

# Exponentially Weighted Moving Average Control Charts for Monitoring Ambient Ozone Levels in Muscat

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**Abstract:** Exponentially Weighted Moving Average (EWMA) control charts are proposed to monitor ambient ozone (O<sub>3</sub>) levels in the city center and industrial areas of Muscat. Weekly averages of 8-hourly concentrations of ozone over a period of one year were used. The EWMA charts showed significant shift in the mean ozone levels at both the sites. However, both the ozone series were found to have significant autocorrelation. Therefore Box-Jenkins autoregressive integrated moving average (ARIMA) models were fitted at the first stage and then residuals were taken to apply EWMA which revealed that the ozone levels in both areas are within natural tolerance limits as well as within the international standard limit.

**Keywords:** ARIMA, EWMA, Quality Control Charts

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## 1. Introduction

The Statistical control charts were primarily developed for quality management of manufacturing processes. However these could be used to monitor the environmental data but the methodology of the construction of such charts should be modified because the environmental data usually exhibit the property of autocorrelation while the control charts are commonly made under the assumption of independence of successive observations [1, 2, 3]. Even small levels of autocorrelation between successive observations can have big effect on the statistical properties of conventional control charts. In the present study we made such analysis by taking air quality data on pollutant concentrations of Ozone (O<sub>3</sub>) to investigate the effect of autocorrelation on the performance of Exponentially Weighted Moving Average (EWMA) control charts.

## 2. Data

Ozone is a gas found in different parts of the atmosphere. Ozone in the upper atmosphere, or stratosphere, helps protect the Earth from the sun's harmful rays. In the lowest level of the atmosphere, the troposphere, ozone is harmful to both human health and the environment. For this reason, ozone is often described as being "good up high and bad nearby".

Although some industrial sources release ozone directly into the environment, most ground-level ozone forms in the air from various chemical reactions. Variations in weather conditions also play an important role in determining ozone levels. Inhalation exposure to ozone has been linked to numerous respiratory health effects. High concentrations of ozone can also affect vegetation and ecosystems. Therefore monitoring and management of ambient concentrations of ozone to an acceptable level should be of high national importance for any country.

In the present study, a time series of 8-hourly weekly average concentrations of ozone over a period of one year were extracted from the records of a gauging station operated by the Ministry of Environment Oman in the city center of Muscat. A similar time series was also derived from the records of a gauging station installed by the Ministry of Environment Oman in an industrial area in the outside of urban area of Muscat. For the data of city center area, the weekly average of O<sub>3</sub> over a year was 13.30 ppb and standard deviation was 3.97 ppb. While for the data of industrial area, it was 13.1 ppb and standard deviation was 2.47 ppb. These averages for both city center area and industrial area were found to be though within the international standard of air quality and a t-test showed a non-significant difference between the average levels at

both the site. These series are presented in Figure #1 below.

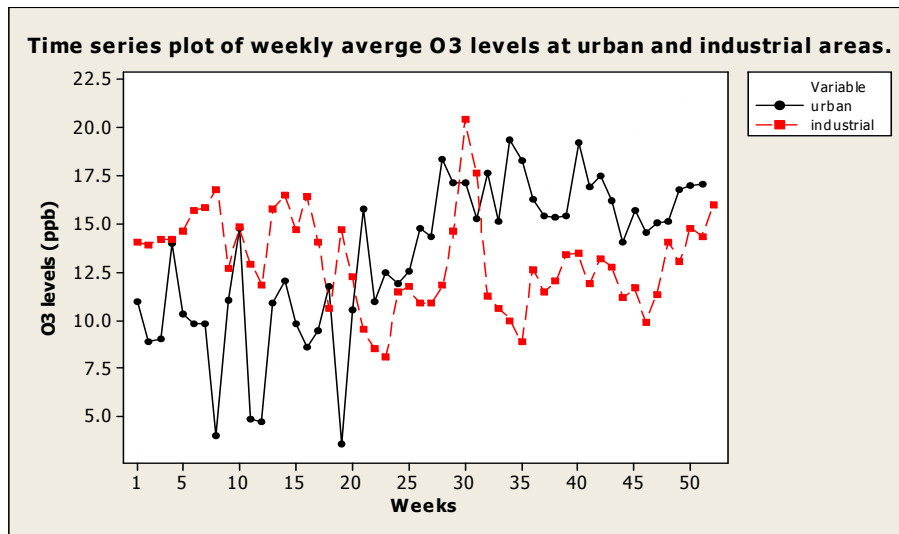


Figure 1. Time series plot of weekly average O3 levels at urban and industrial areas.

### 3. EWMA Control Charts

The Exponentially Weighted Moving Average (EWMA) is a statistic for monitoring the process that averages the data in a way that gives less and less weight to data as they are further removed in time. The statistic that is calculated is:

$$EWMA_t = \lambda Y_t + (1 - \lambda)EWMA_{t-1} \text{ for } t = 1, 2, \dots, n. \quad (1)$$

Where:

- EWMA is the mean of historical data (target)
- $Y_t$  is the observation at time  $t$
- $n$  is the number of observations to be monitored including  $EWMA_0$
- $0 < \lambda \leq 1$  is a constant that determines the depth of memory of the EWMA.

The control limits for EWMA are:

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1 - \lambda)^{2i}]} \quad (2)$$

$$CL = \mu_0 \quad (3)$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)} [1 - (1 - \lambda)^{2i}]} \quad (4)$$

Where the factor  $L$  is set equal 3. The data are assumed to be independent. [5]. The EWMA charts for O3 for urban and industrial area are in Figures #2 (a) and figure#2 (b) respectively as below. Both these charts clearly reveal significant shift in the mean of the process.

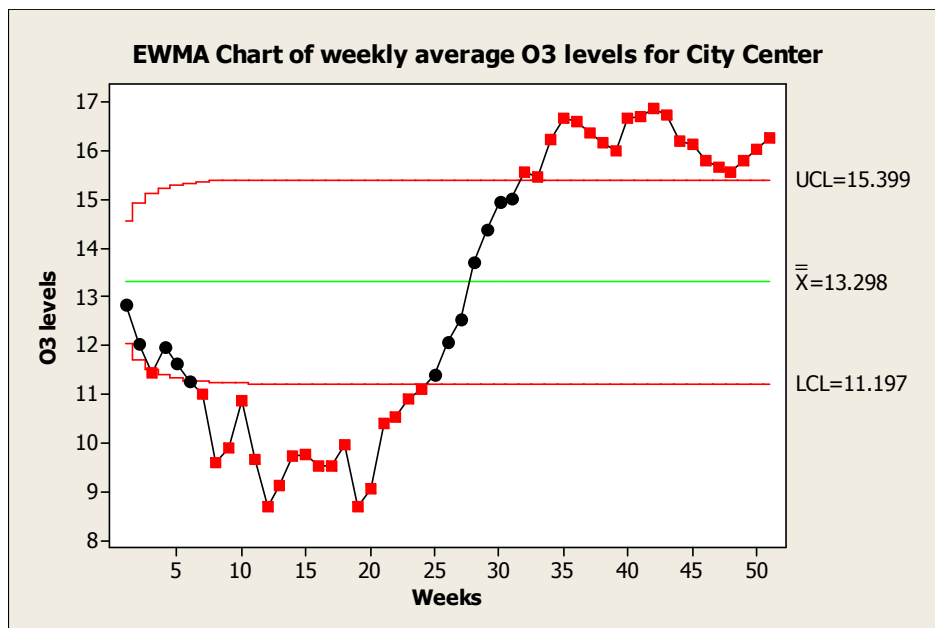


Figure 2(a). EWMA Chart of weekly average O3 levels for City Center.

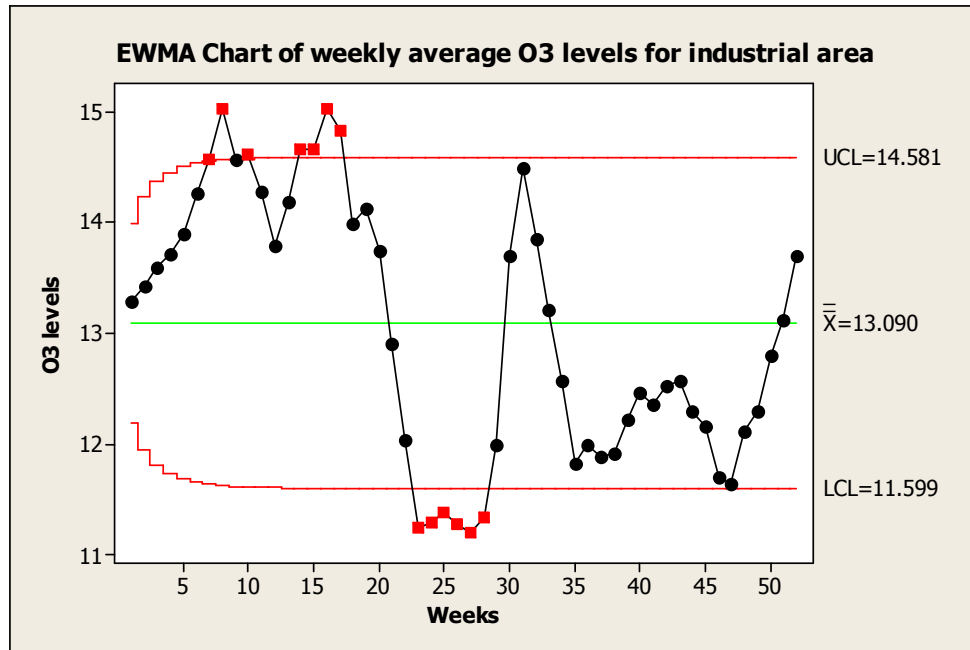


Figure 2(b). EWMA Chart of weekly average O3 levels for industrial area.

#### 4. Control Charts for Autocorrelated Data

If the data are not independent then there are two main approaches to apply Statistical Process Control. In the first approach the control limits are adjusted to account for autocorrelation. In the second approach the series are fitted with a suitable model such as Auto-Regressive Integrated Moving Average (ARIMA) models which has the general form as:

$$\phi_p(B)(1-B)^d Z_t = \theta_0 + \theta_q(B)a_t \quad (5)$$

where  $d$  is positive integer or zero,  $B$  is the backshift operator,  $\phi_p, \theta_q, \theta_0$  are parameters and  $a_t$  is white noise. [6,7,8,9,10]. Then the residuals from these models are used to construct control charts.

The ARIMA(2,1,0) models for both data were identified using Sample Autocorrelation (SAC) and Sample Partial Autocorrelation (SPAC). These models were then fitted to each of the urban and industrial areas ozone series. The residuals from these models were normally distributed. The EWMA control chart is applied to these residuals and are presented in the Figures #3 (a) and figure#3 (b) respectively.

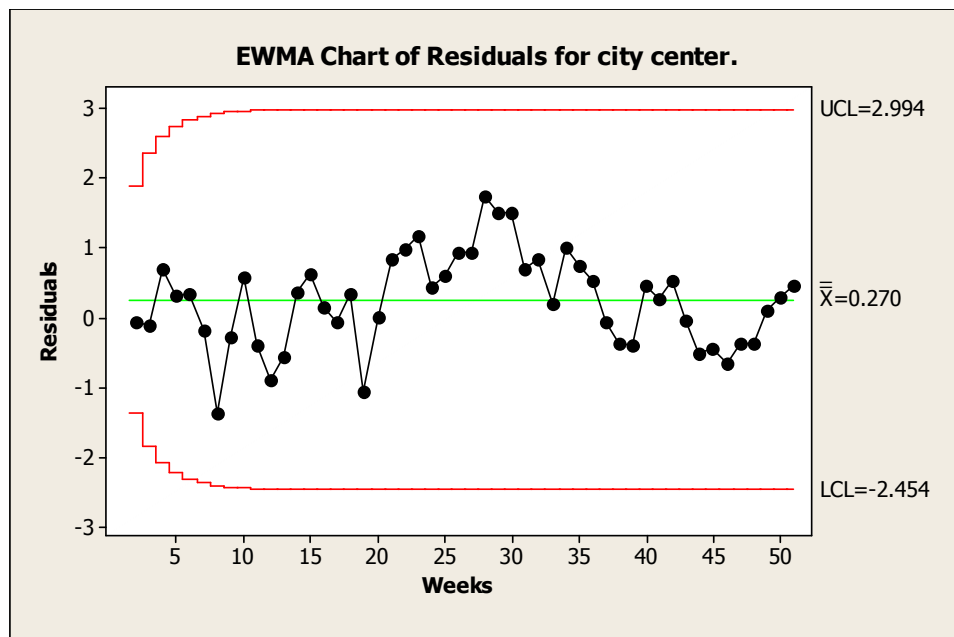


Figure 3(a). EWMA Chart of Residuals for city center.

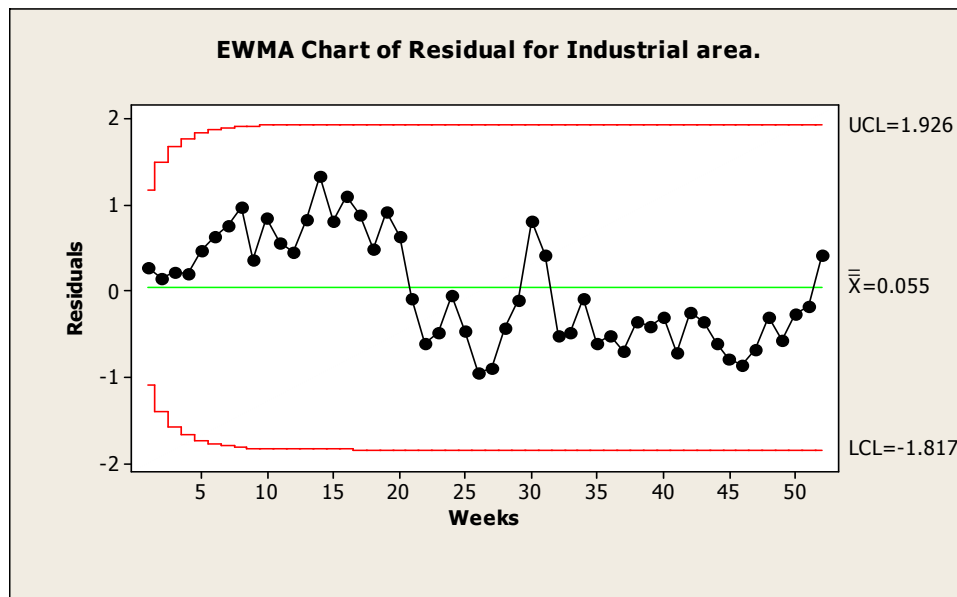


Figure 3(b). EWMA Chart of Residual for Industrial area.

## 5. Conclusion

We found that weekly average ozone levels of both urban area and industrial area had significant autocorrelations. Therefore an appropriate modification of existing statistical quality control techniques, in particular the EWMA chart was very necessary for using it to environmental process management and monitoring. When we used control charts on the assumption of no autocorrelation then we found that there were huge difference in the amount of O<sub>3</sub> between urban area and industrial area since most of the observations were seen to be out of control in city center. However when we applied these control charts by assuming that the data of O<sub>3</sub> were auto-correlated which was done by first fitting appropriate ARIMA models and that model was ARIMA (2, 1, 0) for both the series. After that we draw the EWMA chart for the residual and we found that the observations are within the natural control limit. This leads us to say that there is no evidence that the air quality data of industrial area is different from urban area. This means that the air quality in industrial area has not been affected by pollution alarmingly. Based on our analysis we found that the data in both areas are within the national standard limit.

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