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Partially neighbor balanced designs for circular blocks

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Abstract: A partially neighbor balanced design is a design in which for any fixed treatment, other treatments occur as neighbor λ_i times. This paper generates infinite series of one-dimensional partially neighbor balanced designs for $v = n$ treatments. The blocks used in these designs are considered circular. Designs given here are partially balanced in terms of nearest neighbors and not necessarily in terms of variance. Binary and non-binary concepts have been used for the construction of designs. Theorem 1 generates binary generalized 2-neighbor designs and theorem 2 generates non-binary generalized 3-neighbor designs. These theorems generate designs for $v = n$ treatments i.e., for odd and even number of treatments simultaneously. This concept remains relatively under-explored in the literature. The objective is to decrease error variance due to neighbor effect and reduce computational cost.

Keywords: Non-Binary Blocks, Generalized 2-Neighbor Designs, Generalized 3-Neighbor Designs

1. Introduction

The ouchterlony gel diffusion test is a technique used to identify the relationship of a set of antigens and their individual components. The technique involves cultivating samples of different antigens in a suitable culture medium around an antibody. This test requires suitable ordered sets of antigens. Neighbor circular designs are recommended as usual statistical techniques cannot be applied on this type of observations. Neighbor designs are recommended when response on one plot may be affected by treatments on neighboring plots as well as by the treatment applied to that plot. A neighbor design is called one-dimensional if neighbor effects are controlled in only one way, i.e., either in row or in column direction. One-dimensional neighbor designs are used in circular plates in biometrics and in block design setup in the field of agriculture where each block is a single line of plots and blocks are well separated. In this design setup, if each block is a single line of plots, either extra parameters or bordered plots on both ends of every block for the left and right neighbor effect are needed. These bordered plots are not used for measuring response variables.

Rees (1967) introduced neighbor balanced designs for the ouchterlony gel diffusion test. He suggested more than 40 designs on circular plates but only for odd number of antigens. Hwang (1973) constructed some infinite classes of neighbor designs for $\lambda = 1$ using cyclic methods for (i) $v = 2k+1$ (ii) $v = 2^i k+1$ (iii) $v = 2mk+1$ treatments. Misra et al. (1991) introduced generalized t-neighbor design in which the equality condition on λ as well as on k was relaxed. As per this definition, for a fixed treatment θ, the rest of $(v-1)$ treatments can be divided into groups according to values of λ, independent of the treatments. Any pair of treatments can occur λ_1 or λ_2 or . . . or λ_t times as neighbors but no treatment should occur side by side.

Azais et al. (1993) proposed several methods for constructing circular neighbor designs for $k = v$ and $v-1$ treatments. Preece (1994), putting the condition on parameter λ, developed a quasi-Rees neighbor design in which only one treatment occurs as a neighbor twice and remaining once i.e. $\lambda_1 = 1$ and $\lambda_2 = 2$. Chaure and Misra (1996) constructed generalize t-neighbor designs by arranging the treatments systematically in initial blocks as well as by Galois field and PBIB design. Nutan (2007) constructed a new series of generalized binary and proper neighbor designs, and a series of binary neighbor designs with two distinct block sizes. Using cyclic method, Kedia and Mishra (2008) constructed some new series of generalized 2-neighbour designs and generalized 3-neighbour designs. Hamad et al. (2010) generated non-binary neighbor balanced designs for $v = 2n$ treatment.

2. Partially Neighbor Balanced Designs for Circular Blocks

Neighbor balanced designs become un-necessary when neighbor effect between any two treatments is not important or very costly or has known adverse effect. For example sometimes in agriculture experiment, such response variable is required when certain treatments are repeated side by side more than others. Under such conditions, we prefer partially neighbor balanced designs. In partially neighbor balanced designs, pairs of distinct treatment may appear as neighbor $\lambda_1, \lambda_2, \ldots, \lambda_t$ times, $\lambda_i \geq 0$, $i = 1,2,\ldots,t$. In partially neighbor balanced designs, any two pairs of treatments do not occur for same number of times.

In partially neighbor balanced design, for any fixed treatment, the other treatments are divided into t groups according to values of λ. That is why we can call partially neighbor balanced design a generalized t-neighbor design, $t > 1$. A partially neighbor balanced design with λ_1 and λ_2 is a generalized 2-neighbor design in which treatments are divided in to two groups. Similarly a partially neighbor balanced design with λ_1, λ_2 and λ_3 is a generalized 3-neighbor design in which treatments are divided in to three groups with respect to nearest neighbor-ness. Here in we have constructed a series of binary generalized 2-neighbor and non-binary generalized 3-neighbor designs for circular blocks.

2.1. Construction of Generalized 2-Neighbor and Generalized 3-Neighbor Circular Designs

Let the treatments coming under modulo v be assigned numbers 0 to v-1. Forward and backward differences are used to generate initial block. The difference of first and last number is also included. The other blocks are obtained from the initial block by cycling the treatments. In all blocks there are exactly r replicates of every treatment. The numbers 0 to v-1 in all blocks are arranged in such ways that for a fixed treatment, the rest treatments can occur as neighbor λ_1, λ_2 or λ_1, λ_2, λ_3 times. Let $(j_1, j_2, j_3, \ldots, j_v)$ be the v treatments, not necessarily distinct, of initial block, then forward and backward differences are;

$$\mathrm{F} = \left(j_2 - j_1,\ j_3 - j_2,\ j_4 - j_3, \ldots\ldots\ldots, j_1 - j_v\right) \text{ and}$$

$$\mathrm{B} = \left(j_1 - j_2,\ j_2 - j_3,\ j_3 - j_4, \ldots\ldots\ldots, j_v - j_1\right).$$

The initial block when developed under modulo v, generates a generalized 2-neighbor and generalized 3-neighbor design if:

1. The difference of any two adjacent treatments is from 1 to v-1;
2. Forward and back ward differences are in circular order and in a given direction; say clockwise;
3. The sum of forward and backward differences is equal to zero;
4. Among the totality of forward and backward differences, treatments occur as neighbor either λ_1, λ_2 or λ_1, λ_2, λ_3 times.

3. Main Results

3.1. Theorem 1

Let n be any positive number, $n \geq 6$. Then there exists a generalized 2-neighbor design for $v = n$ treatments. Consider single circular initial block of distinct treatments of size k coming under modulo v;

$$\mathrm{I} = \{j_1, j_2, j_3, \ldots, j_k\}.$$

If treatment number is even then block size, $k = (v+2)/2$ and if number is odd then $k = (v+1)/2$. This initial block generates generalized 2-neighbor binary circular design for;

1. Odd treatments with parameters $k = r = (v+1)/2$, $b = v$, $\lambda_1 = 1$ and $\lambda_2 = 2$. In this design $h_1 = v(k\text{-}2)$ pairs occur as nearest neighbor once and $h_2 = v$ pairs occur as nearest neighbor twice.
2. Even treatments with parameters $k = r = (v+2)/2$, $b = v$, $\lambda_1 = 1$ and $\lambda_2 = 2$. In this design $h_1 = v(k\text{-}3)$ pairs occur as nearest neighbor once and $h_2 = 3(v/2)$ pairs occur as nearest neighbor twice.

Proof

Let k distinct treatments appear in a circular initial block;

$$\mathrm{I} = \{j_1, j_2, j_3, \ldots, j_k\}.$$

The treatments numbers of initial block are given as;

$$\mathrm{I} = [j_1 = j, j_2 = j_1 - (k-1), j_3 = j_1 - 1, j_4 = j_2 + 1, j_5 = j_3 - 1, j_6 = j_4 + 1, \ldots, j_k = j_{k-2} \pm 1]$$

First treatment can be selected at random from $j = 0,1,2,\ldots,v-1$. From the initial block, forward and backward differences are;

$$\pm(j_2 - j_1), \pm(j_3 - j_2), \ldots\ldots\ldots, \pm(j_1 - j_k)$$

The remaining blocks are obtained by incremental cycling of the initial block under modulo v.

In case of odd number of treatments, only $(j_k - j_1)$ appears twice and other differences occur once among the totality of forward and backward differences of all blocks giving $\lambda_1 = 1$ and $\lambda_2 = 2$. In all blocks $h_1 = v(k\text{-}2)$ pairs occur as neighbor once and $h_2 = v$ pairs occur twice as neighbor. The above initial block generates generalized 2-neighbor binary circular design with parameters;

$k = r = (v+1)/2$, $b = v$, $h_1 = v\,(k\text{-}2)$, $\lambda_1 = 1$, $h_2 = v$ and $\lambda_2 = 2$.

In case of even number of treatments, among the totality of forward and back, it appears that only one difference is repeated but remember that in case of even number of treatments, the neighboring treatments whose difference is of size $(v/2)$ actually occur two times, i.e. once as left and once as right neighbor. Thus differences $(j_2 - j_1)$ and $(j_k - j_1)$ appears twice and other differences occur once giving $\lambda_1 = 1$ and $\lambda_2 = 2$. In all blocks $h_1 = v(k\text{-}3)$ pairs occur once and $h_2 = 3(v/2)$ pairs occur twice as neighbor. This initial block yields generalized 2-neighbor binary circular design with parameters;

$$k = r = (v+2)/2,\ b = v,\ h_1 = v(k\text{-}3),\ \lambda_1 = 1,$$

$$h_2 = 3(v/2) \text{ and } \lambda_2 = 2.$$

Note

No attention has been given in literature on the construction of one-dimensional neighbor circular designs for odd number and even number of treatments using same theorem. Ahmed et al. (2009) constructed generalized 2-neighbor design for odd and even number of treatments by two different theorems using cycling shifts. One more difference is construction methodology. No generalized 2-neighbor design is ever developed for odd and even number of treatments (simultaneously) before.

Example 1

Let we have $v = n = 11$ treatments. The initial block for $k = (v+1)/2 = 6$ is;

$$\mathrm{I} = [\, j_1 = j, j_2 = j_1 - (k-1), j_3 = j_1 - 1, j_4 = j_2 + 1, j_5 = j_3 - 1, j_6 = j_4 + 1]$$

Taking $j = 6$ in the initial block we get binary block of size 6 from the theorem is;

$$\mathrm{I} = [\, j_1 = 6,\ j_2 = 6-5 = 1,\ j_3 = 6-1 = 5,\ j_4 = 1+1 = 2,$$
$$j_5 = 5-1 = 4,\ j_6 = 2+1 = 3],$$

$$\mathrm{I} = [\, j_1 = 6,\ j_2 = 1,\ j_3 = 5,\ j_4 = 2,\ j_5 = 4,\ j_6 = 3],$$

$$\mathrm{I} = [6, 1, 5, 2, 4, 3].$$

Forward and backward differences of initial block are;

$$= \pm(1-6), \pm(5-1), \pm(2-5), \pm(4-2), \pm(3-4), \pm(6-3),$$

$$= \pm(-5), \pm(4), \pm(-3), \pm(2), \pm(-1), \pm(3).$$

Sum of all these differences is equal to zero. In the forward and backward differences of initial block, only one difference is repeated where as four differences occur once. In all blocks $h_1 = 44$ pairs occur once and $h_2 = 11$ pairs occur twice as neighbor. The remaining blocks can be obtained by cycling the treatments of initial block under modulo 11, which are as;

(7, 2, 6, 3, 5, 4), (8, 3, 7, 4, 6, 5), (9, 4, 8, 5, 7, 6), (10, 5, 9, 6, 8, 7), (0, 6, 10, 7, 9, 8), (1, 7, 0, 8, 10, 9), (2, 8, 1, 9, 0, 10), (3, 9, 2, 10, 1, 0), (4, 10, 3, 0, 2, 1), (5, 0, 4, 1, 3, 2).

These blocks yield a binary neighbor circular design with parameters:

$$v = 11,\ r = k = 6;\ \mathrm{b} = 11,\ \lambda_1 = 1 \text{ and } \lambda_2 = 2.$$

Example 2

Let there are $v = 2n = 10$ treatments. Putting $n = 5$ and $j = 6$ in the following initial block, we get**;**

$$\mathrm{I} = [\, j_1 = j, j_2 = j_1 - n, j_3 = j_1 - 1, j_4 = j_2 + 1, j_5 = j_3 - 1, j_6 = j_4 + 1],$$

$$\mathrm{I} = [6, 1, 5, 2, 4, 3].$$

Forward and backward differences of initial block are;

$$= \pm(-5), \pm(4), \pm(-3), \pm(2), \pm(-1), \pm(3).$$

The remaining blocks can be obtained under modulo 10 through initial block, which are as;

(7, 2, 6, 3, 5, 4), (8, 3, 7, 4, 6, 5), (9, 4, 8, 5, 7, 6), (0, 5, 9, 6, 8, 7), (1, 6, 0, 7, 9, 8), (2, 7, 1, 8, 0, 9), (3, 8, 2, 9, 1, 0), (4, 9, 3, 0, 2, 1), (5, 0, 4, 1, 3, 2).

We can see that neighboring treatments having difference of size n (in this case it is 5) and difference of 3 occur two times. In all blocks $h_1 = 30$ pairs occur once and $h_2 = 15$ pairs occur twice as neighbor. These blocks yield a binary neighbor circular design with parameters:

$$v = 10,\ r = k = 6;\ \mathrm{b} = 10,\ \lambda_1 = 1 \text{ and } \lambda_2 = 2$$

Corollary

If we delete the last treatment of initial block in theorem 1 then we get generalized 3-neighbor design. This initial block generates generalized 3-neighbor binary circular design with $\lambda_1 = 0$, $\lambda_2 = 1$ and $\lambda_3 = 2$ for $v = n$ treatments. These types of designs can be used in the situations when neighbor effect between certain two treatments is not important. For example in agriculture, certain two varieties are not wanted side by side due to adverse neighbor effect i.e., $\lambda_1 = 0$.

3.2. Theorem 2

Let there are $v = n$ treatments, where $n > 6$. Consider non-binary initial block of size k coming under modulo v;

$$\mathrm{I} = \{ j_1, j_2, j_3, \ldots, j_k \}.$$

If treatment number is even then block size, $k = v\text{-}1$ and for odd $k = v\text{-}2$. This initial block generates generalized 3-neighbor non-binary circular design for odd number of treatments with parameters $k = r = v\text{-}2$, $b = v$, $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$. In this design $h_1 = 2v$ pairs of distinct treatments occur as neighbor once, $h_2 = (k\text{-}5)(v/2)$ pairs occur as neighbor twice and $h_3 = v$ pairs occur as neighbor thrice.

Similarly this initial block generates generalized 3-neighbor non-binary circular design for even number of treatments with parameters $k = r = v\text{-}1$, $b = v$, $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$. In this design $h_1 = v$ pairs occur as neighbor once, $h_2 = (k\text{-}4)(v/2)$ pairs occur as neighbor twice and $h_3 = v$ pairs occur as neighbor thrice.

Proof

Let the k number of treatments appear in a circular initial block;

$$\mathrm{I} = \{ j_1, j_2, j_3, \ldots, j_k \}.$$

Treatment numbers of initial block are;

$$\mathrm{I} = \{ j_1 = j, j_2 = j_1 - 2, j_3 = j_2 - 3, \ldots, j_{(k+1)/2} = j_{(k-1)/2} - (k+1)/2,$$

$$j_{(k+3)/2} = j_{(k+1)/2} + 1, j_{(k+5)/2} = j_{(k+3)/2} + 2, \ldots, j_K = j_{k-1} + (k-1)/2 \}$$

First treatment can be selected at random from

$j = 0,1,2,...,v-1$. From the initial block, forward and backward differences are;

$$\pm(j_2 - j_1), \pm(j_3 - j_2), ..., \pm(j_k - j_1).$$

In case of odd number of treatments, among the totality of forward and backward differences of all blocks, $(j_{(k+1)/2} - j_{(k-1)/2})$ and $(j_{(k+3)/2} - j_{(k+1)/2})$ appears once giving $\lambda_1 = 1$, $(j_k - j_1)$ differences occur thrice giving $\lambda_3 = 3$ and remaining differences occur twice giving $\lambda_2 = 2$. The remaining blocks are obtained by cycling the initial block under modulo v. In all blocks $h_1 = 2v$ pairs of distinct treatments occur as neighbor one time, $h_2 = (k\text{-}5)v$ pairs occur as neighbor two times and $h_3 = v$ pairs occur as neighbor three times. The above initial block generates generalized 3-neighbor non-binary neighbor circular design for odd number of treatments with parameters;

$k = r = 2n\text{-}1$, $b = v$, $h_1 = 2v$, $\lambda_1 = 1$, $h_2 = (k\text{-}5)(v/2)$,

$$\lambda_2 = 2,\ h_3 = v \text{ and } \lambda_3 = 3.$$

In case of even number of treatments, among the totality of forward and backward differences of all blocks, only $(j_{(k+3)/2} - j_{(k+1)/2})$ appears once giving $\lambda_1 = 1$, $(j_k - j_1)$ differences occur thrice giving $\lambda_3 = 3$ and remaining differences occur twice giving $\lambda_2 = 2$. Remaining blocks are obtained by cycling the initial block under modulo v. In all blocks $h_1 = v$ pairs occur as neighbor one times, $h_2 = (k\text{-}4)(v/2)$ pairs occur as neighbor two times and $h_3 = v$ pairs occur as neighbor three times. The above initial block generates generalized 3-neighbor non-binary neighbor circular design for even number of treatments with parameters;

$$k = r = 2n\text{-}1,\ b = v,\ h_1 = v,\ \lambda_1 = 1,\ h_2 = (k\text{-}4)(v/2),$$

$$\lambda_2 = 2,\ h_3 = v \text{ and } \lambda_3 = 3.$$

Example 3

Let we have $v = n = 9$ treatments. Since treatment number is odd so initial block of size $k = v\text{-}2 = 7$ is;

$$\mathrm{I} = \{j_1 = j, j_2 = j_1 - 2, j_3 = j_2 - 3, j_4 = j_3 - 4, j_5 = j_4 + 1, j_6 = j_5 + 2, j_7 = j_6 + 3\}$$

Taking $j = 8$, we get non-binary initial block of size 7 as;

$$\mathrm{I} = \{j_1 = 8, j_2 = 8-2=6, j_3 = 6-3=3, j_4 = 3-4=8, j_5 = 8+1=0, j_6 = 0+2, j_7 = 2+3=5\},$$

$$\mathrm{I}_1 = [8, 6, 3, 8, 0, 2, 5].$$

Forward and backward differences of initial block are;

$$= \pm(6-8), \pm(3-6), \pm(8-3), \pm(0-8), \pm(2-0), \pm(5-2), \pm(8-5),$$

$$= \pm(-2), \pm(-3), \pm(5), \pm(-8), \pm(2), \pm(3), \pm(3).$$

Among forward and backward differences, two differences occurs once, one difference appears twice and one difference is repeated thrice giving $\lambda_1=1$, $\lambda_2=2$ and $\lambda_3=3$.

The remaining blocks can be obtained by cycling the treatments of initial block under modulo 9, which are as;
(0, 7, 4, 0, 1, 3, 6), (1, 8, 5, 1, 2, 4, 7), (2, 0, 6, 2, 3, 5, 8), (3, 1, 7, 3, 4, 6, 0), (4, 2, 8, 4, 5, 7, 1), (5, 3, 0, 5, 6, 8, 2), (6, 4, 1, 6, 7, 0, 3), (7, 5, 2, 7, 8, 1, 4).

In all blocks $h_1 = 18$ neighbor pairs of distinct treatments occur once, $h_2 = 9$ pairs occur twice as neighbor and $h_3 = 9$ pairs occur thrice as neighbor giving $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$. These blocks yield a non-binary partially neighbor balanced circular design with parameters:

$$v = 9;\ r = k = 7;\ \mathrm{b} = 9;\ h_1 = 18;\ \lambda_1 = 1;\ h_2 = 9;$$

$$\lambda_2 = 2;\ h_3 = 9 \text{ and } \lambda_3 = 3.$$

Example 4

Let $v = n = 8$ treatments. Since treatment number is even so initial block of size $k = v\text{-}1 = 7$. Taking $j = 7$ in the theorem, we get non-binary initial block of size 7 as;

$$\mathrm{I} = (7, 5, 2, 6, 7, 1, 4).$$

The remaining blocks can be obtained under modulo 8 through initial block, which are as;
(0, 6, 3, 7, 0, 2, 5), (1, 7, 4, 0, 1, 3, 6), (2, 0, 5, 1, 2, 4, 7), (3, 1, 6, 2, 3, 5, 0), (4, 2, 7, 3, 4, 6, 1), (5, 3, 0, 4, 5, 7, 2), (6, 4, 1, 5, 6, 0, 3).

In all blocks $h_1 = 8$ neighbor pairs of distinct treatments occur once, $h_2 = 12$ pairs occur twice as neighbor and $h_3 = 8$ pairs occur thrice as neighbor giving $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$. These blocks yield a non-binary partially neighbor balanced circular design with parameters:

$$v = 8;\ r = k = 7;\ \mathrm{b} = 8;\ h_1 = 8;\ \lambda_1 = 1;\ h_2 = 12;$$

$$\lambda_2 = 2;\ h_3 = 8 \text{ and } \lambda_3 = 3.$$

No generalized 3-neighbor design is ever developed for odd and even number of treatments simultaneously.

4. Discussion

When observations are not orthogonal then we have to change the experimental technique for the reduction of error variance. Neighbor designs are used for this purpose. Neighbor effects should be controlled in the situation where experimental units are influenced by neighboring units for example in agriculture, the response on a given plot may be affected by treatments on neighboring plots as well as by the treatment applied to that plot. It is first time in literature that partially neighbor balanced designs are developed for odd and even number of treatments simultaneously. The blocks in the designs are either circular in nature or we deliberately make layout of blocks circular. Linear blocks with border plots serve this purpose but these border plots are not used for measuring response variables. The objective is to study the neighboring effect along with the main effect of the treatments.

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