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Karmarkar's Approach for Solving Linear Programming Problem for Profit Maximization in Production Industries: NBC Port-Harcourt Plant

Nsikan Paul Akpan, Okeh Chibuzor Ojoh

Department of Mathematics and Statistics, Faculty of Science, University of Port Harcourt, Port Harcourt, Nigeria

Email address:
nsipaulakpan@gmail.com (N. P. Akpan), okehchibuzorojoh@gmail.com (O. C. Ojoh)

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Abstract: In this work, the Karmarkar's algorithm of the interior point method is compared to the simplex method by ascertaining the effect of interior point algorithm on linear programming problem of high number of variables and study why it is not so popularly used in solving linear programming problems. Six (6) products of coca-cola Hellenic Port Harcourt plant (Coke 50cl, Coke35cl, Fanta 50cl, Fanta 35cl, Sprite 35cl and Schweppes 33cl) and their raw materials (Concentrates, Sugar, Water and Carbon (iv) oxide) respectively were studied. The data were analysed using Scilab 5.5.2 software for Karmarmar's approach and Tora software for Simplex method and the results are compared. The Karmarkar's algorithm gave a maximum profit of ₦70,478,116.00 giving a breakdown that the company should produce 159300 crates of Coke 35cl, 64173 crates of Fanta 50cl, 8419 crates of Coke 50cl, 4876 crates of Fanta 35cl, 1118 crates of Sprite and 7 crates of Schweppes from the available resources to attain the optimal solution. The Simplex gave a maximum profit of ₦107,666,639.51 giving only the production of about 339482 crates of Schweppes 33cl from the available resources to attain the optimal solution.

Keywords: Linear Programming, Interior Point, Karmarkar, Simplex, Duality Gap

1. Introduction

Linear Programming according to Hoffman and Bradley (1995) is a branch of mathematics that deals with an important class of constrained Optimization problems in manufacturing industries for decision making. This is now very common in many areas of applied science. With this vast application, Robere (2012) pointed out that there has been interest in finding suitable algorithm to find the solution to any linear programming problem. Having introduced linear programming by Leonid Kantorovich for solving a production scheduling problem, the first algorithm to solve this problem called the Simplex Algorithm was introduced by Dantzig in 1947. This algorithm has been working well in practice ever since it was introduced but in the 1970s, it was found out that this simplex algorithm work well in all possible linear programs. This lead to the introduction of the first interior point algorithm "Ellipsoid Method" by Khachiyan in 1979 though was criticised because of its inability to meet some requirements. Then in 1984, Karmarkar came up with another interior point algorithm which he named after himself. This algorithm is generally accepted due to its ability to deal with huge number of variables and produce optimal solution. This prompt to the use of Karmarkar's algorithm in this work.

1.1. Purpose of the Study

To obtain the optimal solution of a linear programming problem using Karmarkar's methods of an interior point algorithm not a corner point algorithm.

1.2. Statement of Problem

Linear programming models are concerned with the optimal way in which available raw materials are utilized to produce the different products of a company. Whenever there is raw material to be transformed into finished goods, there is need to minimize the cost of production so as to increase profit on sales. The problem becomes; what quantity of each product to be produced from the available resources (raw materials) to minimize costs of production and maximize profit. This work tends to develop a better optimization tool that can tackle this problem. This will enable companies to take advantage of this opportunity to improve in their production.

2. Literature Review

When the runtime of simplex method is compared with Karmarkar's method, one can observe that neither of them is faster than the other in all problems. The practical efficiency of both Simplex and Karmarkar's methods depend strongly on the details of their implementation. The iterations numbers required by Karmarkar's method is higher than that of Simplex. The simplex method needs lesser iteration like 2*n to* 3*n* iterations, where *n* is the number of primal variables. Thus, generally the Interior Point Methods are better for large-scale problems Szabo and Kovacs (2003).

Karmarkar's algorithm seems to work better on linear programming problems with no initial feasible point (k=0) and also on very large structural problems Lemire (1989). He went ahead to say that the Karmarkar's algorithm is valid and it exhibits polynomial time complexity. Having used the algorithm to solve some problems, he found out that the algorithm produces speed ups over the simplex method. This counter-opposes the statement made by Szabo and Kovacs that both methods have the same speed, none is faster.

Further discussion on the outline of the algorithm and comment on the applications of this class of method by Nemirovski and Todd in 2008 equally revealed that Karmarkar's algorithm is convenient for polynomial time and large constraints with variables which supports the statement made by szabo and kovacs.

Vanderbei et al (1986) Concluded that Karmarkar's algorithm is formulated in the positive orthant instead of the simplex method. This makes it easier to conceptualize and lead to computational simplicity. They said that this method's sliding-objective-function is replaced by a projected gradient search for the optimum. Empirically, this leads to a decrease in the number of iterations the algorithm requires to solve a problem.

Karmarkar (1984) also stated in his work on "A Polynomial-Time Algorithm for Linear Programming" that the algorithm is not necessary to find the exact solution to all optimization problems rather it is useful to distinguish between two types of approximate solutions. He gave an instance that if X_0 is the exact solution, X is the approximate solution and $C^T x$ is the objective function also, that a strong approximation (approximation in solution space) requires that X be close to X_0 in some suitable norm. a weak approximation (approximation in objective function space) requires $C^T x$ be close to $C^T X_0$. This weak approximation is sufficient to Karmarkar's method and it is easier to achieve numerically.

3. Methodology

The method employed in this work is the interior point algorithm of Karmarkar and the Simplex method. The Simplex method is widely known so will not be talked about extensively rather the new Karmarkar method will be discussed.

For a linear programming problem to be solved by Karmarkar's method manually, such problem must be in a homogenous form (Assumption 1 of karmarkar). The second assumption says that the objective function at optimal has the value zero (0).

Consider the problem

$$\text{Min } C^T x$$

$$\text{Subject to } Ax = 0$$

$$1^T x = 1$$

$$x \geq 0$$

In the constraint, we observe that we don't have b (basic solution value) rather, we have 0 and the sum of all variables is equal to 1.

So when the problem is not given in this form, we convert it to Karmarkar's form before we can solve it by karmarkar's method. Having the problem in homogeneous form, we can now solve thus by the following steps:

(a) Set k=0 {initial iteration}

(b) Find $\alpha = \frac{n-1}{3n}$ and $\gamma = \frac{1}{\sqrt{n(n-1)}}$ the product of α and γ $\Rightarrow \alpha\gamma$ (step length)

(c) We transform the x-space problem to a problem in the y-space. $y = x_0 = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$ where $n = no. of\ variables.$ Then we get $D_0 = diag\ (x_0) = diag\ (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$

(d) We get $C^T =$ coefficient of variables at objective function then, A = matrix of the coefficient of the variables at the constraints. $P = \binom{AD_0}{1}$

(e) We get the Projection matrix $\mathbb{p}$

$$\mathbb{p} = [I - P^T (PP^T)^{-1} P]$$

(f) We find the projected steepest descent direction P_c (can equally be C_p).

$$P_c = -\mathbb{p}c$$

(g) The new value of y can now be obtained by $X_1 = Y_{new} = X_0 - \frac{\alpha\gamma\, P_c}{\| P_c \|}$ where $\| P_c \|$ (norm of P_c) $= \sqrt{(P_{c_1})^2 + (P_{c_2})^2 + \cdots + (P_{c_n})^2}$

(h) The Z (objective function) value will now be calculated as $Z = C^T x.$

The termination condition is as stated before: if the duality gap is zero or negative then stop, otherwise obtain the next iteration until the termination condition is met.

3.1. The Duality Gap at Optimal

$$C^T x - b^T \mu$$

Where $\mu = (PX^2P^T)^{-1}PX^2C$ where P takes values of the matrix above $\binom{AD_0}{1}$ and $X^2 = [daig(x)]^2$ and C is as stated above.

If:
$C^T x - b^T \mu = 0$ or negative{strong duality}
Then
$C^T x > b^T \mu${weak duality}

3.2. Converting Linear Programming Problem to Karmarkar's Form

When the problem is in linear programming form or in standard form, we can convert it to Karmarkar's form before we can solve the problem manually by Karmarkar's method. There are steps to be followed in order to achieve this:

Let us consider the LPP below

$$\text{Max } Z = C^T x$$

$Subject\ to\ Ax \leq b$ (i) {primal)

$$x \geq 0$$

1. Firstly, we get the dual of the primal given above as:

$$\text{Min } Z = b^T w$$

$Subject\ to\ A^T w \geq c$ (ii) {dual)

$$w \geq 0$$

2. We introduce the slack and surplus variables in the primal and dual problems, and then combine the problems.
3. We introduce a boundary constraint K such that any feasible solution to the equation gotten in step 2 above will satisfy sum of all variables in the said equation less than or equal to K and then add a slack variable to get something like: $\sum x_i + \sum w_i + s = K$
4. We homogenize the equations gotten in step 3 above by introducing a dummy variable d (subject to the condition $d = 1$). We then replace the equation $\sum x_i + \sum w_i + s = K$ with the following equivalent equations: $\sum x_i + \sum w_i + s - Kd = 0$ and $\sum x_i + \sum w_i + s + d = K + 1$
5. Transform the equations in step 4 above to have one variable in all the equations instead of different variables. By this, we have

$$x_i = (K+1)y_i, i = 1, 2, \dots, (m+n)$$

$$w_i = (K+1)y_{m+n+i}, i = 1, 2, \dots, (m+n)$$

$$s = (K+1)y_{2m+2n+1}, d = (K+1)y_{2m+2n+2}$$

6. An artificial variable is now introduced in all the equations, such that the sum of the coefficients in each homogeneous equation will be zero and the artificial variable in the last equation is one. These artificial variables are to be minimized. The artificial variable to be introduced here is $y_{2m+2n+3}$

4. Data Presentation and Analysis

The data of this work was gotten from the production and commercial department of Coca-cola plant of Port Harcourt. The researcher's interest is to ascertain the products and their quantities to be produced in other to make an optimal profit using karmarkar and comparing to simplex. The Tora software is used to analyse the data for Simplex and the scilab software is used to analyse the data for Karmarkar since the data cannot be analysed manually because of the numerous number of variables. This lead to a matrix 4x9 which is difficult to solve manually.

Table 1. *Available raw materials in stock.*

Raw Materials	Quantity Available
1. Concentrates	4332 (units)
2. Sugar	467012 (kg)
3. Water (H_2O)	1637660 (litres)
4. Carbon (iv) oxide (CO_2)	8796 (vol. per pressure)

Source: Coca-cola Port Harcourt Plant

Table 2. *Available raw materials needed to produce a crate of each product.*

Flavours	Concentrates	Sugar	Water	Carbon (iv) oxide
Coke 50cl (x_1)	0.00359	0.89	7.552	0.0135
Coke 35cl (x_2)	0.0042	1.12	6.539	0.0133
Fanta 50cl (x_3)	0.0021	1.044	7.671	0.007
Fanta 35cl (x_4)	0.00419	0.86	6.822	0.005
Sprite 35cl (x_5)	0.00359	0.73	6.12	0.0149
Schweppes 33cl (x_6)	0.00438	0.23	4.824	0.0156

Source: Coca-cola Port Harcourt Plant

Table 3. *Available cost of production and selling price of a crate of each product.*

Products	Average cost price (₦)	Average selling price (₦)	Profit (₦)
Coke 50cl (x_1)	760.55	1050	289.45
Coke 35cl (x_2)	679.51	980	300.49
Fanta 50cl (x_3)	762.63	1050	287.37
Fanta 35cl (x_4)	691.91	980	288.09
Sprite 35cl (x_5)	689.67	980	290.33
Schweppes 33cl (x_6)	682.85	980	317.15

Source: Coca-cola Port Harcourt Plant

4.1. Karmarkar's Approach of Solving the Problem

Model Formulation:

$$Max\ Z = 289.45X_1 + 300.49X_2 + 287.37X_3 + 288.09X_4 + 290.33X_5 + 317.15X_6\ Subject\ to:$$

$$0.00359X_1 + 0.0042X_2 + 0.0021X_3 + 0.00419X_4 + 0.00359X_5 + 0.00438X_6 \leq 4332$$

$$0.89000X_1 + 1.12000X_2 + 1.04400X_3 + 0.86000X_4 + 0.7300X_5 + 0.2300X_6 \leq 467012$$

$$7.5520X_1 + 6.5390X_2 + 7.67100X_3 + 6.82200X_4 + 6.1200X_5 + 4.82400X_6 \leq 1637660$$

$$0.01350X_1 + 0.01330X_2 + 0.0070X_3 + 0.0050X_4 + 0.01490X_5 + 0.01560X_6 \leq 8796$$

This can be put in Karmarkar's form by the following steps:
Step 1: Get the dual of the above primal as:

$$\text{Min } Z = 4332w_1 + 467012w_2 + 1637660w_3 + 8796w_4$$

$subject\ to$:

$$0.00359w_1 + 0.89w_2 + 7.552w_3 + 0.0135w_4 \geq 289.45$$

$$0.0042w_1 + 1.12w_2 + 6.539w_3 + 0.0133w_4 \geq 300.49$$

$$0.0021w_1 + 1.044w_2 + 7.671w_3 + 0.007w_4 \geq 287.37$$

$$0.00419w_1 + 0.86w_2 + 6.822w_3 + 0.005w_4 \geq 288.09$$

$$0.00359w_1 + 0.73w_2 + 6.12w_3 + 0.0149w_4 \geq 290.33$$

$$0.00438w_1 + 0.23w_2 + 4.824w_3 + 0.0156w_4 \geq 317.15$$

$with$: $all\ variables\ nonnegative$
Step 2: We introduce the slack and surplus variables in the primal and dual problems, and then combine the problems.

$$0.00359X_1 + 0.0042X_2 + 0.0021X_3 + 0.00419X_4 + 0.00359X_5 + 0.00438X_6 + X_7 = 4332$$

$$0.89000X_1 + 1.12000X_2 + 1.04400X_3 + 0.86000X_4 + 0.7300X_5 + 0.2300X_6 + X_8 = 467012$$

$$7.5520X_1 + 6.5390X_2 + 7.67100X_3 + 6.82200X_4 + 6.1200X_5 + 4.82400X_6 + X_9 = 1637660$$

$$0.01350X_1 + 0.01330X_2 + 0.0070X_3 + 0.0050X_4 + 0.01490X_5 + 0.01560X_6 + X_{10} = 8796$$

$$0.00359w_1 + 0.89w_2 + 7.552w_3 + 0.0135w_4 - w_5 = 289.45$$

$$0.0042w_1 + 1.12w_2 + 6.539w_3 + 0.0133w_4 - w_6 = 300.49$$

$$0.0021w_1 + 1.044w_2 + 7.671w_3 + 0.007w_4 - w_7 = 287.37$$

$$0.00419w_1 + 0.86w_2 + 6.822w_3 + 0.005w_4 - w_8 = 288.09$$

$$0.00359w_1 + 0.73w_2 + 6.12w_3 + 0.0149w_4 - w_9 = 290.33$$

$$0.00438w_1 + 0.23w_2 + 4.824w_3 + 0.0156w_4 - w_{10} = 317.15$$

$$289.45X_1 + 300.49X_2 + 287.37X_3 + 288.09X_4 + 290.33X_5 + 317.15X_6 = 4332w_1 + 467012w_2 + 1637660w_3 + 8796w_4$$

$with$: $all\ variables\ nonnegative$

Step 3: Introduce a boundary constraint K such that any feasible solution to the equation gotten in step 2 above will satisfy sum of all variables in the said equation less than or equal to K and then add a slack variable to get something like: $\sum x_i + \sum w_i + s = K$

$$0.00359X_1 + 0.0042X_2 + 0.0021X_3 + 0.00419X_4 + 0.00359X_5 + 0.00438X_6 + X_7 = 4332$$

$$0.89000X_1 + 1.12000X_2 + 1.04400X_3 + 0.86000X_4 + 0.7300X_5 + 0.2300X_6 + X_8 = 467012$$

$$7.5520X_1 + 6.5390X_2 + 7.67100X_3 + 6.82200X_4 + 6.1200X_5 + 4.82400X_6 + X_9 = 1637660$$

$$0.01350X_1 + 0.01330X_2 + 0.0070X_3 + 0.0050X_4 + 0.01490X_5 + 0.01560X_6 + X_{10} = 8796$$

$$0.00359w_1 + 0.89w_2 + 7.552w_3 + 0.0135w_4 - w_5 = 289.45$$

$$0.0042w_1 + 1.12w_2 + 6.539w_3 + 0.0133w_4 - w_6 = 300.49$$

$$0.0021w_1 + 1.044w_2 + 7.671w_3 + 0.007w_4 - w_7 = 287.37$$

$$0.00419w_1 + 0.86w_2 + 6.822w_3 + 0.005w_4 - w_8 = 288.09$$

$$0.00359w_1 + 0.73w_2 + 6.12w_3 + 0.0149w_4 - w_9 = 290.33$$

$$0.00438w_1 + 0.23w_2 + 4.824w_3 + 0.0156w_4 - w_{10} = 317.15$$

$$289.45X_1 + 300.49X_2 + 287.37X_3 + 288.09X_4 + 290.33X_5 + 317.15X_6 - 4332w_1 - 467012w_2 - 1637660w_3 - 8796w_4 = 0$$

$$\sum_{i=1}^{10} X_i + \sum_{i=1}^{10} w_i + S = K$$

$with: all\ variables\ nonnegative$

Step 4: Homogenize the equations gotten in step 3 above by introducing a dummy variable d (subject to the condition $d = 1$). We then replace the equation $\sum x_i + \sum w_i + s = K$ with the following equivalent equations: $\sum x_i + \sum w_i + s - Kd = 0$ and $\sum x_i + \sum w_i + s + d = K + 1$

$$0.00359X_1 + 0.0042X_2 + 0.0021X_3 + 0.00419X_4 + 0.00359X_5 + 0.00438X_6 + X_7 - 4332d = 0$$

$$0.89000X_1 + 1.12000X_2 + 1.04400X_3 + 0.86000X_4 + 0.7300X_5 + 0.2300X_6 + X_8 - 467012d = 0$$

$$7.5520X_1 + 6.5390X_2 + 7.67100X_3 + 6.82200X_4 + 6.1200X_5 + 4.82400X_6 + X_9 - 1637660d = 0$$

$$0.01350X_1 + 0.01330X_2 + 0.0070X_3 + 0.0050X_4 + 0.01490X_5 + 0.01560X_6 + X_{10} - 8796d = 0$$

$$0.00359w_1 + 0.89w_2 + 7.552w_3 + 0.0135w_4 - w_5 - 289.45d = 0$$

$$0.0042w_1 + 1.12w_2 + 6.539w_3 + 0.0133w_4 - w_6 - 300.49d = 0$$

$$0.0021w_1 + 1.044w_2 + 7.671w_3 + 0.007w_4 - w_7 - 287.37d = 0$$

$$0.00419w_1 + 0.86w_2 + 6.822w_3 + 0.005w_4 - w_8 - 288.09d = 0$$

$$0.00359w_1 + 0.73w_2 + 6.12w_3 + 0.0149w_4 - w_9 - 290.33d = 0$$

$$0.00438w_1 + 0.23w_2 + 4.824w_3 + 0.0156w_4 - w_{10} - 317.15d = 0$$

$$289.45X_1 + 300.49X_2 + 287.37X_3 + 288.09X_4 + 290.33X_5 + 317.15X_6 - 4332w_1 - 467012w_2 - 1637660w_3 - 8796w_4 = 0$$

$$\sum_{i=1}^{10} X_i + \sum_{i=1}^{10} w_i + S - Kd = 0$$

$$\sum_{i=1}^{10} X_i + \sum_{i=1}^{10} w_i + S + d = (K + 1)$$

$$with: all\ variables\ nonnegative$$

Step 5: Transform the equations in step 4 above to have one variable in all the equations instead of different variables. By this, we have

$$x_i = y_i, i = 1, 2, \dots, (m + n)$$

$$w_i = y_{m+n+i}, i = 1, 2, \dots, (m + n)$$

$$s = y_{2m+2n+1}, d = y_{2m+2n+2}$$

$$then\ s = y_{21} and\ d = y_{22}$$

$the\ above\ transformations\ yield\ the\ following\ system:$

$$0.00359y_1 + 0.0042y_2 + 0.0021y_3 + 0.00419y_4 + 0.00359y_5 + 0.00438y_6 + y_7 - 4332y_{22} = 0$$

$$0.89000y_1 + 1.12000y_2 + 1.04400y_3 + 0.86000y_4 + 0.7300y_5 + 0.2300y_6 + y_8 - 467012y_{22} = 0$$

$$7.5520y_1 + 6.5390y_2 + 7.67100y_3 + 6.82200y_4 + 6.1200y_5 + 4.82400y_6 + y_9 - 1637660y_{22} = 0$$

$$0.01350y_1 + 0.01330y_2 + 0.0070y_3 + 0.0050y_4 + 0.01490y_5 + 0.01560y_6 + y_{10} - 8796y_{22} = 0$$

$$0.00359y_{11} + 0.89y_{12} + 7.552y_{13} + 0.0135y_{14} - y_{15} - 289.45y_{22} = 0$$

$$0.0042y_{11} + 1.12y_{12} + 6.539y_{13} + 0.0133y_{14} - y_{16} - 300.49y_{22} = 0$$

$$0.0021y_{11} + 1.044y_{12} + 7.671y_{13} + 0.007y_{14} - y_{17} - 287.37y_{22} = 0$$

$$0.00419y_{11} + 0.86y_{12} + 6.822y_{13} + 0.005y_{14} - y_{18} - 288.09y_{22} = 0$$

$$0.00359y_{11} + 0.73y_{12} + 6.12y_{13} + 0.0149y_{14} - y_{19} - 290.33y_{22} = 0$$

$$0.00438y_{11} + 0.23y_{12} + 4.824y_{13} + 0.0156y_{14} - y_{20} - 317.15y_{22} = 0$$

$$289.45y_1 + 300.49y_2 + 287.37y_3 + 288.09y_4 + 290.33y_5 + 317.15y_6 - 4332y_{11} - 467012y_{12} - 1637660y_{13} - 8796y_{14} = 0$$

$$\sum_{i=1}^{21} y_i - Ky_{22} = 0$$

$$\sum_{i=1}^{22} y_i = 1$$

$with: all\ variables\ nonnegative$

Step 6: An artificial variable is now introduced in all the equations, such that the sum of the coefficients in each homogeneous equation will be zero and the artificial variable in the last equation is one. These artificial variables are to be minimized. The artificial variable to be introduced here is $y_{2m+2n+3}$, we then have:

$Minimize: y_{23}$

$Subject\ to:$

$$0.00359y_1 + 0.0042y_2 + 0.0021y_3 + 0.00419y_4 + 0.00359y_5 + 0.00438y_6 + y_7 - 4332y_{22} + 4332.978y_{23} = 0$$

$$0.89000y_1 + 1.12000y_2 + 1.04400y_3 + 0.86000y_4 + 0.7300y_5 + 0.2300y_6 + y_8 - 467012y_{22} + 467008.126y_{23} = 0$$

$$7.5520y_1 + 6.5390y_2 + 7.67100y_3 + 6.82200y_4 + 6.1200y_5 + 4.82400y_6 + y_9 - 1637660y_{22} + 1637621.625y_{23} = 0$$

$$0.01350y_1 + 0.01330y_2 + 0.0070y_3 + 0.0050y_4 + 0.01490y_5 + 0.01560y_6 + y_{10} - 8796y_{22} + 8796.931y_{23} = 0$$

$$0.00359y_{11} + 0.89y_{12} + 7.552y_{13} + 0.0135y_{14} - y_{15} - 289.45y_{22} + 281.869y_{23} = 0$$

$$0.0042y_{11} + 1.12y_{12} + 6.539y_{13} + 0.0133y_{14} - y_{16} - 300.49y_{22} + 293.814y_{23} = 0$$

$$0.0021y_{11} + 1.044y_{12} + 7.671y_{13} + 0.007y_{14} - y_{17} - 287.37y_{22} + 279.646y_{23} = 0$$

$$0.00419y_{11} + 0.86y_{12} + 6.822y_{13} + 0.005y_{14} - y_{18} - 288.09y_{22} + 274.577y_{23} = 0$$

$$0.00359y_{11} + 0.73y_{12} + 6.12y_{13} + 0.0149y_{14} - y_{19} - 290.33y_{22} + 284.475y_{23} = 0$$

$$0.00438y_{11} + 0.23y_{12} + 4.824y_{13} + 0.0156y_{14} - y_{20} - 317.15y_{22} + 313.076y_{23} = 0$$

$$\sum_{i=1}^{21} y_i - Ky_{22} + (K - 21)y_{23} = 0$$

$$\sum_{i=1}^{23} y_i = 1$$

$with: all\ variables\ nonnegative$

This can now be solved using Scilab 5.5.2 software and then result will be obtained.

Analysis from Scilab of Karmarkar's Method

The above data is analysed using scilab 5.5.2 for karmarkar's method and the result is as below:

Table 4. *Result from the analysis by Karmarkar's Approach.*

Variables	Explanations	Values
Iter	No. of iterations	56
Exitflag		1
Fopt	Objective function	70478116
Xopt		No. of crates
x_1	Coke 50cl	8419
x_2	Coke 35cl	159300
x_3	Fanta 50cl	64173
x_4	Fanta 35cl	4876
x_5	Sprite 35cl	1118
x_6	Schweppes 33cl	7

The result from scilab 5.5.2

4.2. Simplex Method of Solving the Problem

Model formation

$$Max\ Z = 289.45X_1 + 300.49X_2 + 287.37X_3 + 288.09X_4 + 290.33X_5 + 317.15X_6 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

$Subject\ to$:

$$0.00359X_1 + 0.0042X_2 + 0.0021X_3 + 0.00419X_4 + 0.00359X_5 + 0.00438X_6 + S_1 + 0S_2 + 0S_3 + 0S_4 = 4332$$

$$0.89000X_1 + 1.12000X_2 + 1.04400X_3 + 0.86000X_4 + 0.7300X_5 + 0.2300X_6 + 0S_1 + S_2 + 0S_3 + 0S_4 = 467012$$

$$7.5520X_1 + 6.5390X_2 + 7.67100X_3 + 6.82200X_4 + 6.1200X_5 + 4.82400X_6 + 0S_1 + 0S_2 + S_3 + 0S_4 = 1637660$$

$$0.01350X_1 + 0.01330X_2 + 0.0070X_3 + 0.0050X_4 + 0.01490X_5 + 0.01560X_6 + S_1 + 0S_2 + 0S_3 + S_4 = 8796$$

This can be put in standard form as

$$Max\ Z = 289.45X_1 + 300.49X_2 + 287.37X_3 + 288.09X_4 + 290.33X_5 + 317.15X_6 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

$Subject\ to$:

$$0.00359X_1 + 0.0042X_2 + 0.0021X_3 + 0.00419X_4 + 0.00359X_5 + 0.00438X_6 + S_1 + 0S_2 + 0S_3 + 0S_4 = 4332$$

$$0.89000X_1 + 1.12000X_2 + 1.04400X_3 + 0.86000X_4 + 0.7300X_5 + 0.2300X_6 + 0S_1 + S_2 + 0S_3 + 0S_4 = 467012$$

$$7.5520X_1 + 6.5390X_2 + 7.67100X_3 + 6.82200X_4 + 6.1200X_5 + 4.82400X_6 + 0S_1 + 0S_2 + S_3 + 0S_4 = 1637660$$

$$0.01350X_1 + 0.01330X_2 + 0.0070X_3 + 0.0050X_4 + 0.01490X_5 + 0.01560X_6 + S_1 + 0S_2 + 0S_3 + S_4 = 8796$$

This can be solved with TORA software and results will be obtained

Analysis from Tora of Simplex Method

Table 5. *Result from the analysis using Simplex Method.*

Variables	Explanations	Values
	No. of iterations	2
	Objective function	107666640
		No. of crates
x_1	Coke 50cl	0.0000
x_2	Coke 35cl	0.0000
x_3	Fanta 50cl	0.0000
x_4	Fanta 35cl	0.0000
x_5	Sprite 35cl	0.0000
x_6	Schweppes 33cl	339481.76
s_{x7}	Slack	2845.07
s_{x8}	Slack	388931.20
s_{x10}	Slack	3500.08

The result from TORA

5. Conclusion

From the result above, it is shown that the production of about 159300 crates of Coke 35cl, 64173 crates of Fanta 50cl, 8419 crates of Coke 50cl, 4876 crates of Fanta 35cl, 1118 crates of Sprite and 7 crates of Schweppes with that available resources in the Port Harcourt plant of the Coca-cola company will make an optimal profit of ₦70,478,116.00 Looking at the result, Coke 35cl is the most significant product that will push up profit while the production of Schweppes is almost insignificant. Then the result shows that the coca-cola plant in Port Harcourt should produce about 339482 crates of Schweppes only with the available resources in other to make an optimal profit of ₦107,666,640.00 By observation, it is noticed that the other products were not put into consideration as to how many crates to be produced.

Recommendations

Since Karmarkar's approach gave a better result from the data analysed, the production team of Coca-cola Port Harcourt is encourage to adopt this method for better decision making. However, it is still uncertain as to how problems that produce dense A matrix or dense factorizations could be solved efficiently using the Karmarkar's algorithm, the direction for future research include (i) development of better methods to determine the initial feasible interior-point and (ii) decomposition methods for essentially sparse problems but with a few dense rows/columns. If these are achieve, solving linear problems by Karmarkar's algorithm manually will be easier and a good alternative to Simplex method would have equally been achieved.

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