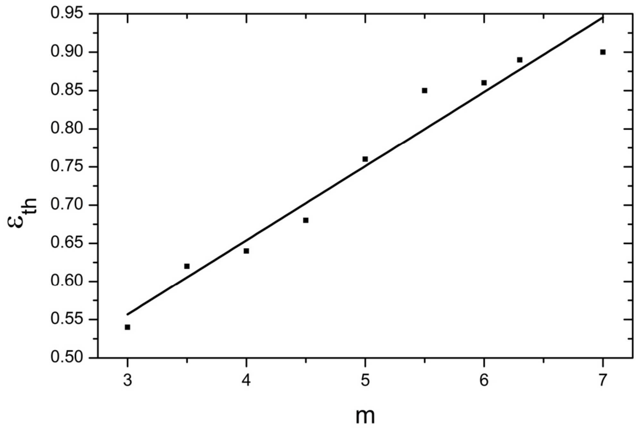

Figure 6. *Threshold values $\varepsilon_{th} = \varepsilon(m)$, below which chimera states cease to exist in the chain structure.*

In the following, we want to explore how the damping parameter ε and the inertia m influence the emergence of chimera states in the chain structure. We have calculated with different values of $m = 3$, 3.5, 4, 4.5, 5, 5.5, 6, 6.3, 7, and plotted the phase diagram of (m, ε_{th}) in Figure 6 with corresponding threshold value $\varepsilon(m) = 0.54$, 0.62, 0.64, 0.68, 0.76, 0.85, 0.86, 0.89, 0.9, respectively. When $\varepsilon > \varepsilon_{th}$, we can observe the chimera states in a continuous parameter space of ε, otherwise not. All the threshold values ε_{th} corresponding to different inertia m fall on roughly a straight line.

4. Conclusions

In this paper, the appearance of chimera states in three populations in the ring and the chain network structures composed of three populations by adjustment of inertia m and damping parameter ε is investigated. In the network of ring structure, chimera states are numerically observed for sufficiently small value of inertia $0 < m < 1$, while chimera states exist in chain structure with the inertia $m > 1$. Clearly, chimera states occur in a narrow interval of inertia for the two different network structures. Our results suggest that the system will vary from chimera states to fully synchronized by increasing m or decreasing ε in ring structure. And with increasing m or decreasing ε in chain structure, there will be more and more oscillators escape from the coherent population, so that the system becomes disordered and chimera states cease to exist. Interestingly, as m increases or ε decreases, the two layered coherent structure appears in both two network structures. Furthermore, we find the corresponding damping thresholds for different value of inertia and demonstrate the nearly linear relationship between m and ε_{th}. This linear relationship has been known to exist for the two populations case. We have also tried to simulate with different numbers of oscillators in each population, such as $N = 32$, $N = 128$, and found that the conclusions applies to larger system as well.

Acknowledgements

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