

# Comparison of parametric and nonparametric item response techniques in determining differential item functioning in polytomous scale

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**Abstract:** This study aims to compare parametric and nonparametric methods based on Item Response Theory in determining differential item functioning in polytomous scales. DIF analysis based on parametric IRT was conducted by using parameters comparison method. For nonparametric IRT analysis, DIF is determined by comparison of area indices pertaining to ICC obtained for reference and focal groups. The Comparisons were conducted on data sets from TIMSS 2011 8th Class students survey where data set pertaining to responses of students to "Attitudes Toward Mathematics" composing of samplings from Turkey and South Korea and it was determined if it incorporated DIF according to country and sex differences. It is observed that parametric and nonparametric methods produce generally similar results for DIF analysis in terms of countries. Nevertheless, DIF analysis results for country based sex groups differed according to techniques based on parametric and nonparametric IRT. Results of the study showed that items incorporating DIF differed as to preferred technique. This indicated importance of choosing the best technique in studies to detect whether scale items incorporates DIF or not.

**Keywords:** Differential Item Functioning, Item Response Theory, Nonparametric Differential Item Functioning, Nonparametric IRT

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## 1. Introduction

Differential Item functioning (DIF), can be defined as differentiation in probability of giving correct response for individuals with the same skills levels but coming from different subgroups (Carter & Zickar, 2011; Cohen, Kim, & Baker, 1993; Finch, 2005; Holland, Wainer, & Service, 1993; Rudner, Getson, & Knight, 1980; Shepard, Camilli, & Williams, 1985; Wang, Tay, & Drasgow, 2013) Differentiation in probabilities may be due to item bias or differences in individual knowledge, skills and traits. In determination of test-item bias Differential item functioning (DIF) analysis is widely used technique. From this point of view, differentiation of parameters of an item for individuals in different sub-populations and in the same skill levels shall indicate existence of DIF. Test items' exhibiting DIF means scale scores obtained from those groups contain systematic error and this issue show up as validity problem of measurement tool in the process of test

development. Therefore DIF analysis are an important part of scale development process.

A number of different theories and statistical techniques are used in detecting DIF. Among them, Classical Test Theory encompasses Item Discrimination power, item difficulty, factor analysis, variance analysis, item difficulty transformation (IDT), Chi-square, Mantel-Haenszel (MH) test statistics, and logistic regression (LR) and Item Response Theory (IRT) embraces signed and unsigned area indices, Lord's Chi-square, item parameters comparison method or comparison of maximum likelihood ratio differences. (Adams & Rowe, 1988; Devine & Raju, 1982; Glickman, Seal, & Eisen, 2009; Hambleton, Swaminathan, & Rogers, 1991; Holland & Thayer, 1986; Holland et al., 1993; Mellenberg, 1983; Raju, 1988; Rodney & Drasgow, 1990; Roju, van der Linden, & Fleeer, 1995; Rudner et al., 1980; Shepard et al., 1985; Zumbo & Hubley, 2003) One of

the criteria, deciding on which technique to be used, is the scale levels of measurement to be analyzed. On account of preponderance of ordinal scale for personality, attitude, achievement or skills measurements in education and psychology, majority of researchers propounded nonparametric methods in DIF analysis for the tools measuring the same traits. (Glickman et al., 2009; Nozawa, 2008; Ramsay, 2000; Sijtsma & Molenaar, 2002).

A review of papers studying to determine DIF in test and items shows that IRT based studies have come into prominence in recent years. Besides it is also observed that researchers widely used multidimensional IRT and nonparametric IRT techniques according to structure of scale and characteristics of measurement results. (Douglas, 1997; Nozawa, 2008; Ramsay, 1991). Those researchers suggested nonparametric DIF rather than parametric IRT techniques due to the fact that scale levels are nominal and ordinal in multi-categorically graded or bi-categorical (1, 0) graded scales. Nonparametric item response theory, used in detecting DIF, is explained below.

### 1.1. Nonparametric IRT

Nonparametric IRT, within the framework of Item Response Theory, focuses on measurements in the level of classification. In parametric IRT models, skills are assumed to have distributions according to normal or logistic distributions and skills scale are constructed over that distribution. In fact this indicates that assumptions determine precisely skill distribution characteristics and shape of skills distributions cannot be predicted. On the other hand, in nonparametric IRT model, skill scale is determined first and then Item Characteristic Curve (ICC) is calculated according to this distribution. In this sense, Nonparametric IRT is less restricted form of traditional IRT. (Nozawa, 2008). Therefore in determining item characteristic curve assumptions of parametric model were not maintained. Instead, nonparametric regression methods such as kernel smoothing, spline regression or spline smoothing are used in estimation of item response functioning.

### 1.2. Kernel Smoothing and Nonparametric Regression

Kernel smoothing approach, suggested by Ramsay(1991), is most widely used nonparametric item characteristics estimation method (ICC) in the literature. In this method, correct answering probability of item  $i$  is estimated as follows:

$$\hat{P}_i(\theta) = \frac{\sum_{j=1}^N K\left(\frac{\theta - \theta_j}{h}\right) u_{ij}}{\sum_{j=1}^N K\left(\frac{\theta - \theta_j}{h}\right)}$$

In the equation,  $N$  denotes number of respondents,  $\theta_j$  and  $j$  denotes skill of respondent, and  $u_{ij}$  denotes binary

response of respondent  $j$  to item  $i$  and  $h$  is bandwidth parameters. (Ramsay, 1991) A typical kernel function is a nonnegative symmetrical function and decreases after reaching maximum value 0 ( $\theta_j = \theta$ ). Bandwidth parameter  $h$  controls the balance between random and systematic error. (Nozawa, 2008; Ramsay, 2000).

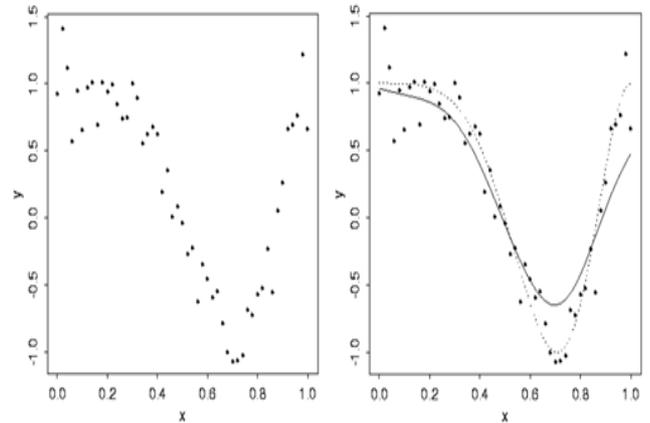


Figure 1. An example for item characteristic curve estimated using Kernel Smoothing.

An example for ICC determination by nonparametric regression method and Kernel smoothing is provided in Figure 1. For the first graphics in the figure, functional relation between independent variable  $x$  and dependent variable  $y$  is estimated. In the second graphics, function curve  $f(x)$  shown by dotted curve is produced from data. Dark curve is estimated by Gaussian Kernel smoothing (Ramsay, 2000). In the second graphics in Figure 1, difference between normal regression curve and the curve after kernel smoothing is clearly seen.

Kernel approach has some advantages over other nonparametric methods. Foremost advantage is the ease of calculation. In calculation of ICC, when weighting mean values approach for observed grades was preferred, there is no agreement on the required convergence in estimation procedure. Kernel method eliminates this uncertainty (Ramsay, 1991; Wand & Jones, 1994). Besides, Kernel smoothing approach can be used with both data obtained from multi-categorical and nominal scale. Most important and third advantage is that Kernel smoothing approach, when skill estimation was in ordinal level, fit consistently. (Douglas, 1997). Douglas noted that when there are sufficient number of items and respondents, and if assumptions of parametric model were not met, Kernel smoothing approach gives better results than parametric IRT.

Multi-categorical nonparametric IRT, is adaptation of analysis of bi-categorical items to multi-categorical items. (Sijtsma & Molenaar, 2002). In this process, Item response Function (IRF) is determined separately for each option of each item. Figures 2 shows separate curves were plotted for an item's each option.

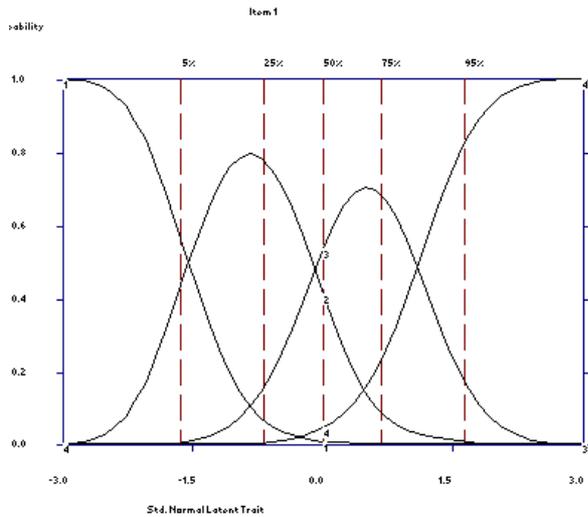


Figure 2. Category response curves, based on nonparametric IRT, belonging to a 4-category item obtained by TestGraf program.

1.3. Determination of Differential Item Functioning in Nonparametric IRT Models

In nonparametric IRT models DIF is determined (Zumbo & Hubley, 2003), like many other IRT models, by calculating areas between item characteristic curves. This area is defined as beta ( $\beta$ ). This value shows the difference between weighted expected grade curves belonging to reference group and focal group which are in the same skills level. TestGraf program, used in this research, estimates  $\beta$  value according to following formula;

$$\beta_{img}(\theta) = \sum_{q=1}^q P_{Fq} [P_{im}^{(F)}(\theta) - P_{im}^{(R)}(\theta)]$$

In above formula,  $P_{im}^{(F)}(\theta)$  and  $P_{im}^{(R)}(\theta)$  show characteristic curves for each option for focal and reference groups. In case focal group has a disadvantage in average,  $\beta$  index takes negative values (Nozawa, 2008).

$\beta$  value, obtained to determine DIF, is normalized by dividing its standard error. This value fits to normal Z distribution and significance of DIF is determined as significance of Z statistics according to Ramsey.

Table 2. Descriptive statistics for DIF comparison groups belonging to TIMSS 2011 8. Class Attitudes toward Mathematics scale

Phrases	Turkey			South Korea			
	Girls		Boys	Girls		Boys	Total
	$\bar{X}$	S	$\bar{X}$ S	$\bar{X}$	S	$\bar{X}$ S	$\bar{X}$ S
m1 ENJOY LEARNING MATHEMATICS	3.23	,935	3.22 ,960	3.22 ,948	2.36 ,848	2.45 ,900	2.40 ,875
m2 WISH HAVE NOT TO STUDY MATH	2.63	1.184	2.63 1.191	2.63 1.187	2.24 ,877	2.38 ,964	2.31 ,923
m3 MATH IS BORING	2.77	1.122	2.72 1.123	2.75 1.123	2.40 ,830	2.46 ,895	2.43 ,862
m4 LEARN INTERESTING THINGS	3.22	,923	3.16 ,985	3.19 ,956	2.44 ,813	2.54 ,868	2.49 ,841
m5 LIKE MATHEMATICS	3.07	1.031	3.07 1.050	3.07 1.040	2.25 ,883	2.38 ,917	2.31 ,902
m6 IMPORTANT TO DO WELL IN MATH	3.82	,525	3.73 ,671	3.77 ,605	3.18 ,811	3.20 ,863	3.19 ,836
Reliability ( $\alpha$ )	0.81			0.86			

Nevertheless, Roussos and Stout (Roussos & Stout, 1996) determined some cut values for  $\beta$  value by their method developed for SIBTEST software program. They suggested that  $|\beta| < .59$  and,  $|\beta| > .59 < .88$  and  $|\beta| > .88$  indicates acceptable, average level and important level DIF respectively.

This study aims to investigate comparatively parametric and nonparametric DIF determination methods, for which theoretical basis were explained above.

To this end, items for Attitudes toward Mathematics in TIMSS 2011 student survey were used as real data. It is investigated if items exhibit DIF or not according to sex and countries. In this study, whether items incorporating DIF and exhibit differences in parametric and nonparametric IRT methods or not, were determined comparatively.

2. Method

2.1. Study Group

Samples for this study obtained from students responding to TIMSS 2011 survey in Turkey and South Korea. Distribution of samplings studied according to DIF comparison groups is provided in Table 1

Table 1. Distribution of samples according to DIF comparison groups

Countries	Sex	N	%	Total
Turkey	Girls	3245	49.5	6561
	Boys	3316	50.5	
South Korea	Girls	2642	51.6	5121
	Boys	2479	48.4	

In deciding for reference and focal groups, South Korea taking the first row in TIMSS 2011 survey is taken as reference group to compare with Turkish sampling.

2.2. Measurement Instrument

In this study 6 items belonging to sub-scale "Attitudes towards Mathematics", for which descriptive statistics were provided in the table below, of TIMSS 2011 8th class student survey. were used as data collection instrument. In Table 2, descriptive statistics pertaining to Scale phrases used in this study and work groups belonging to these phrases are shown.

**2.3. Data Analysis**

DIF analysis by IRT method was conducted using IRTPRO program. This program performed DIF analysis according to Lord’s (F. M. Lord, 1977; F.M. Lord, 1980)parameter comparison technique. Basis of this technique relies on comparison of ICC for focal and reference groups estimated on the basis of item parameters. In other words, this method is based on differences in parameters between groups.

For nonparametric IRT analysis TESTGRAFH program was used. This program detects DIF by comparing indices belonging to ICC obtained on this basis for groups (Ramsay, 2000).

In the study, DIF analysis according to both technique were conducted in three stages.

At the first stage, considering country variables for Turkey and South Korea samplings, DIF analysis were

conducted by parametric and non parametric IRT techniques .

At the second stage, for samples from Turkey and South Korea, DIF analysis according to Sex variable was conducted by parametric IRT techniques.

At the third stage, for samples from Turkey and South Korea, DIF analysis according to Sex variable was conducted by nonparametric IRT techniques.

**3. Results and Discussion**

In this section of study, results obtained by both methods are presented respectively. Besides, the results are discussed in comparison.

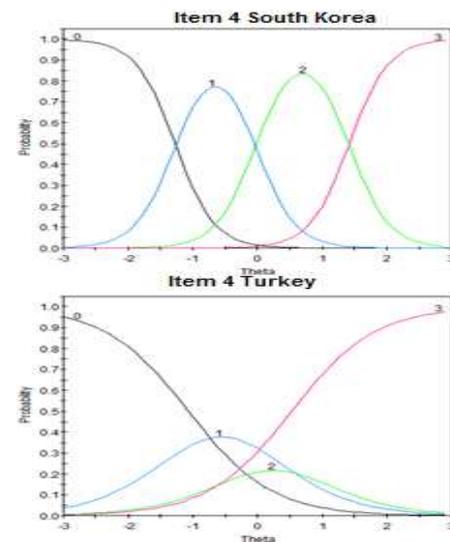
DIF results from IRTPRO program pertaining to Parametric IRT analysis (Turkey - South Korea samplings)

*Table 3. DIF analysis results based on parametric IRT for Turkey and South Korea Samplings*

Item	South Korea				Turkey				$\chi^2_{c/a}$	d.f.	p
	a	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>	a	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>			
1	5.02	-1.02	0.11	1.32	3.71	-1.42	-1	0.01	1550.3	3	0.0001
2	1.83	-1.07	0.31	1.71	1.55	-1.07	-0.04	0.53	1092.7	3	0.0001
3	2.77	-1.2	0.08	1.51	2.23	-1.18	-0.17	0.4	1137.4	3	0.0001
4	3.31	-1.27	-0.03	1.42	1.33	-2.21	-1.35	0.07	1412.4	3	0.0001
5	6.31	-0.84	0.22	1.31	6.17	-1.11	-0.74	0.12	916.8	3	0.0001
6	0.6	-5.09	-2.81	0.66	1.1	-3.84	-3.16	-1.81	1902.5	3	0.0001

Item discrimination and location parameters, estimated using IRTPRO software program for South Korea and Turkey samplings, are given in the table. In last 3 column of Table  $\chi^2$ , degree of freedom, and significance levels pertaining to parameter comparison results belonging to DIF analysis are provided. As shown in Table 3  $\chi^2$ , results are all significant. In other words, in all items DIF, favoring Turkish sampling, was detected. As an example, in item 4, Turkish students reached " disagree", " agree" and " strongly agree" grades, .at lower skill levels than South Korean students. This situation can be observed even better in Figure 3 below.

In Figure 3, Item characteristic curves for item 4 of test calculated over South Korean and Turkish samplings. The figure shows that  $\theta$  level, at which South Korean students reached from 0 level (strongly disagree) to level 1 (disagree), is much lower comparing to Turkish sample. Nevertheless examination of the figures shows that Turkish sample reaches from the level "Agree" to "Strongly Agree "at much lower  $\theta$  level than South Korean sample.



*Figure 3. Item Characteristic curves for item 4 from South Korean and Turkish samplings.*

In the rest of the study, nonparametric IRT analysis for the same scale, were conducted by TestGraf software program. DIF results determined for scale items and levels are provided in Table 4 below.

**Table 4.** DIF analysis results based on nonparametric IRT for Turkish and South Korean Samples

Item	Level	$\beta$	se	z
1	1	0.022	0.015	1.467
	2	-0.05	0.015	-3,333**
	3	-0.061	0.015	-4,067**
	4	0.089	0.015	5,933**
2	1	0.085	0.017	5,000**
	2	0.042	0.017	2,471*
	3	-0.155	0.017	-9,118**
	4	0.027	0.017	1.588
3	1	0.062	0.016	3,875**
	2	0.074	0.016	4,625**
	3	-0.164	0.016	-10,250**
	4	0.027	0.016	1.688
4	1	0.029	0.016	1.813
	2	-0.023	0.016	-1.438
	3	-0.084	0.016	-5,250**
	4	0.078	0.016	4,875**
5	1	0.037	0.015	2,467*
	2	-0.051	0.015	-3,400**
	3	-0.041	0.015	-2,733**
	4	0.054	0.015	3,600**
6	1	0.003	0.015	0,200*
	2	-0.025	0.015	-1,667*
	3	-0.146	0.015	-9,733**
	4	0.169	0.015	11,267**

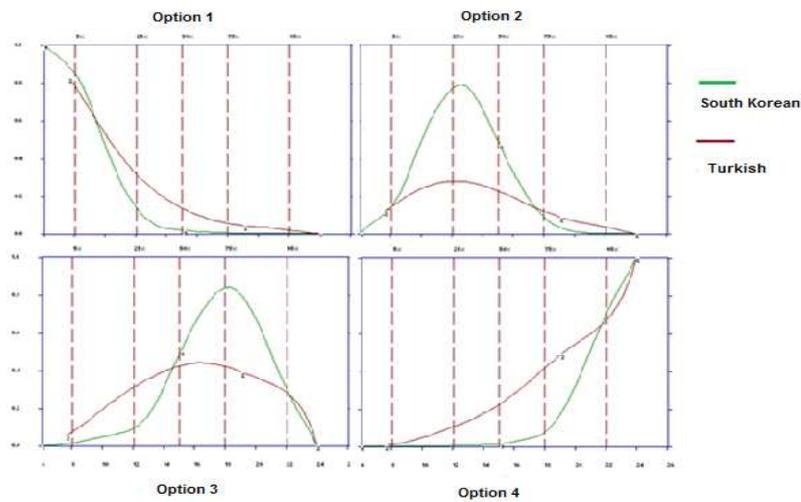
\* Options include DIF

As it can be seen from the Table,  $\beta$  values and their standard errors are calculated separately for each item and its each level pertaining to DIF and significance of DIF was tested using Z statistic. As an example, examining item 4 shows that in agree and strongly agree levels DIF was found statistically significant. Moreover, signs of z values also indicates at the same time DIF direction. In this case, Item 4 Agree level and Strongly Agree levels are reached in lower  $\theta$  levels for focal group and references group respectively. Level (option) characteristic curves obtained by Kernel Smoothing for each level for this item were given in Figure 4.

For this item, it is found that students in Turkish sampling easily reached strongly agree level in the lower skill levels than South Korean students.

DIF analysis results according to sex groups from Turkish and South Korean samplings.

DIF analysis results based on parametric IRT in sex groups in Turkey and sex groups in South Korea are given in Table 5 and Table 6 respectively.



**Figure 2.** DIF analysis results based on nonparametric IRT for Turkish and South Korean Samplings Item 4 categories response curves.

**Table 5.** DIF Analysis results based on parametric IRT for sex groups from Turkish sample.

Item	Girls				Boys				$\chi^2_{c a}$	d.f.	p
	a	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>	a	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>			
1	3.75	-1.44	-1.03	0.02	3.55	-1.46	-1.02	-0.03	5.7	3	0.1284
2	1.83	-1	-0.02	0.49	1.3	-1.2	-0.09	0.56	7.5	3	0.0587
3	2.45	-1.19	-0.17	0.35	2.02	-1.22	-0.18	0.45	6.1	3	0.1045
4	1.18	-2.53	-1.57	0.07	1.47	-2.02	-1.22	0.05	8.8	3	0.032
5	5.56	-1.14	-0.77	0.14	6.98	-1.12	-0.74	0.08	3.5	3	0.3216
6	1.02	-4.57	-3.73	-2.12	1.16	-3.43	-2.82	-1.61	35.4	3	0.0001

Table 6. DIF Analysis results based on parametric IRT for sex groups from South Korean sample.

Item	Girls				Boys				$X^2_{cfa}$	d.f.	p
	a	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>	a	b <sub>1</sub>	b <sub>2</sub>	b <sub>3</sub>			
1	4.97	-1.02	0.18	1.45	4.57	-1	0.18	1.48	1.5	3	0.6812
2	1.79	-1.08	0.45	1.97	1.68	-1.03	0.31	1.8	28.6	3	0.0001
3	2.85	-1.23	0.13	1.63	2.44	-1.14	0.18	1.69	13.8	3	0.0032
4	3.18	-1.31	0.07	1.57	3.07	-1.24	0	1.57	10.3	3	0.016
5	6.07	-0.8	0.31	1.42	5.89	-0.84	0.29	1.48	2.9	3	0.4089
6	0.59	-5.51	-2.88	0.8	0.54	-5.12	-2.92	0.71	16.2	3	0.001

In Table 5,  $\chi^2$  statistics detecting DIF and significance level of this statistics and item parameters estimated by IRTPRO program for groups are provided. Findings indicate that items 4 and 6 in scale incorporate DIF favoring girls in sex groups. In the second stage of analysis, item not exhibiting DIF are taken as connection items and items 4 and 6 were chosen as candidate items exhibiting DIF. Results also showed that items 4 and 6 incorporates DIF favoring girls. The same calculations were repeated for sex groups from South Korean sampling and analysis results are provided in Table 6.

Table 7. DIF analysis results based on nonparametric IRT for sex groups from both Turkish and South Korean Samples

Item	Option	Turkey Sex			South Korea Sex		
		$\beta$	se	z	$\beta$	se	z
1	1	0	0.009	0.000	0.012	0.014	0.857
	2	0.001	0.009	0.111	-0.005	0.014	-0.357
	3	-0.019	0.009	-2,111*	-0.014	0.014	-1.000
	4	0.018	0.009	2,000*	0.007	0.014	0.500
2	1	0.003	0.013	0.231	0.016	0.016	1.000
	2	-0.015	0.013	-1.154	-0.048	0.016	-3,000*
	3	0.004	0.013	0.308	0.004	0.016	0.250
	4	0.008	0.013	0.615	0.027	0.016	1.688
3	1	0.003	0.012	0.250	0.023	0.015	1.533
	2	-0.008	0.012	-0.667	-0.006	0.015	-0.400
	3	0.014	0.012	1.167	-0.024	0.015	-1.600
	4	-0.009	0.012	-0.750	0.007	0.015	0.467
4	1	0.008	0.011	0.727	0.013	0.014	0.929
	2	0.005	0.011	0.455	-0.029	0.014	-2,071*
	3	-0.023	0.011	-2,091*	0.008	0.014	0.571
	4	0.009	0.011	0.818	0.007	0.014	0.500
5	1	-0.002	0.009	-0.222	-0.001	0.014	-0.071
	2	0	0.009	0.000	-0.003	0.014	-0.214
	3	-0.019	0.009	-2,111*	0.003	0.014	0.214
	4	0.021	0.009	2,333*	0.001	0.014	0.071
6	1	0.006	0.008	0.750	0.013	0.017	0.765
	2	0.004	0.008	0.500	-0.006	0.017	-0.353
	3	0.011	0.008	1.375	-0.016	0.017	-0.941
	4	-0.022	0.008	-2,750*	0.009	0.017	0.529

\* Options include DIF

Examining findings in Table 6, In South Korean sample, demonstrated that items 2, 3, 4 and 6 in scale incorporated DIF for sex groups. As shown in Table, item 2 exhibits DIF favoring boys and items 3,4 and 6 exhibit DIF favoring girls. At the second stage of analysis, item not exhibiting DIF are taken as connection items and items 2,3, 4 and 6 were chosen as candidate items exhibiting DIF and analysis was repeated. Analysis results demonstrated again the same item 4 incorporated DIF favoring the same groups.

DIF analysis results based on nonparametric IRT both in sex groups from Turkey and from South Korea were provided in Table 7.

In Table 7, DIF analysis results based on nonparametric IRT for Turkish sample indicated that items 1,4, 5 and 6 in the scale exhibit DIF according to sex groups. According

to Analysis results, 2 an 4 options for item 1, 3 option for item 4, and 3 and 4 options for item 5 and 4 level for item 6 incorporate DIF for sex groups. Signs of parameters can be used to understand which option for an item exhibits DIF favoring a group. For example, girls reached level 3 for item 1 in lower skills levels whereas boys may reach level 4 in lower skills levels. In South Korean sample, DIF analysis results based on nonparametric IRT indicated DIF in one level for only item 2 and 4.

### 4. Conclusions

In the scope of this study, parametric and nonparametric DIF determination methods were examined comparatively. To this end, Attitude towards mathematics items in TIMSS 2011 student survey were used as real data and it is determined if items incorporated DIF or not in terms of sex and country variables. It is noted that parametric and nonparametric methods produced generally similar results for DIF analysis in which countries were labeled as reference and focal groups,

Nevertheless, DIF analysis results for country based sex groups demonstrated differences according to techniques based on parametric and nonparametric IRT. DIF analysis based on parametric IRT for Sex groups on Turkish sampling, determined 2 items incorporated DIF whereas DIF analysis based on nonparametric IRT detected DIF in 4 items. Nonparametric methods, using Turkish sampling, determined DIF in two levels per items 1 and 5 according to sex groups whereas parametric methods did not detect DIF

for neither of these items. This indicated different techniques produced different results for DIF analysis. For South Korean sampling, similar results were observed for sex groups.

A review of relevant field studies reveals that measurements regarding structures in education and psychology provided information at the level of ordering scale. (Allen & Yen, 1979; Murphy & Davidshofer, 1998; Torgerson, 1958). Besides, Sijtsma and Molenaar (2002), on account of preponderance of rating scale level for personality, attitude, achievement or skills measurements in education and psychology, propounded nonparametric methods in DIF analysis for the tools measuring the same traits.

Results of the study showed that items incorporating DMF exhibit difference according to preferred technique. This indicated importance of choosing best fit technique in studies investigating whether scale items incorporated DMF or not. In DIF analysis, which is an important stage in developing and validating scales, researchers should consider level of scale for the measurement and whether assumptions of methods were satisfied or not and chose the best technique accordingly.

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