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A new calculation technique of characteristics of multichannel measurer

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Abstract: A new calculation technique of accuracy characteristics of multichannel measurer was offered, ensuring essentially more exact results in comparison with the known ones. The expressions for the signal parameter maximum likelihood estimate bias and variance were found, in case of the multichannel reception of the quasidetermined or random signal and while the prior distribution of the informative parameter and the number of channels can be any. The influence of the value of the prior interval length and the number of channels on the accuracy of this estimate was investigated. The procedures were specified of the minimal channel number choice with the predetermined error of measurement. By statistical modeling methods the range of applicability of the theoretical formulae was established for the estimate characteristics, with the different numbers of channels and signal-to-noise ratios.

Keywords: Multichannel Reception, Maximum Likelihood Estimate, Measurement Characteristics, Statistical Modeling

1. Introduction

The optimal (according to a maximum likelihood method) signal parameter measurer receiving a realization of observable data in a kind of signal and hindrance sum generates the output effect which is proportional to the functional of likelihood ratio (FLR) or its logarithm [1, 2]. It is supposed that values of output effect are formed within the whole prior definition interval $[L_1, L_2]$ of the estimated parameter l_0. As a result, l_m the position of an absolute maximum of $M(l)$, logarithm of FLR is an estimate that should be determined. However it is seldom possible to make the optimal measurer with continuous change of current parameter value. In most cases it is necessary to resort to the multichannel scheme with discrete like values of unknown parameter [2, 3].

In case of the multichannel scheme the measurer yields $M(l)$ samples in ν equidistant points l_i, $i=1, 2, \ldots, \nu$ of the prior interval $[L_1, L_2]$. As an estimate the value l_k is accepted, corresponding to the channel with the greatest output signal.

Statistical characteristics of an estimate l_k of the signal parameter l_0 are considered in [2, 3], when the measurer is used with the odd number of channels. It was assumed that the output signal-to-noise ratio (SNR) does not depend on value l_0 (i.e. the parameter l_0 is nonpower), and prior distribution of the estimated parameter l_0 obeys to the uniform law. However, the theoretical formulae received there for bias (systematic error) and variance (mean square error) of multichannel estimate leave out of account all possible conditions and results of measurements, and thereof they may produce a major error. Besides, there is a big class of signals and unknown parameters for which assumptions mentioned above are not carried out. Also the number of channels in the measurer can be both odd and even. Thereby correction and generalization of the results [2, 3] are of interest, in case of the estimate of the arbitrary (power or nonpower) signal parameter l_0 with the prior distribution $w_{pr}(l_0)$, generally distinct from the uniform, if the multichannel scheme with any number of channels is used.

2. Characteristics of the Signal Parameter Estimate In Case of the Multichannel Reception

Let us suppose that the optimal (maximum likelihood) estimate l_m (in the receiver with a continuous change of a current value of the measurable parameter l_0) is described by the probability density $w(l_m | l_0)$. We designate the number of the channel, nearest to l_0, as n, and the distance between two adjacent channels as $\Delta = l_i - l_{i-1}$. Then with the fixed value of l_0 it is possible to present the conditional bias $b_\nu(l_k|l_0) = \langle l_k \rangle - l_0$ and variance $V_\nu(l_k|l_0) = \langle (l_k - l_0)^2 \rangle$ of the estimate l_k in the multichannel measurer as follows:

$$b_\nu(l_k|l_0) = \sum_{i=-(n-1)}^{\nu-n} (i\Delta)\, P_{0i}\,,\; V_\nu(l_k|l_0) = \sum_{i=-(n-1)}^{\nu-n} (i\Delta)^2\, P_{0i} \quad (1)$$

Where

$$P_{0i} = P\left[l_k - n\Delta = L_1 + (i - 1/2)\Delta\right] = \int_{L_1+(n+i-1)\Delta}^{L_1+(n+i)\Delta} w(l_m | l_0)\, dl_m\,,$$

and $\langle\ \rangle$ designates the averaging operation for all the possible realizations of the observable data.

If the parameter l_0 is described by the prior probability density $w_{pr}(l_0)$, then, similarly to Eq. (1), for the unconditional estimate bias $b_\nu(l_k)$ and variance $V_\nu(l_k)$, we have

$$b_\nu(l_k) = \sum_{i=-(\nu-1)}^{\nu-1} (i\Delta)\, P_i\,,\;\; V_\nu(l_k) = \sum_{i=-(\nu-1)}^{\nu-1} (i\Delta)^2\, P_i\,. \quad (2)$$

Here

$$P_i = \sum_{n=1}^{\nu} P\left[l_k - n\Delta = L_1 + (i - 1/2)\Delta\right] =$$
$$= \sum_{n=1}^{\nu} \int_{L_1+(n-1)\Delta}^{L_1+n\Delta} w_{pr}(l_0) \int_{L_1+(n+i-1)\Delta}^{L_1+(n+i)\Delta} w(l_m | l_0)\, dl_m dl_0\,.$$

Formulae (2) become essentially simpler, if the probability density $w(l_m | l_0)$ of the maximum likelihood estimate (MLE) l_m depends on a difference of arguments: $w(l_m | l_0) = w(l_m - l_0)$ – i.e. the parameter l_0 is nonpower [1, 2], and the prior distribution $w_{pr}(l_0)$ is described by the uniform distribution law: $w_{pr}(l_0) = 1/\nu\Delta$. Then for the probability P_i there can be written down

$$P_i = \frac{1}{\Delta} \int_0^\Delta \left[F(x + i\Delta) - F(x + (i-1)\Delta)\right] dx \quad (3)$$

Where $F(x) = \int_{L_1}^{x} w(l_m)\, dl_m$ is the distribution function of MLE l_m.

Substituting Eq. (3) in the Eq. (2) results in

$$b_\nu(l_k) = \sum_{i=-(\nu-1)}^{\nu-1} i \left[\int_{i\Delta}^{(i+1)\Delta} F(x)\, dx - \int_{(i-1)\Delta}^{i\Delta} F(x)\, dx\right],$$

$$V_\nu(l_k) = \Delta \sum_{i=-(\nu-1)}^{\nu-1} i^2 \left[\int_{i\Delta}^{(i+1)\Delta} F(x)\, dx - \int_{(i-1)\Delta}^{i\Delta} F(x)\, dx\right].$$

If the distribution of MLE l_m is symmetric concurrently, i.e. $F(x) = 1 - F(-x)$, then the estimate l_k in the multichannel measurer is unbiased – $b_\nu(l_k) = 0$, and its variance is

$$V_\nu(l_k) = 2\Delta \sum_{i=1}^{\nu-1} i^2 \int_0^\Delta \left[F(x + i\Delta) - F(x + (i-1)\Delta)\right] dx \quad (4)$$

Under conditions of high posterior accuracy of MLE l_m, when the digitization errors are comparable with the potential ones, caused by the hindrance effect, it is possible to confine ourselves to the first summand in the sum (4). We designate the variance of MLE l_m as σ_0^2. Then, considering that $F(x + \Delta) \approx 1$ under $\Delta >> \sigma_0$ ($\Delta \geq 2 \div 3\sigma_0$), and extending the top limit of integration to infinity, we find the known result [2, 3]:

$$V_\nu(l_k) \cong 2\Delta \int_0^\infty \left[1 - F(x)\right] dx$$

Let us now consider the applicability of the formulae (2) in the practical applications.

3. Estimate of Rectangular Video Pulse Duration

Let an additive mix

$$x(t) = s(t, \tau_0) + n(t) \quad (5)$$

Be received by the measurer input.

$$s(t, \tau_0) = \begin{cases} a\,, & 0 \leq t \leq \tau_0\,, \\ 0\,, & t < 0,\ t > \tau_0\,, \end{cases} \quad (6)$$

Here is the useful signal with amplitude and duration τ_0, while $n(t)$ is Gaussian white noise with one-sided spectral density N_0. With the observable realization (5), it is necessary to estimate the parameter τ_0, which values are from the prior interval $[T_1, T_2]$.

In compliance with [1, 2], we write down the logarithm of FLR in terms of

$$M(\tau) = \frac{2a}{N_0} \int_0^\tau x(t)\, dt - \frac{a^2 \tau}{N_0}\,,\ \tau \in [T_1, T_2] \quad (7)$$

Then the estimate τ_k of the duration τ_0 of the pulse (6) in the maximum likelihood measurer with ν channels, is now defined as

$$\tau_k = \arg\sup M(\tau_i)\,,\ i = 1,\ 2, \ldots,\ \nu\,. \quad (8)$$

Here $\tau_i = T_1 + (i - 1/2)\Delta$ is the pulse duration, to which i-th channel is tuned, and $\Delta = (T_2 - T_1)/\nu$ is the above stated distance between the two adjacent channels.

According to [4], the probability density of MLE $\tau_m = \arg\sup M(\tau)$, $\tau \in [\mathrm{T}_1, \mathrm{T}_2]$ in the measurer with the continuous change of the current value of the estimated parameter can be represented as

$$w(\tau_m|\tau_0) = f\left(\frac{z_s^2|\tau_m - \tau_0|}{2\tau_s}\right) \Bigg/ \int_{\mathrm{T}_1}^{\mathrm{T}_2} f\left(\frac{z_s^2|\tau_m - \tau_0|}{2\tau_s}\right) d\tau_m \quad (9)$$

$$f(x) = \Phi\left(\sqrt{x/2}\right) - 1 + \exp(2x)\left[1 - \Phi\left(3\sqrt{x/2}\right)\right]$$

Where $z_s^2 = 2a^2\tau_s/N_0$ is the output SNR for middle pulse duration $\tau_s = (\mathrm{T}_1 + \mathrm{T}_2)/2$, and

$\Phi(x) = \int_{-\infty}^{x} \exp(-t^2/2)\, dt/\sqrt{2\pi}$ is the probability integral.

Then characteristics of the estimate (8) for the specified prior probability density $w_{pr}(\tau_0)$ of the parameter τ_0 can be found from Eq. (2), with $w_{pr}(\tau_0)$ and $w(\tau_m|\tau_0)$ (9) substituting $w_{pr}(\mathrm{l}_0)$ and $w(\mathrm{l}_m|\mathrm{l}_0)$, respectively.

In Figs. 1, 2 the theoretical dependences are shown of the normalized variance $\tilde{V}_\nu = V_\nu(\tau_k)/\tau_s^2$, calculated by the formulae (2), (9), with

$$w_{pr}(\tau_0) = 1/(\mathrm{T}_2 - \mathrm{T}_1) \quad (10)$$

and the prior interval length value of either $(\mathrm{T}_2 - \mathrm{T}_1)/\tau_s = 1$ (Fig. 1), or $(\mathrm{T}_2 - \mathrm{T}_1)/\tau_s = 2$ (Fig. 2). Here the curves 1 correspond to $\nu = 2$; 2 – $\nu = 3$; 3 – $\nu = 6$.

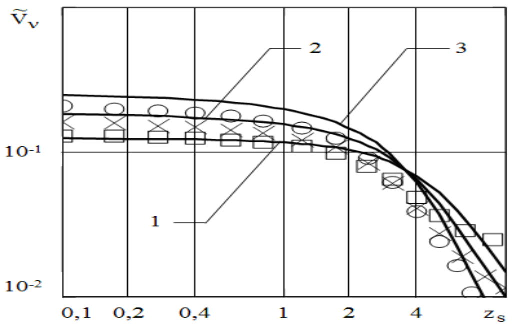

Figure 1. *Normalized variance of duration estimate of rectangular video pulse under $(T_2 - T_1)/\tau_S=1$*

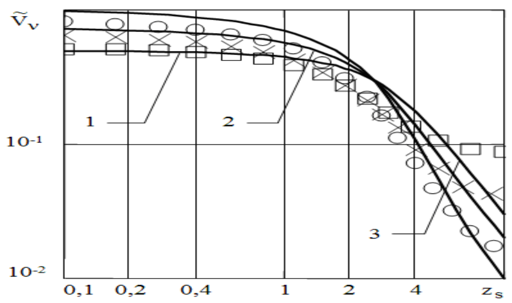

Figure 2. *Normalized variance of duration estimate of rectangular video pulse under $(T_2 - T_1)/\tau_S=2$.*

Experimental characteristics of the estimate (8) were found by means of statistical computer modeling. During the modeling process, the samples $M(\tau_i)$ of the functional $M(\tau)$ (7) were formed, from the prior interval $[\mathrm{T}_1, \mathrm{T}_2]$ and with the specified values of SNR z_s and durations τ_0, τ_i:

$$M(\tau_i) = z_s^2\left[\min(\tilde{\tau}_i, \tilde{\tau}_0) - \tilde{\tau}_i/2\right] + z_s W_{\tilde{\tau}}(\tilde{\tau}_i) \quad (11)$$

Here $\tilde{\tau}_i = \tau_i/\tau_s$, $\tilde{\tau}_0 = \tau_0/\tau_s$, and $W_{\tilde{\tau}}(\tilde{\tau}_i)$ is the sample of the standard Wiener process with zero mathematical expectation and dispersion $\tilde{\tau}_i$.

Samples of the Wiener process $W_{\tilde{\tau}}(\tilde{\tau}_i)$ in Eq. (11) were obtained as [5]

$$\begin{cases} W_{\tilde{\tau}}(\tilde{\tau}_i) = \alpha_1\sqrt{\tilde{\tau}_1}, & i = 1, \\ W_{\tilde{\tau}}(\tilde{\tau}_i) = W_{\tilde{\tau}}(\tilde{\tau}_{i-1}) + \alpha_i\sqrt{\tilde{\tau}_i - \tilde{\tau}_{i-1}}, & i > 1, \end{cases} \quad (12)$$

Where α_i are independent Gaussian random numbers with zero mathematical expectations and unit dispersions? Formation of Gaussian numbers α_i with parameters (0, 1) was carried out in terms of the independent random numbers ϑ_n, uniformly distributed within the interval [0, 1] by the Cornish-Fisher method [6]:

$$\alpha_i = \mathrm{A}_i + \frac{\mathrm{A}_i^3 - 3\mathrm{A}_i}{20N}, \quad A_i = \sqrt{\frac{12}{N}} \sum_{n=1}^{N}\left[\vartheta_{N(i-1)+n} - 0{,}5\right]. \quad (13)$$

As follows from [6], the number of summands N in the sum (13) was chosen equal to 5.

For each realization of $M(\tau)$, generated by Eqs. (11)-(13), both the estimate τ_k was determined according to Eq. (8) and its characteristics were calculated. It was supposed that duration τ_0 of the useful signal (6) in each testing was a uniformly distributed random value with a probability density (10). In Figs. 1, 2 experimental dependences of the variance $\tilde{V}_\nu$ for $\nu = 2$ (squares), $\nu = 3$ (crosses) and $\nu = 6$ (circles) are presented, obtaining from the statistical modeling. Each experimental $\tilde{V}_\nu$ value was found as a result of the processing of at least 10^5 realizations of $M(\tau)$. Consequently, confidence intervals boundaries deviate from experimental values no more than up to 5...10 % with the probability of 0.9.

According to Figs. 1, 2, the theoretical dependences for the variance of the multichannel duration estimate (8) approximate the experimental data in a satisfactory manner, at least, for $z_s \geq 0{,}1$, $\nu \geq 2$.

4. Estimate of Time and Power Parameters of Random Radio Pulse

Let us suppose now that the useful signal received against white noise $n(t)$ is random radio pulse, so that realization of the observable data has the appearance.

$$x(t) = s(t, \lambda_0, \tau_0, D_0) + n(t), \quad t \in [0, T], \quad (14)$$

Where

$$s(t,\lambda_0,\tau_0,D_0)=\xi(t)\; I\left(\frac{t-\lambda_0}{\tau_0}\right),\quad I(x)=\begin{cases}1\,, & |x|\le 1/2\,,\\ 0\,, & |x|>1/2\,,\end{cases} \tag{15}$$

λ_0 Is time delay, τ_0 is duration of the pulse, and $\xi(t)$ is centered stationary Gaussian random process possessing spectral density

$$G(\omega)=\frac{\pi D_0}{\Omega}\left[I\left(\frac{\theta-\omega}{\Omega}\right)+I\left(\frac{\theta+\omega}{\Omega}\right)\right] \tag{16}$$

In Eq. (16) it is designated: θ – band center, Ω – bandwidth and D_0 – dispersion of the process $\xi(t)$.

With the observable realization (14) it is necessary to estimate unknown parameters $\lambda_0\in[\Lambda_1,\Lambda_2]$, $\tau_0\in[\mathrm{T}_1,\mathrm{T}_2]$, $D_0\in[0,\infty)$. It is assumed that the condition $\Lambda_1-\mathrm{T}_2/2\le\lambda_0-\tau_0/2<\lambda_0+\tau_0/2\le\Lambda_2+\mathrm{T}_2/2$ is satisfied, so the pulse (15) is always situated within the observation interval $[0,T]$.

We now consider a practically important case when the process $\xi(t)$ fluctuations are "fast", i.e. $\mu=\tau_0\Omega/2\pi>>1$. Then MLEs λ_m, τ_m, D_m of time delay λ_0, duration τ_0 and dispersion D_0 are defined as [7]

$$(\lambda_m,\tau_m)=\underset{\lambda\in[\Lambda_1,\Lambda_2]\,,\ \tau\in[\mathrm{T}_1,\mathrm{T}_2]}{\arg\sup}\; L(\lambda,\tau)$$

$$D_m=\max\left[0;\ M(\lambda_m,\tau_m)/\tau_m-E_N\right] \tag{17}$$

Here

$$L(\lambda,\tau)=\tau E_N\left\{M(\lambda,\tau)/\tau E_N-\ln\left[M(\lambda,\tau)/\tau E_N\right]-1\right\} \tag{18}$$

$$M(\lambda,\tau)=\int_{\lambda-\tau/2}^{\lambda+\tau/2} y^2(t)\,dt\qquad y(t)=\int_{-\infty}^{\infty}x(t')\,h(t-t')\,dt$$

Is observable realization (14) response of the filter with transfer function $H(\omega)$, satisfying a condition $|H(\omega)|^2=I[(\theta-\omega)/\Omega]+I[(\theta+\omega)/\Omega]$, and $E_N=\Omega N_0/2\pi$ is a signal band mean noise power.

It is easy to see that the measurer (17) can be practically realized only as a multichannel device, so that the estimates λ_k, τ_k D_k of parameters λ_0, τ_0 D_0 are defined as

$$\tau_k=\underset{\tau_i}{\arg\sup}\left[\sup_{\lambda\in[\Lambda_1,\Lambda_2]}L(\lambda,\tau)\right],\ \lambda_k=\underset{\lambda\in[\Lambda_1,\Lambda_2]}{\arg\sup}L(\lambda,\tau_k) \tag{19}$$

$$D_k=\max\left[0;\ M(\lambda_k,\tau_k)/\tau_k-E_N\right]$$

Where $\tau_i=\mathrm{T}_1+(i-1/2)\Delta$, $\Delta=(\mathrm{T}_2-\mathrm{T}_1)/\nu$, $i=\overline{1,\nu}$ and ν is the number of the measurer channels.

Let us consider characteristics of joint estimates (19). We designate

$$|\delta|=(\mathrm{T}_2-\mathrm{T}_1)/2\tau_s=(\mathrm{T}_2-\mathrm{T}_1)/(\mathrm{T}_2+\mathrm{T}_1) \tag{20}$$

for the signal duration (15) maximum possible absolute deviation from the average duration $\tau_s=(\mathrm{T}_2+\mathrm{T}_1)/2$. Then, in terms of the results received in [8] for the maximum variance of time delay estimate λ_k, we can state that

$$\begin{aligned}V(\lambda_k|\lambda_0)&=P_0V_0(\lambda_k|\lambda_0)+\\&+(1-P_0)\left[(\Lambda_1^2+\Lambda_1\Lambda_2+\Lambda_2^2)/3-\lambda_0(\Lambda_1+\Lambda_2)+\lambda_0^2\right].\end{aligned} \tag{21}$$

Here

$$\begin{aligned}V_0(\lambda_k|\lambda_0)&=(\tau_s^2/3)\Big\{3\delta_\nu^2/8+H\left[z_s\sqrt{2\psi(2-\psi)\,\delta_\nu}\right]\times\\&\times\left[4\psi(2-\psi)^2(4\psi-5)\,z_s^2\,\delta_\nu^3+2(2-\psi)(5-3\psi)\,\delta_\nu^2+\right.\\&\left.+1+3(16\psi-29)\,\delta_\nu/\psi z_s^2+39/\psi^2z_s^4\right]+\sqrt{(2-\psi)\,\delta_\nu/\pi\psi}\times\\&\times\left[2\psi(2-\psi)(5-4\psi)\,z_s\delta_\nu^2+(22\psi-35)\,\delta_\nu/z_s+39/\psi z_s^3\right]\Big\},\end{aligned} \tag{22}$$

$$\begin{aligned}P_0=\frac{1}{\sqrt{2\pi}}\int_1^\infty&\exp\left[-\frac{\tilde m x}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)\right]\left\{\sqrt{\frac{\tilde m_S(2-\tilde m_S)}{2\pi}}\,\zeta(x)\times\right.\\&\left.\times\exp\left[-\frac{\zeta^2(x)}{2-\tilde m_S}\right]+\left[2+\tilde m_S\zeta^2(x)-\tilde m_S\right]\exp\left(-\frac{\zeta^2(x)}{2}\right)\right\}dx\end{aligned}$$

are the maximum variance and probability of a reliable estimate λ_k respectively, and $z_s^2=\mu_s q_0^2/(1+q_0)^2$ is SNR for a pulse (15), the duration of which is equal to average duration τ_s,

$$\mu_s=\tau_s\Omega/2\pi\,,\quad q_0=D_0/E_N\,,\quad H(x)=\exp(x^2/2)\,\left[1-\Phi(x)\right],$$

$$\zeta(x)=\left[x-z_s(1+q_0)\,(1-\delta_\nu)\right]/\sqrt{1+q_0(2+q_0)(1-\delta_\nu)}\,,$$

$$\tilde m_S=\delta_\nu/\left[1+q_0(2+q_0)\,(1-\delta_\nu)\right],\qquad \delta_\nu=|\delta|/\nu\,,$$

$$\psi=2(1+q_0)^2/\left[1+(1+q_0)^2\right],\quad \tilde m=(\Lambda_2-\Lambda_1)/\tau_s\,.$$

As reliable estimate λ_k [1, 2], the estimate is understood, found on the assumption that $|\lambda_k-\lambda_0|\le(\tau_k+\tau_0)/2$, when mean value (signal function) of the sufficient statistics $M(\lambda,\tau)$ (18), with $\lambda=\lambda_k$ and $\tau=\tau_k$, is distinct from τ_kE_N.

According to [7], the maximum variance of estimate D_k is defined as

$$V(D_k|D_0)=\max\left\{V(D_q|D_0)\Big|_{\substack{\tau_0=\tau_s(1-\delta_\nu)\\ \tau^*=\tau_s}},\ V(D_q|D_0)\Big|_{\substack{\tau_0=\tau_s(1+\delta_\nu)\\ \tau^*=\tau_s}}\right\}, \tag{23}$$

where $V(D_q|D_0)$ is variance of the dispersion estimate D_q, synthesized by the likelihood method, on the assumption that signal duration (15) is known inaccurately and is equal to τ^* ($\tau^*\ne\tau_0$) [7]:

$$V(D_q|D_0)=2E_N^2\int_0^\infty(x-q_0)\left[1-F_S^*(x)\,F_N^*(x)\right]dx+D_0^2\,. \tag{24}$$

In Eq. (24) we state that

$$F_S^*(x) = \Psi_0\left[\zeta_S(x), m_S\right],$$

$$\zeta_S(x) = \left(\sqrt{\mu}/\sigma\right)\left[x(1+\delta_\tau) - q_0\left(1+\min(0;\delta_\tau)\right)\right],$$

$$\sigma^2 = \begin{cases}(1+q_0)^2(1+\delta_\tau), & \delta_\tau < 0;\\ (1+q_0)^2+\delta_\tau, & \delta_\tau > 0;\end{cases} \quad m_S = |\delta_\tau| \begin{cases}1/(1+\delta_\tau), & \delta_\tau < 0;\\ 1/\sigma^2, & \delta_\tau > 0;\end{cases}$$

$$\Psi_0(u,\rho) = \int_{-\infty}^{u} \Phi\left[\frac{u - x(1-\rho)}{\sqrt{\rho(2-\rho)}}\right]\exp\left(-\frac{x^2}{2}\right)\frac{dx}{\sqrt{2\pi}} -$$
$$-\frac{\rho u}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right)\Phi\left(u\sqrt{\frac{\rho}{2-\rho}}\right) - \frac{\sqrt{\rho(2-\rho)}}{2\pi}\exp\left(-\frac{u^2}{2-\rho}\right),$$

$$F_N^*(x) = \begin{cases}\exp\left\{-\dfrac{mx\sqrt{\mu}}{\sqrt{2\pi(1+\delta_\tau)}}\exp\left[-\dfrac{x^2\mu\,(1+\delta_\tau)}{2}\right]\right\}, & x \ge 1\big/\sqrt{\mu\,(1+\delta_\tau)}\,;\\ 0, & x < 1\big/\sqrt{\mu\,(1+\delta_\tau)}\,;\end{cases}$$

$m = (\Lambda_2 - \Lambda_1)/\tau_0$, and $\delta_\tau = (\tau^* - \tau_0)/\tau_0$ is a relative detuning on the pulse duration.

For the probability density $w(\tau_m|\tau_0)$ of the duration estimate τ_m in the measurer with the continuous change of current values of the estimated parameters in terms of the results [9] and by analogy to [3], we have

$$w(\tau_m|\tau_0) = \int_{\Lambda_1}^{\Lambda_2} w_2(\lambda_m,\tau_m|\lambda_0,\tau_0)\,d\lambda_m, \tag{25}$$

where

$$w_2(\lambda,\tau|\lambda_0,\tau_0) = $$
$$= \frac{w_1\left[\lambda-\lambda_0-(\tau-\tau_0)/2\right]w_1\left[\lambda_0-\lambda+(\tau_0-\tau)/2\right]}{\int_{\Lambda_1}^{\Lambda_2}\int_{T_1}^{T_2} w_1\left[\lambda-\lambda_0-(\tau-\tau_0)/2\right]w_1\left[\lambda_0-\lambda+(\tau_0-\tau)/2\right]d\lambda d\tau},$$

and

$$w_1(x) = \frac{2}{\tau_0}\begin{cases}z_1^2\,w\left(2z_1^2x/\tau_0, 1/R\right), & x \ge 0;\\ z_2^2\,w\left(2z_2^2x/\tau_0, R\right), & x < 0;\end{cases}$$

$$z_1^2 = \frac{\mu}{q_0^2}\left[q_0 - \ln(1+q_0)\right]^2,\; z_2^2 = \frac{\mu}{q_0^2}\left[(1+q_0)\ln(1+q_0) - q_0\right]^2,$$

$$R = (1+q_0)\left[(1+q_0)\ln(1+q_0)-q_0\right]\big/\left[q_0 - \ln(1+q_0)\right],$$

$$w(x,u) = \Phi\left(\sqrt{|x|/2}\right) - 1 + \left[(2+u)/u\right]\exp\left[|x|(1+u)/u^2\right]\times$$
$$\times\left\{1 - \Phi\left[\sqrt{|x|/2}\,(2+u)/u\right]\right\}.$$

Then, under the specified prior probability density $w_{pr}(\tau_0)$ of the parameter τ_0, unconditional bias and variance of the estimate τ_k (19) can be found from Eq. (2), substituting $w_{pr}(\tau_0)$ and $w(\tau_m|\tau_0)$ (25) for $w_{pr}(l_0)$, $w(l_m|l_0)$, respectively.

To find out the experimental estimate characteristics, the statistical computer modeling of functional $M(\lambda,\tau)$ (18) was carried out. For reduction of computational burden it was supposed that the band limitedness condition of a kind $\vartheta \gg \Omega$ for the process $\xi(t)$ is satisfied. It allows to use representation of the filter response $y(t)$ (18) through its low-frequency quadratures [5] and to form sufficient statistics $M(\lambda,\tau)$ (18) as the sum of the two independent random processes:

$$M(\lambda,\tau) = \frac{1}{2}\left[M_1(\lambda,\tau) + M_2(\lambda,\tau)\right],\; M_j(\lambda,\tau) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} y_j^2(t)\,dt \tag{26}$$

$$y_j(t) = \int_{-\infty}^{\infty} x_j(t')\,h_0(t-t')\,dt',\; x_j(t) = s_j(t) + n_j(t),\; j = 1,2.$$

Here $s_j(t) = \xi_j(t)\,I\left[(t-\lambda_0)/\tau_0\right]$, $\xi_j(t)$ $n_j(t)$ are statistically independent centered Gaussian random processes with the spectral densities $G_0(\omega) = (2\pi D_0/\Omega)\,(\omega/\Omega)$ and N_0, respectively, while the spectrum $H_0(\omega)$ of the function $h_0(t)$ satisfies a condition $|H_0(\omega)|^2 = I(\omega/\Omega)$.

During modeling within the interval $\left[\tilde\Lambda_1, \tilde\Lambda_2\right]$, $\tilde\Lambda_{1,2} = \Lambda_{1,2}/\tau_s$, with discretization step $\Delta\tilde t$ in normalized time $\tilde t = t/\tau_s$, $\tilde y_{jn}$ samples were formed of normalized random process realizations $\tilde y_j(\tilde t) = y_j(t)\sqrt{\tau_s/N_0}$, $j = 1,2$ (26). With Eqs. (27) In mind, we can achieve the step approximation of the normalized sufficient statistics $\tilde M(\tilde\lambda,\tilde\tau_i) = M(\lambda,\tau_i)/N_0$ (18), presented as:

$$M(\tilde\lambda,\tilde\tau_i) = \frac{\Delta\tilde t}{2}\sum_{n=N_{\min}}^{N_{\max}}\left(\tilde y_{1n}^2 + \tilde y_{2n}^2\right) \tag{27}$$

Here $N_{\min} = \mathrm{int}\left\{\left(\tilde\lambda - \tilde\tau_i/2\right)/\Delta\tilde t\right\}$, $N_{\max} = \mathrm{int}\left\{\left(\tilde\lambda + \tilde\tau_i/2\right)/\Delta\tilde t\right\}$, $\tilde\lambda = \lambda/\tau_s$ represent normalized current value of time delay; $\tilde\tau_i = \tau_i/\tau_s$ is normalized duration to which the i-th measurer channel is tuned; and $\mathrm{int}\{\cdot\}$ is integer part.

With $\Delta\tilde t = 0{,}05/\mu_s$, the mean square error of the step approximation (27) of a continuous realization (26) did not exceed 10 %. Samples of processes y_{jn}, $j = 1,2$ were formed in terms of the sequence of independent Gaussian random numbers by a moving summation method [5]:

$$\tilde y_{jn} = \frac{1}{\sqrt{\Delta\tilde t}}\sum_{m=n-p}^{n+p}\alpha_{im}H_{nm} + \sum_{m=\max(m_{\min},n-p)}^{\min(m_{\max},n+p)}\xi_{jm}H_{nm}, \tag{28}$$

$$\xi_{jm} = \frac{1}{\pi}\sqrt{\frac{q_0}{\Delta\tilde{t}}}\sum_{n=0}^{2p} H_{np}\beta_{j\,n+m+1}$$

Here $m_{\min} = \operatorname{int}\{(\tilde{\lambda}_0 - \tilde{\tau}_0/2)/\Delta\tilde{t}\}$, $m_{\max} = \operatorname{int}\{(\tilde{\lambda}_0 + \tilde{\tau}_0/2)/\Delta\tilde{t}\}$, $\tilde{\lambda}_0 = \lambda_0/\tau_s$, $\tilde{\tau}_0 = \tau_0/\tau_s$, α_{jm} , β_{jm} are independent Gaussian random numbers with zero expectations and unit dispersions, $H_{nm} = \sin[2\pi\mu_s\Delta\tilde{t}(n-m)]/[\pi(n-m)]$.

In the sums (28) number of summands corresponds to the value $p = 100$. It provides a relative deviation of the generated sample dispersion from the modeled process dispersion to be no more than 5 %. Formation of independent Gaussian numbers with parameters (0, 1) was carried out following Eqs. (13).

In confinement with the Eqs. (26)-(28), realization of the process $\tilde{M}(\tilde{\lambda},\tilde{\tau}_i)$ was produced, and normalized estimates $\tilde{\lambda}_k = \lambda_k/\tau_s$, $\tilde{\tau}_k = \tau_k/\tau_s$, $q_k = D_k/E_N$ were defined according to Eq. (19) and their variances were found.

In Figs. 3-5 some results of statistical modeling are presented where corresponding theoretical dependences are shown also. Each experimental value was received as a result of processing of not less than 10^4 realizations $\tilde{M}(\tilde{\lambda},\tilde{\tau}_i)$, with $\mu_s = 150$, $q_0 = 1$, $\tilde{\Lambda}_1 = 0$, $\tilde{\Lambda}_2 = \tilde{m} = 17$, $\tilde{\lambda}_0 = (\tilde{\Lambda}_2 + \tilde{\Lambda}_1)/2$, $w_{pr}(\tau_0)$ (10). Thus with probability of 0.9 confidence intervals boundaries deviate from experimental values no more than for 10... 15 %. In Fig. 3 continuous lines represent dependences of the normalized variance $\tilde{V}_\lambda = 12V(\tilde{\lambda}_k|\tilde{\lambda}_0)/\tilde{m}^2$ of estimate $\tilde{\lambda}_k$ from parameter δ (20). Curves 1 are calculated via formulae (21), (22), with $\nu = 2$, 2 $-\nu = 3$, 3 $-\nu = 5$. By squares, crosses and rhombuses the experimental values of estimate variance $\tilde{V}_\lambda$ are designated for $\nu = 2$, 3 and 5. In Fig. 4, 5 the similar dependences are presented of maximum variance $\tilde{V}_q = V(q_k|q_0)$ of estimate q_k and of unconditional variance $\tilde{V}_\tau = V(\tilde{\tau}_k)$ of estimate $\tilde{\tau}_k$ plotted by formulae (23), (24) and (2), (25), respectively. Designations in Fig. 4, 5 are the same as in Fig. 3.

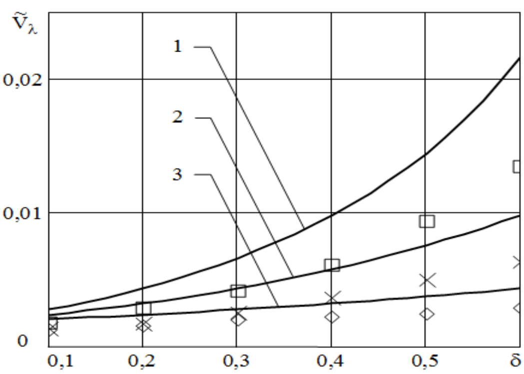

Figure 3. *Normalized variance of time delay estimate of random radio pulse.*

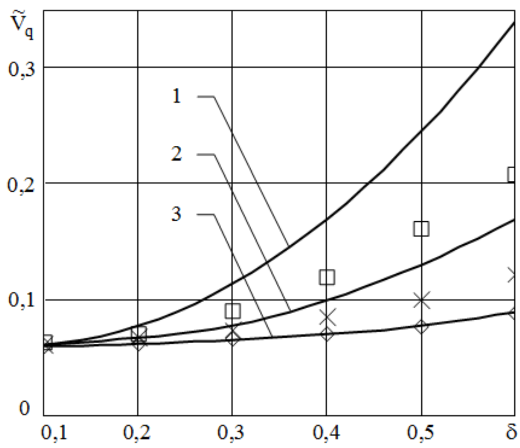

Figure 4. *Normalized variance of dispersion estimate of random radio pulse.*

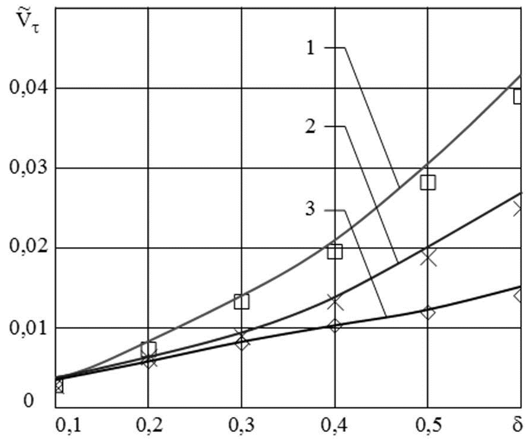

Figure 5. *Normalized variance of duration estimate of random radio pulse.*

As follows from Figs. 3-5 and the conducted modeling, the theoretical dependences (21)-(24) for the maximum variances of estimates λ_k and D_k can be used for practical calculations, if , at least, $\mu_s \geq 150$, $z_s \geq 1$, $\nu \geq 2$; and their matching with the values of real variance improves with the increasing ν and decreasing δ. So, with $\delta \leq 0.6$, $\nu \geq 2$ maximum variances of estimates λ_k and D_k no more than 2 times higher than the true ones, and when $\delta \leq 0.3$, $\nu \geq 3$, they are no more than 1.1 times higher. The formulae (2), (25) for unconditional variance of signal duration estimate τ_k adequately describes the corresponding experimental values for $z_s \geq 1$ and $\nu \geq 2$, and they are substantially more accurate than the already known formulae, describing the multichannel reception signal parameters estimates characteristics [2, 3].

5. Conclusion

In order to define the multichannel measurer characteristics, it is possible to use the expressions for the unconditional bias and variance of a decided estimate, found in the present work. The introduced expressions are correct for any type of signal and its estimated parameter, and they also have essentially greater precision in comparison with the formulae for accuracy characteristics of multichannel reception, adduced in the known literature [2, 3]. The conducted computer experiments show that the received new formulae adequately describe experimental data in a wide range of output SNR values and with any number of measurer channels.

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