American Journal of Physics and Applications
2022; 10(2): 24-32
http://www.sciencepublishinggroup.com/j/ajpa
doi: 10.11648/j.ajpa.20221002.11
ISSN: 2330-4286 (Print); ISSN: 2330-4308 (Online)

A Damage Mechanics Analysis on Rheological Failure of Rocks Under High Temperatures and Pressures

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To cite this article:
Zuan Chen, Zhihe Jin, Xiaoge Huang, Shengwen Qi. A Damage Mechanics Analysis on Rheological Failure of Rocks Under High Temperatures and Pressures. *American Journal of Physics and Applications*. Vol. 10, No. 2, 2022, pp. 24-32.
doi: 10.11648/j.ajpa.20221002.11

Received: February 16, 2022; **Accepted**: March 3, 2022; **Published**: March 11, 2022

Abstract: During rheological process of rocks at high temperatures and pressures, damage occurs when the visco-plastic strains accumulate to a certain level. Damage is assumed to start accumulating at the onset of tertiary creep. The evolution of damage can lead to localized deformation and eventual failure of the rock. This paper develops a damage constitutive relation and damage evolution equation for rheological failure of rocks based on the theory and method of damage mechanics. A method of determining the material constants in the constitutive relation and damage evolution equation is proposed and employed to estimate the parameters for marble based on the experiment results. One numerical example explaining deep earthquake occurrence is presented to illustrate the application of the constitutive relation and damage evolution equation. The numerical results indicate that the proposed constitutive and damage equations are capable of predicting earthquake occurrence based on the shear stress evolution. The proposed damage constitutive relation and damage evolution equation for rheological failure of rocks provide a theoretical base for numerical calculation simulating geodynamics process inside earth interior.

Keywords: Rock Rheology, Damage, Constitutive Equation, Evolution Equation

1. Introduction

Mechanical properties of geological materials depend on temperature and pressure. Under low temperatures and pressures, rocks deform elastically and undergo failure in a brittle manner. Under high temperatures and pressures, rocks undergo rheological deformations and fail in a ductile manner. Under intermediate temperature and pressure conditions, rocks exhibit deformation and failure behavior between the two extreme cases [37].

Rocks in the upper lithosphere of Earth contain many defects such as dislocations, grain boundaries and microcracks. Under certain conditions of temperature and pressure, material damage will take place around the defects. It is thus necessary to use constitutive equations considering damage in analyzing deformation and failure processes in Earth's interior. The theory of damage mechanics has been well developed and has been used to investigate rock behavior at shallow depths. In deep interior of Earth, research works on geodynamics process using damage mechanics are also found in literatures. It is generally understood [3, 8, 25, 29] that damage represents the characteristics of microcrack/microvoid distribution in the material. Consequently, damage may be described as the damaged section divided by the original intact section [7] and be used to define the effective stress in a damaged material [26]. However, damage evolution in rocks under high temperatures and pressures is still poorly understood. The existing models on rock damage considered several aspects including thermodynamics of partial melting materials [1-2, 24, 33], phenomenological theory of material softening [17, 34], and brittle damage in crustal rocks [4-5, 14-15, 29-31]. Those models may be applied to investigate the behavior of geological materials undergoing elastic deformations. The models, however, may not be applied to study permanent elasto-viscoplastic deformations that occur in rocks under high temperatures and pressures. Karrech et al. [23] developed a continuum damage theory that can deal with nonlinear visco-plastic deformations. Bercovici and Ricard [2] discussed grain-damage hysteresis and plate tectonic states. However, under conditions of high temperature, high pressure and low differential stress, theoretical and experimental investigations are mostly concerned with steady state process of rock rheology which is generally described by the power law creep relation. Tertiary creep which leads to final failure has mostly been ignored.

Material constants in the creep constitutive equations are usually determined using creep and constant strain-rate tests. In a constant strain-rate test, the relation between the differential stress and strain in a specimen is obtained with strain rate kept as a constant. In a creep test, the relation between strain and time is obtained with differential stress kept as a constant [16]. Figure 1A illustrates a creep curve at constant differential stresses. For a given constant stress, an initial strain is first produced by the stress. The strain then continuously increases with time. In the first stage after loading, the strain rate decreases with time and this period is called primary creep. In the second stage, the strain rate is a constant and the region is called steady state creep or secondary creep. In the final stage of creep called tertiary creep, the strain rate increases with time and creep failure or rupture occurs at the end of tertiary creep. Figure 1B shows a stress-strain curve at a constant strain rate. In this case, the stress increases linearly with strain before reaching a yield point, which is followed by a region of strain hardening. The curve also shows that the strain hardening weakens at higher strain levels until steady-state flow is attained. Rupture of the sample may terminate steady-state flow at high strains, but a region of strain softening may intervene between steady-state flow and rupture. These results show the correlation between constant strain rate and creep tests as discussed in Heard [16].

As we know, the steady-state process of creep is universal in Earth's interior. Tertiary creep and creep failure are also common and very crucial in some geological events, for example, rock failure associated with magma transport in upper mantle [6]. The present work is concerned with the constitutive equations that describe tertiary creep behavior of rocks under high temperatures and pressures as well as low differential stress. First, a damage constitutive relation and damage evolution equation for rheological failure process of rocks are presented based on the theory and analytical method of damage mechanics which deals with dissipation process of rock deformations. Second, a method to determine the physical parameters in the equations is described. Finally, one numerical example is presented to illustrate the application of the model to creep failure of rocks. The research work provides a basis for simulating creep deformation and failure in interior Earth.

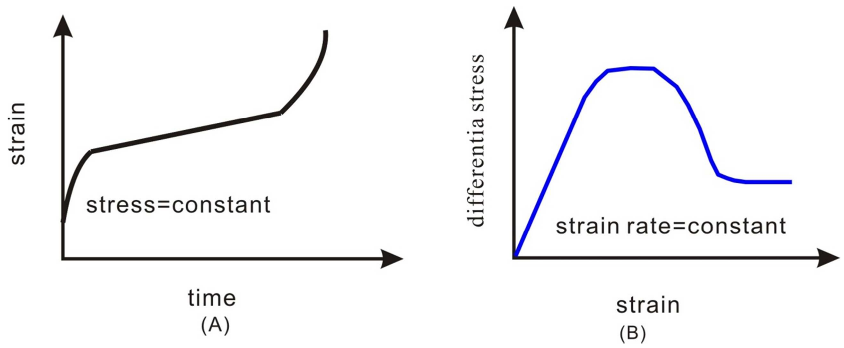

Figure 1. *Schematic relations between constant strain-rate and creep tests. (A) creep curves plotted on usual strain-time coordinates; (B) stress-strain curves for constant strain-rate tests.*

2. Damage Constitutive Equation for Rocks in Rheological Failure Process

Under high temperature and pressure conditions, creep of crustal and upper mantle rocks is often described by the following constitutive relation [38]:

$$\dot{\varepsilon} = A\sigma^{n} \exp\left(-\frac{E + pV}{RT} \right) \tag{1}$$

in which ε $\dot{\varepsilon}$ is the shear strain rate, σ the differential stress, E the activation energy, p the pressure, V the activation volume, R the universal gas constant, T absolute temperature, and A and n material constants.

Equation (1) describes the rheological deformation of steady state creep (also called secondary creep) for constant strain rate under constant differential stress. While the steady state creep generally represents the creep stage of the longest duration, rock failure occurs after tertiary creep that follows the secondary creep. During tertiary creep, the strain rate increases rapidly until rock failure and deformations of rock at the microscopic level are localized, which lead to severe microscopic damages. It is thus essential to include damage in the constitutive relations at this stage of creep to precisely predict the failure behavior of rocks under high temperature and pressure conditions.

Consider elasto-viscoplastic deformation of a rock. The total strain increment consists of the elastic and inelastic strain increments as follows [22]:

$$\mathrm{d}\varepsilon = \mathrm{d}\varepsilon^{\mathrm{e}} + \mathrm{d}\varepsilon^{\mathrm{in}} \tag{2}$$

where ε represents strain tensor, and the superscripts *e* and *in* represent the elastic and inelastic strains, respectively. This also is the assumption of the Maxwell visco-elastic model. The elastic strain tensor is related to the stress by the generalized Hooke's law as follows:

$$\sigma = C : \varepsilon^e \tag{3}$$

Where σ is the Cauchy stress tensor and C is the fourth-order elasticity tensor for the damaged material. In a damaged material, an "effective" stress applied on the actual undamaged skeleton is defined under the state of uniaxial stress as follows [20, 25, 29-30]

$$\tilde{\sigma} = \frac{\sigma}{1-\omega} \tag{4}$$

Where σ is the uniaxial stress and ω is the so-called damage variable which can be understood as the ratio of damaged portion of the section to the entire section on which the stress is applied. In this work we assume isotropic damage and use a linear elastic damage model of Simo [36]. Eqs. (3) and (4) can now be employed to obtain the following stress-elastic strain-damage relationship:

$$\sigma = C : \varepsilon^e = (1-\omega)C^0 : \varepsilon^e \tag{5}$$

Where C^0 is the fourth-order elasticity tensor for the original intact material. For isotropic rocks, the elasticity tensor has the coefficients:

$$C_{ijkl} = \left(K - \frac{2}{3}G\right)\delta_{ij}\delta_{kl} + G\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$

where K is the bulk modulus, G the shear modulus, δ_{ij} the Kronecker symbol, and the indices (i, j, k, l) have the range of 1, 2 and 3.

The relation between the inelastic strain and stress may be derived using a dissipation potential Φ. The potential depends on both observable variables (such as strain and temperature) and internal variables (such as damage, elastic strain, and other dissipation quantities). According to the thermodynamics theory for irreversible deformation processes, the dissipation potential Φ may be expressed as follows:

$$\Phi = \Phi(\dot{\boldsymbol{\varepsilon}}^{in}, \dot{\omega}, \frac{\boldsymbol{q}}{T}; \boldsymbol{\varepsilon}^{in}, \omega, T) \tag{6}$$

Where ε^{in} is the inelastic strain tensor, ω is the damage variable, T is temperature, and q is the heat flux. Then, we may obtain the following constitutive relationship

$$\begin{aligned}
&\boldsymbol{\sigma} = \frac{\partial \varphi}{\partial \boldsymbol{\varepsilon}^{in}}; \quad \text{or } \sigma_{ij} = \frac{\partial \varphi}{\partial \varepsilon_{ij}^{in}}, i, j = 1,2,3 \\
&Y = \frac{\partial \varphi}{\partial \dot{\omega}}; \\
&\nabla T = -\frac{\partial \varphi}{\partial \left(\dfrac{\boldsymbol{q}}{T}\right)}; \quad \text{or } \frac{\partial T}{\partial x_i} = \frac{\partial \varphi}{\partial \left(\dfrac{q_i}{T}\right)}, i = 1,2,3
\end{aligned} \tag{7}$$

Where σ is the stress tensor, or a force driving irreversible deformation; Y is the conjugate of the damage variable or the strain energy density release rate.

Based on Legendre transformation, dual dissipative complementary potential ψcan be defined as follows:

$$\psi = \psi(\boldsymbol{\sigma}, Y, \nabla T; \boldsymbol{\varepsilon}^{in}, \omega, T) \tag{8}$$

Similarly, the following constitutive relationship may be obtained:

$$\begin{aligned}
&\dot{\boldsymbol{\varepsilon}}^{in} = \frac{\partial \psi}{\partial \boldsymbol{\sigma}}; \quad \text{or } \dot{\varepsilon}_{ij}^{in} = \frac{\partial \psi}{\partial \sigma_{ij}}, i, j = 1,2,3 \\
&\dot{\omega} = \frac{\partial \psi}{\partial Y}; \\
&\frac{\boldsymbol{q}}{T} = -\frac{\partial \psi}{\partial (\nabla T)}; \quad \text{or } \frac{q_i}{T} = -\frac{\partial \psi}{\partial \left(\dfrac{\partial T}{\partial x_i}\right)}, i = 1,2,3
\end{aligned} \tag{9}$$

For simplicity, we assume that both the dissipative potential and dissipative complementary potential are uncoupled among various dissipative mechanisms for damaged rocks. Therefore, the dissipative complementary potential can be partitioned into two parts, i.e., damage dissipative complementary potential ψ_ω and inelastic visco-plastic dissipative complementary potential ψ_n as follows:

$$\psi = \psi_n + \psi_\omega(\sigma, Y) \tag{10}$$

Where σ is the stress tensor and Y is the strain energy density release rate related to damage rate as shown as equation (7). Because we only consider the rheological deformation process of rocks at constant temperature, the internal variable T (temperature) can be dropped out in Eq. (10).

The damage dissipative complementary potential and visco-plastic dissipative complementary potential for time-dependent visco-plastic problems may be expressed in the following form [28] (Liu and Hao, 2011):

$$\begin{aligned}
\psi_\omega &= \frac{1}{2}\frac{Y_\Pi^2}{S_0}\frac{\dot{P}}{(1-\omega)^{\alpha_0}} \\
\psi_n &= \frac{K_0(1-\omega)}{N+1}\left[\frac{\sigma_{ep}}{K_0(1-\omega)}\right]^{N+1}
\end{aligned} \tag{11}$$

where S_0, α_0, K_0, n are material constants; Y_Π is a damage dual force for isotropic materials; and:

$$\sigma_{ep} = \left[\frac{3}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}'\right]^{\frac{1}{2}}; \boldsymbol{\sigma}' = \left[\boldsymbol{\sigma} - \frac{1}{3}tr(\boldsymbol{\sigma})I\right] \tag{12}$$

$$\dot{P} = \sqrt{\frac{2}{3}(\dot{\boldsymbol{\varepsilon}}^{in} : \dot{\boldsymbol{\varepsilon}}^{in})} = \sqrt{\frac{2}{3}(\dot{\varepsilon}_{ij}^{in} : \dot{\varepsilon}_{ij}^{in})}, i.j = 1,2,3$$

Where σ_{ep} is the effective stress and $\dot{P}$ is the effective inelastic strain rate.

Then the constitutive equations for rock creep and damage can be written as follows:

$$\dot{\boldsymbol{\varepsilon}}^{in} = \frac{\partial \psi_n}{\partial \boldsymbol{\sigma}} = \left[\frac{\sigma_{ep}}{K_0(1-\omega)}\right]^{n} \frac{3}{2}\frac{\sigma'}{\sigma_{ep}} \tag{13}$$

$$\dot{P} = \left[\frac{\sigma_{ep}}{K_0(1-\omega)}\right]^{n} \tag{14}$$

The damage evolution equation may be taken in the following form:

$$\dot{\omega} = \frac{\partial \psi_{\omega}}{\partial Y_{\Pi}}\frac{\partial Y_{\Pi}}{\partial Y} = \frac{\partial \psi_{\omega}}{\partial Y_{\Pi}} = \frac{Y_{\Pi}}{S_0(1-\omega)^{\alpha_0}}\dot{P} \tag{15}$$

If we define the damage variable as the ratio of damaged portion of the section to the entire section on which the stress is applied, the expression of Y_{Π} postulated by Lemaitre [33] under low temperature and low pressure conditions are given as follows:

$$Y_{\Pi} = \frac{\sigma_{ep}^2 R_v}{2E_0(1-\omega)^2} \tag{16}$$

Where E_0 is the elastic module, ν is Poisson's ratio, Rv is given by

$$R_v = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_m}{\sigma_{ep}}\right)^2 \tag{17}$$

in which:

$$\sigma_m = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \tag{18}$$

It is necessary to point out that the relationship for R_v is valid only at the condition of $\sigma_m < \sigma_{ep}$, which implies that the rock undergoes accelerated creep deformation.

In this paper we are concerned with damage evolution in rocks under high temperatures and high pressures. Eqs. (16) and (17) of Lemaitre's model are thus not applicable. In addition, stresses will satisfy a condition that $\sigma_m > \sigma_{ep}$ under high pressures, which implies that the creep undergoes three phases, i.e., transient creep, steady creep and tertiary creep. In this case, damage is assumed to start accumulating at the onset of tertiary creep. An obvious fact is that higher pressures result in smaller increases in the rate of damage. Hence we must modify R_v in Eq. (17) so that the model can be used under high temperature and pressure conditions. We introduce a parameter R'_v related to the confining pressure. The proposed damage evolution equation has the following form:

$$\dot{\omega} = \frac{\sigma_{ep}^2 R'_v}{2E_0 S_0(1-\omega)^{2+\alpha_0}}\left[\frac{\sigma_{ep}}{K_0(1-\omega)}\right]^n \log\left(\lambda_0 + \frac{T}{T_0}\right) \tag{19}$$

where the material constants are defined as follows

$$\begin{aligned} R'_v &= \left(\frac{\sigma_{ep}}{\sigma_m}\right)^2 \\ K_0^{-n} &= A \\ \alpha_0 &= -1, \\ B &= \frac{A}{2E_0 S_0} \end{aligned} \tag{20}$$

In the above equations, T is the temperature; A, B, λ_0 and T_0 are material constants. It is seen that the damage evolution rate may be decreased by higher pressure (associated with σ_m) and low devitoric stress (associated with σ_{ep}) through the parameter R_v'. The relationship between damage accumulation and temperature is proposed here based on some experimental results on granite under high temperatures [9]. Finally, the rheological constitutive equation and damage evolution equation of rock become

$$\begin{aligned} \dot{\varepsilon}_n &= A\left(\frac{\sigma_{ep}}{1-\omega}\right)^n \exp\left(-\frac{E+PV}{RT}\right)\frac{\sigma'}{\sigma_{ep}}, \\ \dot{\omega} &= \frac{B\sigma_{ep}^2 R'_v}{(1-\omega)}\left(\frac{\sigma_{ep}}{1-\omega}\right)^n \log\left(\lambda_0 + \frac{T}{T_0}\right) \end{aligned} \tag{21}$$

The above constitutive equation with damage variable shows that the strain rate satisfies the Lemaitre strain equivalence principle, i.e., the strain tensor induced by stress tensor acting on the damaged material is equivalent to that induced by the effective stress tensor acting on the intact material.

In general damage is not significant during secondary creep at low differential stress levels. We thus can assume that the damage variable is zero, i.e., ω=0, at the onset of tertiary creep although energy dissipative process with other mechanisms such as dislocation and diffusion exist in secondary creep. During tertiary creep, damage starts to evolve from ω=0 to $\omega=\omega_c$ which is the critical damage value at final rupture of material. More detailed analysis about material parameters related to the damage in the rheological process of rocks will be discussed in the next section.

3. Method for Determination of Material Parameters

The material parameters in the damage constitutive equation may be obtained by rheological experiments of rock samples. The parameters in the damage evolution equation and the condition for damage initiation are very difficult to determine because of limitations in time duration in creep tests under conditions of high temperatures and pressures.

Damage and rupture of rocks are closely related to the dissipative energy during the deformation process. We assume that damage begins to occur when the dissipative energy accumulates to a critical value $W_{d,}$. When the dissipative energy further increases to W_p, complete rupture takes place. W_d and W_p may be estimated using the stress-strain curve under constant strain rate. W_d is the plastic work done by the stress before peak point on the stress-strain curve, and W_p is the dissipative work corresponding to the softening part of the stress-strain curve after the peak point. W_p is also called fracture energy. Figure 2 schematically shows the areas that correspond to the two energies, respectively.

Plastic work for pseudo tri-axial rheological test may be described as following:

$$W = \int_0^t \sigma d\varepsilon^{in} = \int_0^t \sigma \frac{d\varepsilon^{in}}{dt} dt \tag{22}$$

where σ and ε represent differential stress and axial strain, respectively, in a creep test. We assume that no damage occurs (i.e., ω=0) and only visco-plastic deformation takes place during secondary creep of the rock. Substituting the constitutive equation with ω=0 into Eq. (22), we have

$$W = \int_0^t \sigma \cdot A\sigma^n \exp\left(-\frac{E+PV}{RT}\right) dt = A\sigma^{n+1} \exp\left(-\frac{E+PV}{RT}\right) t \tag{23}$$

Where parameter A contains a factor $\left(\frac{3}{2}\right)^{n-1}$. Damage in the rock is initiated when $W=E_p$. Using this condition and Eq. (23), the time at which damage begins to occur, t_d, may be determined as follows

$$t_d = \frac{E_P}{A\sigma^{n+1}} \exp\left(\frac{E+PV}{RT}\right) \tag{24}$$

During tertiary creep, damage in the rock grows continuously. Substituting Eq. (21) into Eq. (22), we have:

$$\begin{aligned} W &= \int \sigma \cdot A\left(\frac{\sigma}{1-\omega}\right)^n \exp\left(-\frac{E+PV}{RT}\right) dt \\ &= \frac{2A\exp\left(-\frac{E+PV}{RT}\right)(1-\omega/2)\omega}{3B\sigma R'_V \log\left(\lambda_0 + \frac{T}{T_0}\right)} \end{aligned} \tag{25}$$

Creep failure occurs when $W=E_f$. The critical damage ω_c is determined by the following formula:

$$\omega_c = 1 - \sqrt{1 - \frac{2DE_f\sigma R'_V}{A}} \tag{26}$$

where

$$D = \frac{3B\log\left(\lambda_0 + \frac{T}{T_0}\right)}{2\exp\left(-\frac{E+PV}{RT}\right)} \tag{27}$$

The relationship between creep failure time t_c and ω_c may be obtained from damage evolution equation as follows:

$$\int_0^{\omega_c} \left[-(1-\omega)^{n+1}\right] d(1-\omega) = \int_{t_d}^{t_c} B\sigma^{n+2} R'_V \log\left(\lambda_0 + \frac{T}{T_0}\right) dt \tag{28}$$

Integrating the above equation gives the relation between time t_c and ω_c as follows:

$$\omega_c = 1 - \left[1 - (n+2)BR'_V\sigma^{n+2} \log\left(\lambda_0 + \frac{T}{T_0}\right)(t_c - t_d)\right]^{\frac{1}{n+2}} \tag{29}$$

The material parameter B in the damage evolution equation may be estimated by the softening part of the stress-strain curve under constant strain rate experiment for rocks under low confining pressure. When strain rate is constant, the following relationship may be obtained through rheological constitutive equation and damage evolution equation:

$$\dot{\varepsilon}^n = \frac{A(1-\omega)\dot{\omega}}{B\sigma_{ep}^2 R'_V} \tag{30}$$

The relation between damage ω and time t is obtained as follows:

$$\omega = 1 - \left[1 - (n+2)BR'_V\sigma_s^{n+2} \log\left(\lambda_0 + \frac{T}{T_0}\right)(t - t_d)\right]^{\frac{1}{n+2}} \tag{31}$$

Where σ_s is the peak stress on the stress-strain curve. The constant B may be determined using Eqs. (25) and (26) and curve fitting of the test data.

In summary, the parameters in the damage constitutive equation may be estimated using the creep test results, and the parameters in the damage evolution equation may be estimated using the stress-strain curve of the material.

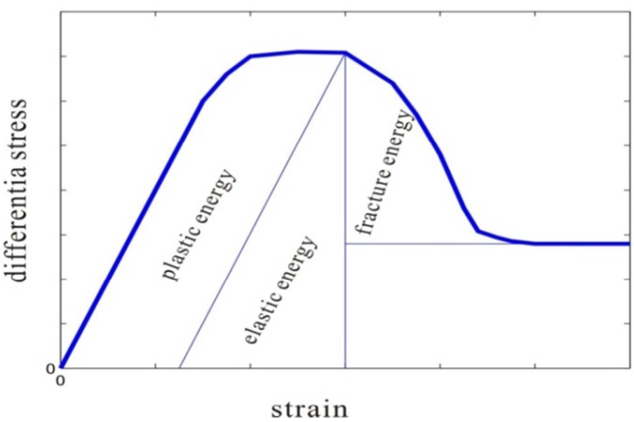

Figure 2. *Sketch of various energy regions.*

4. Determination of Material Parameters Using the Test Results for Marble

In this section, we use marble as an example to show how to determine the material parameters in the constitutive relation and damage evolution equation in Eq. (21).

There are many test results on marble since 1960's [11, 16, 19, 35, 39]. In most of the sixty-four tests reported by Heard [35], jacketed cylinders of Yule marble (oriented parallel, at 45°C and normal to the foliation) were extended 10 percent at temperatures from 25°C to 500°C under 5 kb confining pressure. In the tests of Schmid et al.[35], specimens of Carrara marble were experimentally deformed at temperatures between 600°C and 1050°C and strain rates between 10^{-2} and 10^{-6} see^{-1}. Jin et al. [19] reported their room-temperature experimental results on Yexian marble under cyclic loading with confining pressures ranging from 50 to 200 MPa and a constant strain rate $5 \times 10^{-5}\mathrm{s}^{-1}$. In the work of Rutter [39], Carrara marble was deformed to very large tensile and compressive strains at temperatures ranging between 500°C and 1000°C, and confining pressures of 200 and 300 MPa. Schmid et al. [35] summarized and reviewed some experimental data.

Figure 2 schematically shows the stress-strain curve and associated plastic energy and fracture energy. These energies depend on confining pressure and temperature. The plastic energy increases with increasing confining pressure. The fracture energy increases with increasing confining pressure under low level of pressures, but decreases with increasing confining pressure under higher level of pressures. We propose the following expression to approximate the fracture energy E_f:

$$\mathrm{E_f} = \mathrm{H_1}\exp(-\mathrm{H_2}\sigma_c) - \mathrm{H_3}\exp(-\mathrm{H_4}\sigma_c) + \mathrm{H_5} - \mathrm{k}_\alpha \mathrm{PV_0}(\mathrm{T} - \mathrm{T_r}) \tag{32}$$

Where T is the absolute temperature;k_α is the coefficient of thermal expansion; V_0 is the specific volume of rock at zero pressure; $T_r$is room temperature which is selected as 298.15 K in this paper, σ_c is the confining pressure in MPa, and H_i (i=1, 2, 3, 4, 5) are material constants. The last term in equation (32) is the thermal energy at temperature T, which reduces total fracture energy.

The material parameters H_i (i=1, 2, 3, 4, 5) in equation (32) are determined using the experimental results in Heard [16] and Jin et al. [19]. A non-linear least-squares method is employed and the results are given as follows:

$$\mathrm{H_1} = 0.384\mathrm{MPa}; \mathrm{H_2} = 0.0075\mathrm{MPa}^{-1}; \mathrm{H_3} = 4.0995\mathrm{MPa}; \mathrm{H_4} = 0.2352\mathrm{MPa}^{-1}; \mathrm{H_5} = 0.6242\mathrm{MPa}$$

We propose the following relationship between the plastic energy and confining pressure:

$$\mathrm{E}_p = R_1 \sigma_c^{R_2} T^{R_3} \tag{33}$$

WhereR$_1$, R$_2$ and R$_3$ are material constants. Again by using the test results of Heard [16] and Jin et al. [19] and a non-linear least-squares method, these constants are determined as follows:

$$\mathrm{R}_1 = 7.0\times10^2 \mathrm{M}Pa^{1-R_2}K^{-R_3}; R_2 = 3.67; R_3 = -3.33 \tag{34}$$

From the test data at different confining pressures and temperatures, we found that the parameters in the rheological constitutive equation for steady state creep vary greatly. For example, when confining pressure and temperature are 600MPaand 800°C, respectively, the rheological parameters are given as follows:

$$\mathrm{E_0} = 96770.0\ \mathrm{MPa},\ \ \mathrm{E{=}232.35kJmol^{-1}, A{=}10^3MPa^{-4.2}\ s^{-1}, n{=}4.2, V{=}357}\mu\mathrm{m}^3 mol^{-1} \tag{35}$$

Using Eqs. (25), (32), and (35), parameters B, λ_0 and T0 at the same condition with equation (35) can be estimated as $\mathrm{B} = 3.15 \times 10^{-14}\mathrm{MPa}^{-6.2}$, λ_0=1.0003759, and T_0=298.15K, respectively.

Figure 3 shows the stress-strain curve at a constant strain rate $10^{-5}\mathrm{s}^{-1}$ under a confining pressure of 600MPa and temperature of 800°C based on the parameters determined above and equations (21) and (33).

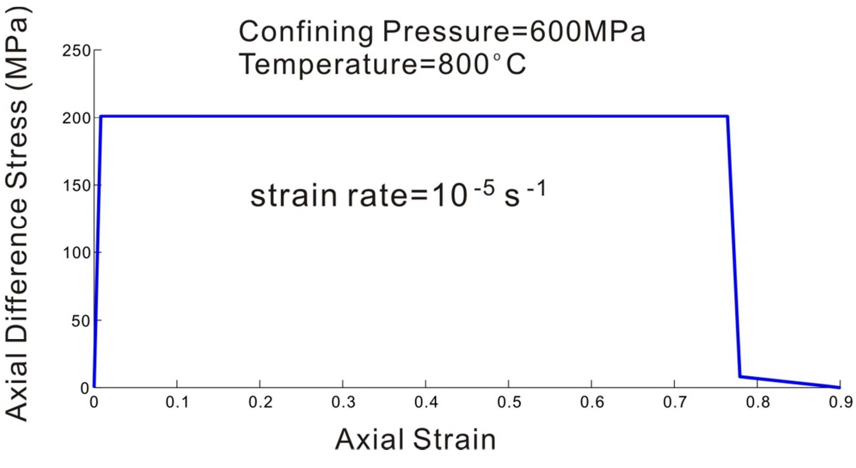

Figure 3. *Stress-strain curve at constant strain rate 10-5s-1 under confining pressure with 600MPa and temperature with 800℃ for marble is calculated based on experimental results (Heard, 1963; Jin et al., 1991).*

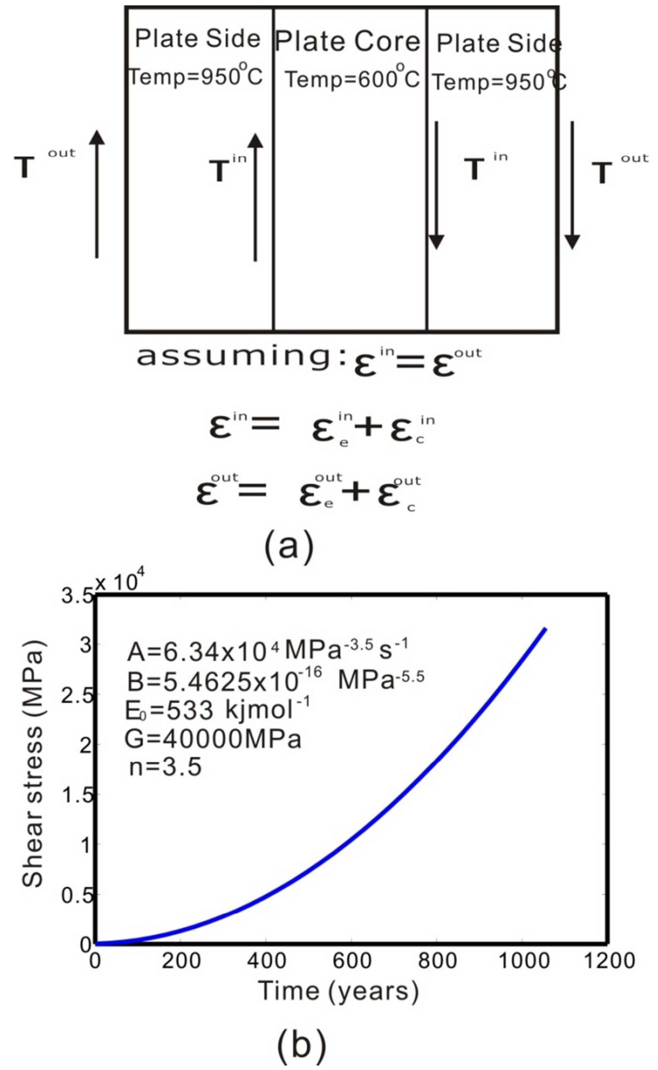

Figure 4. *A calculating model for deep earthquakes mechanism (a) Sketch of model for numerical calculation; (b) The shear stress change in plate central part with time.*

5. One Numerical Example

In this section, we present one example to illustrate applications of the constitutive relation and damage evolution equation developed in this paper and to discuss the geodynamic implications of rock rhological failure under high temperatures and pressures.

One of the mechanisms for explaining deep earthquake occurrence is plastic instability due to excessive heat and high strain rate [12, 13, 18, 32]. The damage constitutive equations presented in this paper can be used to calculate stresses at a fault and to describe the deformation process and damage evolution leading to rheological failure.

As we know, when a plate subducts into the upper mantle, the temperature in the outside layer is higher than that in the central part of the plate as schematically shown in Figure 4a. It is assumed that deformations in both parts are the same. The stresses in the two parts are thus different due to different rheological properties caused by different temperatures. The calculation procedure for the stresses is described as follow:

1) The initial shear stresses and temperatures are denoted by τ_0^{out} and T^{out} in the outside part and τ_0^{in} and T^{in} in the inside part. Assume

$$\tau_0^{out} = \tau_0^{in} = \tau_0; \qquad T^{out} > T^{in}$$

2) Calculate the shear creep strain in the outside part caused by the shear stress in the outside part. The total strain is the sum of the elastic shear strain and creep shear strain. The shear stresses and creep strains at time t are denoted by τ_t^{out} and ε_t^{out} in the outside part and τ_t^{in} and ε_t^{in} in the inside part, respectively. Now

$$\tau_t^{out} = \tau_0; \qquad \tau_t^{in} = \tau_0 + G(\varepsilon_t^{in} - \varepsilon_t^{out})$$

3) Repeating the steps above, we may obtain the shear stress as a function of time in the central part of the plate.

The material parameters are basically based on the experimental results for olivine. One of the parameters, B, is artificially assumed because of lack of experiment data. The initial shear stress is assumed as 50MPa, confining pressure is 24 GPa, which corresponds to a depth of 650km within Earth interior. The shear stress variation with time in the central part is shown in figure 4b. It is found that the shear stress may reach to $3\text{x}10^4$MPa. This stress level satisfies the requirement for fault sliding at depth of 650 km [21].

6. Conclusion

Deformations of rocks in Earth's interior are a slow time-dependent process under actions of various mechanical and thermal forces. Damage and creep mechanics is a convenient tool to describe and simulate this process. In this paper we develop a damage constitutive model for the rheological failure process of rocks on the basis of the theory and analysis method of damage mechanics concerning the dissipative process of damage. Different from the existing studies on damages in rocks under high temperatures and pressures, this work assumes that damage is negligible during secondary creep and starts accumulating at the beginning of tertiary creep. In addition, it puts forward a method for determining the material parameters in the constitutive and damage evolution equations. The material parameters for marble are estimated using the method and the available experimental test results.

One example explaining deep earthquake occurrence is given to illustrate the application of the damage constitutive equation and damage evolution equation under high temperatures and high pressures The numerical results indicates that our proposed constitutive model is capable of predicting damage evolution and rheological failure in rocks under high temperatures and pressures, i.e., earthquake occurrence based on the shear stress evolution.

Acknowledgements

This research was supported by Yunnan Major Science and Technology Special Project (202002AF080003) and the National Natural Science Foundation of China (Grant No. 41825018 and Grant No. U20B2014).

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