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Correlation Matrices Design in the Spatial Multiplexing Systems

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Abstract: Channel correlation is closely related to the capacity of the multiple-input multiple-output (MIMO) correlated channel. Indeed, the high correlated channel degrades the system performance and the quality of wireless communication systems in terms of the capacity. Thus, we design an inverse-orthogonal matrix such as Toeplitz-Jacket matrix to design transmit and receive correlation matrices to mitigate the channel correlation of the MIMO systems. The numerical and simulation results are performed for both uncorrelated and correlated channel capacities in the case of single sided fading correlations.

Keywords: Transmit and Receive Correlation Matrices, The Correlated MIMO Channel, Inverse-Orthogonal Matrices Toeplitz -Jacket Matrices, The Channel Capacity, The Spatial Correlation

1. Introduction

MIMO communication offers significant capacity gains as well as improved diversity advantage [1], [2]. Multipath fading environment denotes a remarkable challenge in the implementation of reliable wireless MIMO systems. More recently, the development community has organized that using multiple antennas at the transmitter and the receiver can help overcome the detrimental effects of fading using a technique known as antenna diversity. In recent year studies report that in single-user, point-to-point links, using multiple-element arrays (MEAs) at both transmitter and receiver increases the capacity significantly over single-antenna systems [3], [4]. The information-theoretic capacity of MEA systems in a narrow-band Rayleigh-fading environment have analyzed in [3]. They consider independent and identically distributed (i.i.d.) fading at different antenna elements, and assume that the transmitter does not know the channel. Due to the channel variability of time and frequency where circumstances permit coding across many channel coherence intervals, the achievable rate scales as $\min(M_T, M_R)\log(1+SNR)$ since M_T and M_R are the numbers of transmit and receive antennas, respectively and SNR is the Signal to Noise Ratio.

Thus, the channel capacity of a multiple antenna system can be increased by the factor of $\min(M_T, M_R)$ without using additional transmits power or spectral bandwidth in [5].

However, on the Shannon capacity of multi-antenna wireless systems, the seminal work of Telatar in [6] has attracted a lot of attention. A single-user MIMO system has started the development with the investigation of the channel capacity. Many results on the capacity for different types of channel state information at the transmitter and/or receiver are known or unknown. Thus, we discuss in some detail about input and output correlation matrices such as transmit and receive correlation matrices and their effects on the channel capacity, with focus on the case of multiple numbers of antennas. The impact of correlation of the channel matrix on the achievable capacity in [4], [7], [8]. Most of the related works are using the assumption, that the channel covariance matrix is the Kronecker product of the covariance matrices of transmit and receive antennas [9], [10]. Many publications dealing with MIMO channel modeling aim at describing the spatial correlation properties of MIMO channels directly, e.g., [11], [12], [13], [14], [15], [16]. Their common approach is to model the correlation at the receiver and the transmitter independently, neglecting the statistical interdependence of both link ends. It may be desirable to study capacity and error rate performance accounting for spatial-correlation effects, due to the propagation channel and the transmit/receive arrays. Typically derived of spatially correlated MIMO channels under certain assumptions about the scattering in the propagation environment. One fashionable correlation model like the stochastic model has been used to investigate the capacity of MIMO radio channels in [11].

The assumption of narrowband channel [17] can be easily extended to include the discrete Fourier transform (DFT) unitary matrices [18] whose defining property is being unitarily equivalent (same singular values) to a channel with independent non-identically distributed entries. The unitary independent unitary channel model in [7] can also revert to the separable correlation model [10, 13, and 19]. In [20], [21], the transmit and receive antennas with orthogonal polarizations may provide low levels of correlation include minimum or antenna spacing while making communication link robust to polarization rotations in the channel. Therefore, we are interested in the correlated MIMO channel with inverse-orthogonal transmit and receive correlation matrices like Toeplitz-Jacket matrices in this paper.

The Toeplitz matrix is introduced in [22], [23] and the Jacket matrix is currently proposed by M. H. Lee in [24], [25]. A Jacket matrix in which all entries are of modulus 1 is called a complex Hadamard matrix in [26], [27]. The Jacket matrices are a generalization of complex Hadamard matrices. A Hadamard matrix is a square matrix whose entries are either +1 or -1 and whose rows are mutually orthogonal. A class of Jacket matrices are motivated by the center weighted Hadamard matrices available in signal processing, image compression, numerical analysis and communications. The researchers have made a considerable amount of effort to develop various kinds of orthogonal transforms. Since the orthogonal transform with the independent parameters can carry many different characterizations of digital signals, it is interesting to investigate the possibility of generalization of Hadamard and DFT and so on.

This paper is organized as follows:

In Section 2 and Section 3 gives some details about the system model and Toeplitz structure of transmit and receive correlation matrices design, respectively. In Section 4 we design Toeplitz-Jacket structure of transmit and receive correlation matrices. In Section 5, we describe the capacity of the MIMO deterministic channel. We analyze the numerical result in Section 6. Finally, we investigate the simulation results and conclusions are presented in Section 7 and Section 8.

2. System Model

Let us consider a narrowband point-to-point MIMO system with M_T transmit and M_R receive antennas which are modeled as:

$$\mathbf{y}_k = \sqrt{\frac{E_x}{M_T}}\mathbf{H}\mathbf{x}_k + \mathbf{n}_k,\quad k = 1,2... \tag{1}$$

where $\mathbf{y}_k \in \mathbb{C}^{M_R \times 1}$ and $\mathbf{x}_k \in \mathbb{C}^{M_T \times 1}$ are the received and transmitted signal vectors at time k, E_x is the energy of the transmitted signals and $\mathbf{n}_k$ is the zero-mean independent and identically distributed (i.i.d.) complex Gaussian noise vector at time k include covariance matrix $\sigma_z^2\mathbf{I}_{M_R}$. The matrix $\mathbf{H} \in \mathbb{C}^{M_R \times M_T}$ is assumed $M_R \times M_T$ MIMO channel matrix to be Rayleigh fading i.e. $vec(\mathbf{H}) \sim CN(0, \mathbf{R})$.

The channel matrix $\mathbf{H}$ for the cases in which we have correlated transmit and outputs receive antennas is approximately modeled as [7, 8, 9, 10, 11].

$$\mathbf{H} = \mathbf{R}_{M_R}^{1/2}\mathbf{H}_{iid}\left(\mathbf{R}_{M_T}^{1/2}\right)^H \tag{2}$$

where $\mathbf{R}_{M_T} \in \mathbb{C}^{M_T \times M_T}$ and $\mathbf{R}_{M_R} \in \mathbb{C}^{M_R \times M_R}$ are the deterministic transmit and receive correlation matrices, respectively while $\mathbf{H}_{iid}$ is i.i.d., Rayleigh-faded channel. The operator $(\cdot)^H$ is called Hermitian. The deterministic transmit and receive side correlation matrices can be defined as:

$$\mathbf{R}_{M_T} = \frac{E\left[\mathbf{H}^H\mathbf{H}\right]}{Tr\left[\mathbf{R}_{M_R}\right]} = \frac{E\left[\mathbf{H}^H\mathbf{H}\right]}{Tr\left[\mathbf{H}\mathbf{H}^H\right]} = \frac{1}{M_R}E\left[\mathbf{H}^H\mathbf{H}\right], \tag{3}$$

$$\mathbf{R}_{M_R} = \frac{E\left[\mathbf{H}\mathbf{H}^H\right]}{Tr\left[\mathbf{R}_{M_T}\right]} = \frac{E\left[\mathbf{H}\mathbf{H}^H\right]}{Tr\left[\mathbf{H}^H\mathbf{H}\right]} = \frac{1}{M_T}E\left[\mathbf{H}\mathbf{H}^H\right], \tag{4}$$

and

$$E\left[Tr\left\{\mathbf{H}\mathbf{H}^H\right\}\right] = M_R M_T \tag{5}$$

Therefore, we can write a $M_R \times M_T$ MIMO channel correlation matrix as follows:

$$\mathbf{R}_H = \mathbf{R}_{M_T} \otimes \mathbf{R}_{M_R} \tag{6}$$

where the Kronecker product of $\mathbf{R}_{M_T} \otimes \mathbf{R}_{M_R}$ of $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ is defined as:

$$\mathbf{R}_H = \begin{bmatrix} \rho_{11}^{Tx}\mathbf{R}_{M_R} & \rho_{12}^{Tx}\mathbf{R}_{M_R} & \cdots & \rho_{1M_T}^{Tx}\mathbf{R}_{M_R} \\ \rho_{21}^{Tx}\mathbf{R}_{M_R} & \rho_{22}^{Tx}\mathbf{R}_{M_R} & \cdots & \rho_{2M_T}^{Tx}\mathbf{R}_{M_R} \\ \vdots & \cdots & \ddots & \vdots \\ \rho_{M_T1}^{Tx}\mathbf{R}_{M_R} & \rho_{M_T2}^{Tx}\mathbf{R}_{M_R} & \cdots & \rho_{M_TM_T}^{Tx}\mathbf{R}_{M_R} \end{bmatrix} \tag{7}$$

where as, we can be defined the transmit and receive matrices, $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ are

$$\mathbf{R}_{M_T} = \begin{bmatrix} \rho_{11}^{Tx} & \rho_{12}^{Tx} & \cdots & \rho_{1M_T}^{Tx} \\ \rho_{21}^{Tx} & \rho_{22}^{Tx} & \cdots & \rho_{2M_T}^{Tx} \\ \vdots & \cdots & \ddots & \vdots \\ \rho_{M_T1}^{Tx} & \rho_{M_T2}^{Tx} & \cdots & \rho_{M_TM_T}^{Tx} \end{bmatrix},$$

And

$$\mathbf{R}_{M_R}=\begin{bmatrix}\rho_{11}^{Rx} & \rho_{12}^{Rx} & \cdots & \rho_{1M_R}^{Rx}\\ \rho_{21}^{Rx} & \rho_{22}^{Rx} & \cdots & \rho_{2M_R}^{Rx}\\ \vdots & \cdots & \ddots & \vdots\\ \rho_{M_R1}^{Rx} & \rho_{M_R2}^{Rx} & \cdots & \rho_{M_RM_R}^{Rx}\end{bmatrix}$$

For an example 1: If 2×2 transmit and receive correlation matrices as follows [11]:

Let, $\mathbf{R}_{M_R}=\begin{bmatrix}1 & \rho\\ \rho^* & 1\end{bmatrix}$ and $\mathbf{R}_{M_T}=\begin{bmatrix}1 & \mu\\ \mu^* & 1\end{bmatrix}$,

So the MIMO correlation matrix becomes 4×4 matrix as:

$$\mathbf{R}_H=\begin{bmatrix}1\times\begin{bmatrix}1 & \rho\\ \rho^* & 1\end{bmatrix} & \mu\times\begin{bmatrix}1 & \rho\\ \rho^* & 1\end{bmatrix}\\ \mu^*\times\begin{bmatrix}1 & \rho\\ \rho^* & 1\end{bmatrix} & 1\times\begin{bmatrix}1 & \rho\\ \rho^* & 1\end{bmatrix}\end{bmatrix}=\begin{bmatrix}1 & \rho & \mu & \mu\rho\\ \rho^* & 1 & \mu\rho^* & \mu\\ \mu^* & \mu^*\rho & 1 & \rho\\ \mu^*\rho^* & \mu^* & \rho^* & 1\end{bmatrix} \tag{8}$$

However, the general treatment of $\mathbf{H}$ matrix we refer the reader to [7]. Under the virtual channel condition in [28], the use of uniform linear arrays (ULAs) at the transmitter and the receiver makes $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ are an approximately Toeplitz. Thus, we will discuss about an approximately Toeplitz structure of transmit and receive correlation matrices, $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ are in the next Section 3.

3. Toeplitz Structure of Transmit and Receive Correlation Matrices Design

Definition of Toeplitz Matrix: It is well known that a Toeplitz matrix $\mathbf{T}_M=\{t_{i,j}\}$ is a $M\times M$ matrix where $t_{i,j}=t_{j-i}$ for very $1\le i,j\le M$, i.e., form in [22, 23]

$$\mathbf{T}_M=\begin{bmatrix}t_0 & t_1 & t_2 & \cdots & t_{M-1}\\ t_{-1} & t_0 & t_1 & & \\ t_{-2} & t_{-1} & t_0 & & \vdots\\ \vdots & & & \ddots & \\ t_{-M+1} & & \cdots & & t_0\end{bmatrix} \tag{9}$$

By the definition of (9), we recall *Theorem 1* as

Theorem 1: A square $M\times M$ correlation matrix $\mathbf{R}_M$ is an approximately Toeplitz.

Proof of Theorem 1: Let $M_T=M_R=M$ and the $M\times1$ observation vector $\mathbf{x}_k$ denote the elements of the time series $x_k, x_{k-1},\ldots,x_{k-M+1}$. To show the composition of the vector $\mathbf{x}_k$ explicitly, we write,

$$\mathbf{x}_k=\left[x_k,x_{k-1},\ldots.,x_{k-M+1}\right]^T \tag{10}$$

Now we define the correlation matrix of a stationary discrete-time stochastic process denoted by the time series as the expectation of the outer product of the observation vector $\mathbf{x}_k$ with itself. Let $\mathbf{R}_M$ is a $M\times M$ correlation matrix, reflecting the correlations between the rows or column vectors of $\mathbf{H}$ and define in this way. Thus we obtain from [29]

$$\mathbf{R}_M=E\left[\mathbf{x}_k\mathbf{x}_k^H\right] \tag{11}$$

where the operator $(\cdot)^H$ represents Hermitian transposition, i.e., the operation of transposition combined with complex conjugation. By substituting "(10)" in "(11)"

$$\mathbf{R}_M=\begin{bmatrix}r_0 & r_1 & r_2 & \cdots & r_{M-1}\\ r_{-1} & r_0 & r_1 & & \\ r_{-2} & r_{-1} & r_0 & & \vdots\\ \vdots & & & \ddots & \\ r_{-M+1} & & \cdots & & r_0\end{bmatrix} \tag{12}$$

Hence, we claim "(12)" is an approximately $M\times M$ Toeplitz matrix. Similarly we can rearranged backward of an observation vector $\mathbf{x}_k^T=\left[x_{k-M+1},\ldots.,x_{k-1},x_k\right]$ is given by

$$\mathbf{R}_M^T=\begin{bmatrix}r_0 & r_{-1} & r_{-2} & \cdots & r_{-M+1}\\ r_1 & r_0 & r_{-1} & & \\ r_2 & r_1 & r_0 & & \vdots\\ \vdots & & & \ddots & \\ r_{M-1} & & \cdots & & r_0\end{bmatrix} \tag{13}$$

In "(12)" and "(13)" shows that the element r_0 on the main diagonal is always real valued. For complex valued data, the remaining elements of $\mathbf{R}_M$ assume complex values. The definition of "(9)" and "(12)", we can claim transmit and receive correlation matrix, $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ is an approximately Toeplitz i.e.,

$$\mathbf{R}_{M_T}=\begin{bmatrix}r_0 & r_1 & r_2 & \cdots & r_{M_T-1}\\ r_{-1} & r_0 & r_1 & & \\ r_{-2} & r_{-1} & r_0 & & \vdots\\ \vdots & & & \ddots & \\ r_{-M_T+1} & & \cdots & & r_0\end{bmatrix}, \tag{14}$$

$$\mathbf{R}_{M_R}=\begin{bmatrix}r_0 & r_1 & r_2 & \cdots & r_{M_R-1}\\ r_{-1} & r_0 & r_1 & & \\ r_{-2} & r_{-1} & r_0 & & \vdots\\ \vdots & & & \ddots & \\ r_{-M_R+1} & & \cdots & & r_0\end{bmatrix} \tag{15}$$

Therefore, the correlation matrix $\mathbf{R}_M$ plays a key role in the statistical analysis and design of discrete-time filters in [29]. It is therefore important that we understand its various properties and their applications. Especially, the definition of "(11)", we find that the correlation matrix of a stationary discrete-time stochastic process the following properties as [29]:

Case1: Correlation matrix $\mathbf{R}_M$ of a stationary discrete-time stochastic process is Hermitian.

So a complex-valued matrix is Hermitian if it is equal to its conjugate transpose. We may thus express the Hermitian properties of the correlation matrix $\mathbf{R}_M$ by writing

$$\mathbf{R}_M = \mathbf{R}_M^H \tag{16}$$

where this property follows directly from the definition of "(12)". Thus the Hermitian property of the $\mathbf{R}_M$ is to write from "(12)"

$$r_{-k} = r_k^* \tag{17}$$

where the operator $(\cdot)^*$ is called conjugate transpose and r_k is the auto-correlation function of the stochastic process x_k for a lag of k. Accordingly, for a wide-sense stationary process we only need M_T or M_R values of the auto-correlation function r_k for $k = 0,1,...,M-1$ in order to completely define the $\mathbf{R}_M$ matrix. Thus, we may rewrite from "(12)" as follows:

$$\mathbf{R}_{M_T} = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{M_T-1} \\ r_1^* & r_0 & r_1 & & \\ r_2^* & r_1^* & r_0 & & \vdots \\ \vdots & & & \ddots & \\ r_{M_T-1}^* & & \cdots & & r_0 \end{bmatrix} \tag{18}$$

Case2: Correlation matrix $\mathbf{R}_M$ of a stationary discrete-time stochastic process is Toeplitz.

By "(12)", we can say, a $M \times M$ correlation matrix $\mathbf{R}_M$ is Toeplitz, if all elements on its main diagonal are equal and if the elements on any other diagonal parallel to the main diagonal are also equal. From the expanded form of R_M given in "(18)", we see that all the elements on the main diagonal are equal to r_0, all the elements on the first diagonal above the main diagonal are equal to r_1, all the elements along the first diagonal below the main diagonal are equal to r_1^*, and so on for the other diagonals. Therefore, we conclude that transmit and receive correlation matrices, $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ is an approximately Toeplitz which has shown in "(14)" and "(15)".

4. Design Toeplitz - Jacket Structure of Transmit and Receive Correlation Matrices

Let a square $M \times M$ correlation matrix $[\mathbf{R}_M]_{ij} = r_{ij}$ is called *Jacket matrix* in [30] or *inverse-orthogonal* Toeplitz matrix in [31, 32], or type II matrix in [33], if its inverse matrix satisfies:

$$\left[\mathbf{R}_M^{-1}\right]_{i,j} = \frac{1}{M r_{j,i}}, \tag{19}$$

i.e., the inverse matrix can be obtained by taking element-wise inverse and transposition up to a negligible constant factor. Equivalently these matrices satisfy the following relations [31], [32] as

$$\sum_{k=1}^{M} \frac{r_{i,k}}{r_{j,k}} = M\delta_{i,j}, \qquad i,j = 1,2,...M \tag{20}$$

where $\delta_{i,j}$ is the Kronecker delta- a function of two variables usually integers,

$$\delta_{i,j} = \begin{cases} 1 & if\ \ i = j \\ 0 & if\ \ i \neq j \end{cases} \tag{21}$$

If C is nonzero constant then the definition of Jacket matrix in [30] can be rewritten as follows: A square $M \times M$ matrix

$$\mathbf{R}_M = \begin{pmatrix} r_{0,0} & r_{0,1} & \cdots & r_{0,M\text{-}1} \\ r_{1,0} & j_{1,1} & \cdots & r_{1,M\text{-}1} \\ \vdots & \ddots & \ddots & \vdots \\ r_{M\text{-}1,0} & r_{M\text{-}1,1} & \cdots & r_{M\text{-}1,M\text{-}1} \end{pmatrix}, \tag{22}$$

is called a Jacket matrix if it's normalized element-inverse transposed

$$\mathbf{R}_M^T = \frac{1}{C}\begin{pmatrix} 1/r_{0,0} & 1/r_{0,1} & \dots & 1/r_{0,M\text{-}1} \\ 1/r_{1,0} & 1/r_{1,1} & \dots & 1/r_{1,M\text{-}1} \\ \vdots & \vdots & \ddots & \vdots \\ 1/r_{M\text{-}1,0} & 1/r_{M\text{-}1,1} & \dots & 1/r_{M\text{-}1,M\text{-}1} \end{pmatrix}^T \tag{23}$$

Satisfies,

$$\mathbf{R}_M \mathbf{R}_M^T = M\mathbf{I}_M \tag{24}$$

where operator $(\cdot)^T$ is called transpose inverse.

For an example 2: Let a, b and c is nonzero complex numbers then Toeplitz-Jacket matrix of order 4 becomes from [31],

$$[\mathbf{R}_4]_{ij}(a,b,c) = \begin{bmatrix} a & b & c & -\frac{bc}{a} \\ -\frac{ab}{c} & a & b & c \\ \frac{a^2}{c} & -\frac{ab}{c} & a & b \\ \frac{a^2 b}{c^2} & \frac{a^2}{c} & -\frac{ab}{c} & a \end{bmatrix}, \tag{25}$$

and the element-wise inverse of $[\mathbf{R}_4]_{ij}(a,b,c)$ is given by

$$[\mathbf{R}_4]_{ij}^{-1}(a,b,c) = \begin{bmatrix} \frac{1}{a} & \frac{1}{b} & \frac{1}{c} & \frac{1}{-\frac{bc}{a}} \\ \frac{1}{-\frac{ab}{c}} & \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{\frac{a^2}{c}} & \frac{1}{-\frac{ab}{c}} & \frac{1}{a} & \frac{1}{b} \\ \frac{1}{\frac{a^2 b}{c^2}} & \frac{1}{\frac{a^2}{c}} & \frac{1}{-\frac{ab}{c}} & \frac{1}{a} \end{bmatrix} \quad (26)$$

where $[\mathbf{R}_4]_{ij}(a,b,c)[\mathbf{R}_4]_{ij}^{-1}(a,b,c) = 4I_4$ (see in [34]). Particularly, a real Hadamard matrix of order $M > 2$ with the Toeplitz structure is either circulant or negacyclic in [31, corollary 3.1]. So we can write by "(12)" and [31]:

$$\frac{r_{-1}}{r_{M-1}} = \pm 1 \quad (27)$$

We see that when the "(27)" becomes, $r_{-1}/r_{M-1} = +1$, then by "(12)", we get $r_{-1} = r_{M-1}$ for $l = 1,2,...,M-1$ i.e., then matrix $\mathbf{R}_M$ is circulant. Otherwise $r_{-1}/r_{M-1} = -1$, then by "(12)", we get $r_{-1} = -r_{M-1}$ hence i.e., the matrix $\mathbf{R}_M$ is negacyclic. The well-known example of circulant complex Hadamard matrices, see e.g. [35]. Therefore, when a be an arbitrary of a $\mathbf{R}_M$ matrix and b be a nonzero complex number, then $\mathbf{R}_M$ matrix of order M is given by

$$\begin{aligned} \mathbf{R}_M^{'} &= adiag\left(1,\ b^{-1},\ b^{-2},\ ...,\ b^{-M+1}\right) \\ &\quad \mathbf{R}_M\ diag\left(1,\ b,\ b^2,\ ...,\ b^{M-1}\right) \end{aligned} \quad (28)$$

where the $(i,j)-th$ entry of $\mathbf{R}_M^{'}$ reads $r_{ij}\ ab^{j-i}$.

Hence Hadamard matrices have both theoretical applications ranging from harmonic analysis [36] to quantum information theory [32]; as well as applications in signal processing [30, 37]. Thus, we recall *Theorem 2* as follows:

Theorem 2: Toeplitz-Jacket structure of correlation matrices $[\mathbf{R}_M]_{i,j}$ is approximately circulant one.

Proof of Theorem 2: Let, $s = \sqrt[M]{r_{-1}/r_{M-1}}$ from "(12)", where the operator $\sqrt[M]{\cdot}$ represents the principal $M-th$ root, the matrix, $\mathbf{R}_M$ is given by "(27)"

$$\tilde{\mathbf{R}}_M = \mathbf{D}_M^{-1}\mathbf{R}_M\mathbf{D}_M \quad (29)$$

where $\mathbf{D}_M = diag\left\{1\ \ s^1\ \ s^2\ \ \cdots\ \ s^{M-1}\right\}$ and $\mathbf{D}_M^{-1} = diag\left\{1\ \ s^{-1}\ \ s^{-2}\ \ \cdots\ \ s^{-M+1}\right\}$ is a $M \times M$ diagonal matrix, respectively and we claim Toeplitz-Jacket structure of correlation matrix $\tilde{\mathbf{R}}_M$ is a $M \times M$ circulant matrix as in [31]. Finally, setting "(29)" in "(2)", then the renovated channel matrix becomes

$$\bar{\mathbf{H}} = \bar{\mathbf{R}}_{M_R}^{1/2}\bar{\mathbf{H}}_{iid}\left(\bar{\mathbf{R}}_{M_T}^{1/2}\right)^H \quad (30)$$

where $\bar{\mathbf{R}}_{M_R}^{1/2} = \mathbf{D}_{M_R}^{1/2}\tilde{\mathbf{R}}_{M_R}^{1/2}$, $\bar{\mathbf{R}}_{M_T}^{1/2} = \tilde{\mathbf{R}}_{M_T}^{H/2}\mathbf{D}_{M_T}^{-H/2}$ and $\bar{\mathbf{H}}_{iid} = \mathbf{D}_{M_R}^{-1/2}\mathbf{H}_{iid}\mathbf{D}_{M_T}^{H/2}$.

5. Capacity Analysis of MIMO Deterministic Channel

The well-known formula of the capacity of a deterministic channel is defined as [38-39]

$$\mathbf{C} = \max_{f(\mathbf{x})} I(\mathbf{x};\mathbf{y})\ \ bits/channel\ use \quad (31)$$

where $f(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^k\left\{\det(\mathbf{R}_{xx})\right\}}}\exp\left[-\frac{1}{2}\mathbf{x}^H\mathbf{R}_{xx}^{-1}\mathbf{x}\right]$ is the probability density function (PDF) of the transmit signal vector $\mathbf{x}$, and $I(\mathbf{x};\mathbf{y})$ is the mutual information of random vector $\mathbf{x}$ and $\mathbf{y}$ is given by

$$\begin{aligned} I(\mathbf{x};\mathbf{y}) &= \mathbf{H}(\mathbf{y}) - \mathbf{H}(\mathbf{y/x}) \\ &= \mathbf{H}(\mathbf{y}) - \left\{\mathbf{H}(\mathbf{x/x}) + \mathbf{H}(\mathbf{n/x})\right\} \\ &= \mathbf{H}(\mathbf{y}) - \mathbf{H}(\mathbf{n}) \\ &= \log_2\det\left(I_{M_R} + \frac{E_x}{M_T N_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \end{aligned} \quad (32)$$

$\mathbf{H}(\mathbf{y}) = \log_2\det(\pi e\mathbf{R}_{yy})$ is the differential entropy of $\mathbf{y}$ with covariance $\mathbf{R}_{yy}$ in [6], since $\mathbf{R}_{yy}$ is obtained by

$$\begin{aligned} \mathbf{R}_{yy} &= E\left\{\mathbf{y}\mathbf{y}^H\right\} \\ &= \frac{E_{xx}}{M_T}\mathbf{H}E\left[\mathbf{x}\mathbf{x}^H\right]\mathbf{H}^H + E\left[\mathbf{n}\mathbf{n}^H\right], \\ &= \frac{E_{xx}}{M_T}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H + N_0\mathbf{I}_{M_R} \end{aligned}$$

$\mathbf{H}(\mathbf{y/x})$ is the conditional entropy of $\mathbf{y}$ when $\mathbf{x}$ is given and $\mathbf{H}(\mathbf{n}) = \log_2\det(\pi e N_0\mathbf{I}_{M_R})$ is a constant. In "(32)" shows that the mutual information is maximized when $\mathbf{H}(\mathbf{y})$ is maximized. Therefore, when CSI is present at transmitter side, the capacity of deterministic MIMO channel is given by

$$\mathbf{C} = \max_{tr(\mathbf{R}_{xx})=M_T}\log_2\det\left(I_{M_R} + \frac{E_{xx}}{M_T N_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right)\ bps/Hz \quad (33)$$

When the MIMO channel matrix $\mathbf{H}$ is randomly changed, then the channel capacity is also randomly time-varying [39] since the random channel is an ergodic process. Therefore, the average capacity is calculated by "(33)" for known channel with CSI at the transmitter side,

$$E[C] = E\left[\max_{tr(\mathbf{R}_{xx})=M_T}\log_2\det\left(I_{M_R} + \frac{E_{xx}}{M_T N_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right)\right]bps/Hz \quad (34)$$

5.1. Analysis of Uncorrelated Channel Capacity

For real channel realization, the transmitter has no CSI and its power is equally allocated to each transmitting element. Thus, the matrix $\mathbf{R}_{xx}$ is given by

$$\mathbf{R}_{xx} = \mathbf{I}_{M_T} \tag{35}$$

In this case, the channel capacity "(33)" is obtained by "(35)"

$$\mathbf{C} = \log_2 \det\left(I_{M_R} + \frac{E_x}{M_T N_0} \mathbf{H}\mathbf{H}^H \right) bps / Hz \tag{36}$$

The SVD expansion of a $\mathbf{H} \in \mathbb{C}^{M_R \times M_T}$ matrix is

$$\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^H \tag{37}$$

where $\mathbf{U} \in \mathbb{C}^{M_R \times M_R}$ $\mathbf{V} \in \mathbb{C}^{M_T \times M_T}$ is an unitary matrices which means that

$$\mathbf{U}\mathbf{U}^H = \mathbf{V}\mathbf{V}^H = \mathbf{I} \tag{38}$$

and the diagonal matrix, $\Lambda \in \mathbb{C}^{M_R \times M_T}$ is given by

$$\Lambda = diag\left\{ \sqrt{\lambda_1}, \ \sqrt{\lambda_2}, \ \cdots, \ \sqrt{\lambda_{M_{\min}}}, \ 0, \ \cdots, \ 0 \right\} \tag{39}$$

where $M_{\min} = \min(M_T, M_R)$. So the capacity (36), expressed in terms of the singular values

$$\mathbf{C} = \sum_{i=1}^{M_{\min}} \log_2 \left(1 + \frac{E_x}{M_T N_0} \lambda_i \right) bps / Hz \tag{40}$$

The "(40)" has lower and upper bound as follows [5]

$$\begin{aligned} &\log_2 \det\left(I_{M_R} + \frac{E_x}{M_T N_0} Tr\left(\mathbf{H}\mathbf{H}^H\right) \right) \\ &\le C \le M_{\min} \cdot \log_2 \det\left(I_{M_R} + \frac{E_x}{M_T N_0} \frac{Tr\left(\mathbf{H}\mathbf{H}^H\right)}{M_{\min}} \right) \end{aligned} \tag{41}$$

Using "(5)" in "(41)" becomes

$$\begin{aligned} &\log_2 \det\left(I_{M_R} + \frac{E_x}{N_0} M_R \right) \\ &\le C \le M_{\min} \log_2 \det\left(I_{M_R} + \frac{E_x}{M_T N_0} \max\left(M_T, M_R\right) \right) \end{aligned} \tag{42}$$

Therefore, for low SNR case whereas consider $\gamma_0 = \frac{E_x}{N_0}$ is a fixed SNR value, the capacity "(36)" becomes

$$\mathbf{C}_{\gamma_0 \to 0} \approx \frac{\gamma_0 Tr\left(\mathbf{H}\mathbf{H}^H\right)}{M_T \ln 2} \approx \frac{\gamma_0 M_R}{\ln 2} \tag{43}$$

In "(43)" expression is independent of M_T, and thus, even under the most favorable propagation conditions the multiplexing gains are lost, and from the perspective of the capacity, multiple transmit antennas are of no value.

If $M_T \gg M_R$ and as a consequence [5]

$$\left(\frac{Tr\left(\mathbf{H}\mathbf{H}^H\right)}{M_T} \right)_{M_T \gg M_R} \approx \mathbf{I}_{M_R} \tag{44}$$

Thus, the capacity "(36)" is given by

$$\begin{aligned} \mathbf{C}_{M_T \gg M_R} &\approx \log_2 \det\left(I_{M_R} + \gamma_0 I_{M_R} \right) \\ &= M_R \cdot \log_2 \left(1 + \gamma_0\right) \end{aligned}, \tag{45}$$

which matches the upper bound "(42)". Similarly, If $M_T \ll M_R$ and as a result

$$\left(\frac{Tr\left(\mathbf{H}^H\mathbf{H}\right)}{M_R} \right)_{M_T \ll M_R} \approx \mathbf{I}_{M_T} \tag{46}$$

With the equality $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$, combines "(32)" and "(46)", yields

$$\begin{aligned} \mathbf{C}_{M_T \ll M_R} &= \log_2 \det\left(I_{M_R} + \frac{\gamma_0}{M_T} \mathbf{H}^H\mathbf{H} \right) \\ &\approx M_T \cdot \log_2 \left(1 + \gamma_0 \frac{M_R}{M_T} \right) \end{aligned} \tag{47}$$

which matches the upper bound "(42)".

5.2. Analysis of Correlated Channel Capacity

When spatial correlation is applied in "(36)", then using "(2)" in "(36)", the channel capacity is given by [39]

$$\mathbf{C} = \log_2 \det\left(I_{M_R} + \frac{E_x}{M_T N_0} \mathbf{R}_{M_R}^{1/2} \mathbf{H}_{iid} \left(\mathbf{R}_{M_T}^{1/2}\right)^H \mathbf{R}_{M_T}^{1/2} \mathbf{H}_{iid}^H \left(\mathbf{R}_{M_R}^{1/2}\right)^H \right) \tag{48}$$

With the equality $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$, this can be written to

$$\mathbf{C} = \log_2 \det\left(I_{M_R} + \frac{E_x}{M_T N_0} \mathbf{H}_{iid} \mathbf{R}_{M_T} \mathbf{H}_{iid}^H \mathbf{R}_{M_R} \right) bps / Hz \tag{49}$$

By setting the condition $\mathbf{R}_{M_R} = \mathbf{I}_{M_R}$ and $M_T \gg M_R$ in "(49)", that means no correlation exists between the receive antennas. Thus, the capacity "(49)" becomes,

$$\mathbf{C}_{M_T \gg M_R} = M_R \cdot \log_2 \left(1 + \frac{E_x}{M_T N_0} \mathbf{H}_{iid} \mathbf{R}_{M_T} \mathbf{H}_{iid}^H \right) bps / Hz \tag{50}$$

which matches as "(44)".

By setting another condition $\mathbf{R}_{M_T} = \mathbf{I}_{M_T}$ and $M_T \ll M_R$ in "(49)", that means no correlation exists between the transmit antennas. Then, the capacity "(49)" becomes,

$$\mathbf{C}_{M_T \ll M_R} = M_T \cdot \log_2\left(1+\frac{E_x}{M_T N_0}\mathbf{H}_{iid}^H\mathbf{H}_{iid}\mathbf{R}_{M_R}\right) bps/Hz\,, \quad (51)$$

If $M_T = M_R = M$, $\mathbf{R}_{M_T}$ and $\mathbf{R}_{M_R}$ is full rank and when the SNR is high, the channel capacity "(49)" can be approximated as [39]

$$\begin{aligned} C_{M_T=M_R=M} \approx{}& \log_2 \det\left(\frac{E_{xx}}{M_T N_0}\mathbf{H}_{iid}\mathbf{H}_{iid}^H\right) \\ &+\log_2 \det\left(\mathbf{R}_{M_T}\right)+\log_2 \det\left(\mathbf{R}_{M_R}\right) \end{aligned} \quad (52)$$

where term $\log_2 \det\left(\mathbf{R}_{M_T}\right)+\log_2 \det\left(\mathbf{R}_{M_R}\right)$ is always negative by the fact that $\log_2 \det\left(\mathbf{R}_M\right) \le 0$ for any correlation matrix $\mathbf{R}_M$. So the MIMO channel capacity has been reduced in "(52)". Since the determinate of a unitary matrix is unity, so the determinate of correlation matrix is given by

$$\det\left(\mathbf{R}_M\right) = \prod_{i=1}^{M}\lambda_i\,, \quad (53)$$

and the geometric mean is bounded by the arithmetic mean, that is,

$$\left(\prod_{i=1}^{M}\lambda_i\right)^{\frac{1}{M}} \le \frac{1}{M}\sum_{i=1}^{M}\lambda_i = 1 \quad (54)$$

From "(53)" and "(54)", it is obvious that,

$$\log_2 \det\left(\mathbf{R}_M\right) \le 0 \quad (55)$$

The equality in "(55)" holds when the correlation matrix is identity matrix. Thus, the quantity in term, $\log_2 \det\left(\mathbf{R}_{M_T}\right)+\log_2 \det\left(\mathbf{R}_{M_R}\right)$ are all negative. As a result, we assume an approximately Toeplitz-Jacket structure of transmit and receive correlation matrices as "(29)" to reduce the reflection of input and output correlation matrices of the MIMO correlated channel as in "(30)". Similarly, by setting the values of "(29)" and "(30)" in "(50)", "(51)" and "(52)" and can be written:

$$\mathbf{C}_{M_T \gg M_R} = M_R \cdot \log_2\left(1+\frac{E_x}{M_T N_0}\bar{\mathbf{H}}_{iid}\bar{\mathbf{R}}_{M_T}\bar{\mathbf{H}}_{iid}^H\right) bps/Hz\,, \quad (56)$$

$$\mathbf{C}_{M_T \ll M_R} = M_T \cdot \log_2\left(1+\frac{E_x}{M_T N_0}\bar{\mathbf{H}}_{iid}^H\bar{\mathbf{H}}_{iid}\bar{\mathbf{R}}_{M_R}\right) bps/Hz \quad (57)$$

and

$$\mathbf{C}_{M_T=M_R=M} \approx \log_2 \det\left(\frac{E_x}{M_T N_0}\bar{\mathbf{H}}_{iid}\bar{\mathbf{R}}_{M_T}\bar{\mathbf{H}}_{iid}^H\bar{\mathbf{R}}_{M_R}\right) bps/Hz \quad (58)$$

6. The Numerical Analysis

The channel correlation of a MIMO system is mainly due to two components such as spatial correlation and the antenna mutual coupling. Except the spatial correlation will contribute to the correlation, antenna mutual coupling will also contribute for MIMO system in [40]. In the transmitter antenna mutual coupling causes the input signals being coupled into neighboring antennas. The antenna mutual coupling influences both the spatial correlation and SNR, is taken into account by means of the impedance matrix in [5], [40].

In this paper, we will ignore the antenna mutual coupling and assume the spatial correlation scheme for simple example of enumeration that the channels are Gaussian random channels with a unit variance and a zero mean. For a measured MIMO i.i.d channel $\mathbf{H}_{iid} \in \mathbb{C}^{M_R \times M_T}$ has the following form:

$$\mathbf{H}_{iid} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M_T} \\ h_{21} & h_{22} & \cdots & h_{2M_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R 1} & h_{M_R 2} & \cdots & h_{M_R M_T} \end{bmatrix} \quad (59)$$

In general, we define the spatial correlation coefficient between the channels as [40], [41], [42]

$$\begin{aligned} \rho_{ij,pq} &= \frac{E\left[h_{ij}h_{pq}^*\right]}{\sqrt{E\left[h_{ij}h_{ij}^*\right]E\left[h_{pq}h_{pq}^*\right]}} \\ &= \rho_{ip}^{Tx}\rho_{jq}^{Rx} \\ &= J_0\left(pd_t\left|q-j\right|\right)J_0\left(pd_r\left|p-i\right|\right) \end{aligned} \quad (60)$$

where $i,p = 1,2,...,M_R$, $j,q = 1,2,....,M_T$ and the operator $J_o(\cdot)$ is a called zero order Bessel function, p is the transmit branch, q is the receive branch, the antenna separations at the transmitter and receiver are d_t and d_r , respectively.

For example 3: Let us consider the antenna separations at the transmitter and receiver are $d_t = 0.24\lambda$, $d_r = 0.15\lambda$ in [40] and the i.i.d. MIMO channel $\mathbf{H}_{iid} \in \mathbb{C}^{4\times 8}$ is designed a 4×8 MIMO system where $M_T \gg M_R$ such as $M_T = 8$ and $M_R = 4$ equipped with dipole antenna aligned as uniform linear arrays (ULAs) as follows:

$$\mathbf{H}_{iid} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} & h_{17} & h_{18} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} & h_{27} & h_{28} \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & h_{36} & h_{37} & h_{38} \\ h_{41} & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} & h_{47} & h_{48} \end{bmatrix} \quad (61)$$

Since the transmit $M_T \times M_T$ correlation matrix, ρ_{M_T} with fix receiving antenna can be calculated as:

$$\boldsymbol{\rho}_{M_T} = \begin{bmatrix} 1 & 0.5074 & -0.2654 & -0.3149 & 0.1594 & 0.2608 & -0.1019 & -0.2319 \\ 0.5074 & 1 & 0.5074 & -0.2654 & -0.3149 & 0.1594 & 0.2608 & -0.1019 \\ -0.2654 & 0.5074 & 1 & 0.5074 & -0.2654 & -0.3149 & 0.1594 & 0.2608 \\ -0.3149 & -0.2654 & 0.5074 & 1 & 0.5074 & -0.2654 & -0.3149 & 0.1594 \\ 0.1594 & -0.3149 & -0.2654 & 0.5074 & 1 & 0.5074 & -0.2654 & -0.3149 \\ 0.2608 & 0.1594 & -0.3149 & -0.2654 & 0.5074 & 1 & 0.5074 & -0.2654 \\ -0.1019 & 0.2608 & 0.1594 & -0.3149 & -0.2654 & 0.5074 & 1 & 0.5074 \\ -0.2319 & -0.1019 & 0.2608 & 0.1594 & -0.3149 & -0.2654 & 0.5074 & 1 \end{bmatrix} \quad (62)$$

By using "(26)" and "(62)" is given by

$$\frac{r_{-1}}{r_{M_R-1}} = \frac{0.5074}{-0.2319} = -2.1880 \tag{63}$$

which matches the negative values as "(26)" and the transmit correlation matrix "(62)" becomes negacyclic. Thus, we get, $s_{M_T} = \sqrt[4]{-2.1880} = 1.0189 + 0.4220i$ the operator $\sqrt[4]{\cdot}$ represents the principal $8-th$ root of 8×8 transmit correlation matrix. Then, the transmit correlation matrix "(62)" becomes a new transmit correlation matrices $\tilde{\boldsymbol{\rho}}_{M_T}$ is given by:

$$\tilde{\boldsymbol{\rho}}_{M_T} = \mathbf{D}_{M_T}^{-1}\boldsymbol{\rho}_{M_T}\mathbf{D}_{M_T} \tag{64}$$

where,

$$\mathbf{D}_{M_T} = diag\left\{1 \quad s_{M_T}^1 \quad s_{M_T}^2 \quad s_{M_T}^3 \quad s_{M_T}^4 \quad s_{M_T}^5 \quad s_{M_T}^6 \quad s_{M_T}^7\right\},$$

$$\mathbf{D}_{M_T}^{-1} = diag\left\{1 \quad s_{M_T}^{-1} \quad s_{M_T}^{-2} \quad s_{M_T}^{-3} \quad s_{M_T}^{-4} \quad s_{M_T}^{-5} \quad s_{M_T}^{-6} \quad s_{M_T}^{-7}\right\}$$

and $\tilde{\boldsymbol{\rho}}_{M_T}\tilde{\boldsymbol{\rho}}_{M_T}^{-1} = \mathbf{I}_{M_T}$ which matches the "Jacket" conditions as in [30], [34].

Similarly, the antennas are dipoles, so the channel correlation matrix ρ_{M_R} at receiver with fix transmitting antenna can be calculated as:

$$\boldsymbol{\rho}_{M_R} = \begin{bmatrix} 1 & 0.7900 & 0.2906 & -0.1962 \\ 0.7900 & 1 & 0.7900 & 0.2906 \\ 0.2906 & 0.7900 & 1 & 0.7900 \\ -0.1962 & 0.2906 & 0.7900 & 1 \end{bmatrix}, \tag{65}$$

Similarly, by (65) and (26), we get

$$\frac{r_{-1}}{r_{M_R-1}} = \frac{0.7900}{-0.1962} = -4.0265 \tag{66}$$

which matches the negative values as "(26)" and the receive correlation matrix "(89)" becomes negacyclic. Thus, we get, $s_{M_R} = \sqrt[4]{-4.0265} = 1.0017 + 1.0017i$ the operator $\sqrt[4]{\cdot}$ represents the principal $4-th$ root of 4×4 receive correlation matrix. Then, the receive correlation matrix "(65)" becomes new receive correlation matrices $\tilde{\boldsymbol{\rho}}_{M_R}$ is given by:

$$\tilde{\boldsymbol{\rho}}_{M_R} = \mathbf{D}_{M_R}^{-1}\boldsymbol{\rho}_{M_R}\mathbf{D}_{M_R} \tag{67}$$

where

$$\mathbf{D}_{M_R} = diag\left\{1 \quad S_{M_R}^1 \quad S_{M_R}^2 \quad S_{M_R}^3\right\},$$

and $\mathbf{D}^{-1} = diag\left\{1 \quad S_{M_R}^{-1} \quad S_{M_R}^{-2} \quad S_{M_R}^{-3}\right\}$.

and $\tilde{\boldsymbol{\rho}}_{M_R}\tilde{\boldsymbol{\rho}}_{M_R}^{-1} = \mathbf{I}_{M_R}$ which matches the "Jacket" conditions as in [30, 34].

7. Simulation Results

In this section, all Tables and Figures show the channel average and total capacity among the transmitter and receiver side correlation at 20 [dBs] and 30 [dB] SNR including 4x8 MIMO system, respectively. The Table 1 illustrates the average channel capacity in three different channel model at one side correlation like transmitter side correlation and 20 [dBs] SNR include 4x8 MIMO systems. It is evident that the i.i.d., channel always provides higher channel capacity between correlated channels. When i.i.d., channel generates the average channel capacity 13.4026 bps/Hz at 20 [dBs] SNR, then the correlated channel produces 12.4258 bps/Hz and 13.1042 bps/Hz at the same power. The Table 1 also shows the percentage of average channel capacity between ordinary and proposed channel model at transmitter side correlation. The percentage of proposed correlated channel model is 5.45% larger than ordinary channel model. When the receiver side correlation is applied in 4x8 MIMO systems at the equal power, the average channel capacity is significantly decreased at correlated channel model. In Table 2 shows 3.51% of average channel capacity at receives side correlation and 20 [dBs] SNR values. Similarly, we can depict Table 3 and Table 4 using transmitter and receiver side correlation at 30 [dBs] SNR values. However, while 30 [dBs] SNR values apply in 4x8 MIMO systems, the percentage of average channel capacity (4.10%) inherently increases at transmitter side correlation moreover the remarkable degrades the percentage of average capacity (2.03%) at receiver side correlation which has been shown in Table 3 and Table 4 in this paper.

Table 1. *Average Channel Capacity in [bps/Hz] with 4x8 MIMO System at 20 [dBs] SNR (When $M_T \gg M_R$ and Transmitter Side Correlation).*

Channel Model	Transmitter Side Correlation only	Percentage (%) at Correlated Channel
i.i.d. Channel"(36)"	13.4026 bps/Hz	
Conv. Corr. Channel "(50)" and "(62)"	12.4258 bps/Hz	5.45%
Prop. Corr. Channel "(56)" and "(64)"	13.1042 bps/Hz	

Table 2. *Average Channel Capacity in [bps/Hz] with 4x8 MIMO System at 20 [dBs] SNR (When $M_T \gg M_R$ and Receiver Side Correlation).*

Channel Model	Receiver Side Correlation only	Percentage (%) at Correlated Channel
i.i.d. Channel "(36)"	13.3698 bps/Hz	
Conv. Corr. Channel "(50)" and "(65)"	9.7379 bps/Hz	3.51%
Prop. Corr. Channel "(56)" and "(67)"	10.0800 bps/Hz	

Table 3. *Average Channel Capacity in [bps/Hz] with 4x8 MIMO System at 30 [dBs] SNR (When $M_T \gg M_R$ and Transmitter Side Correlation).*

Channel Model	Transmitter Side Correlation only	Percentage (%) at Correlated Channel
i.i.d. Channel "(36)"	19.4997 bps/Hz	
Conv. Corr. Channel "(50)" and "(62)"	18.3098 bps/Hz	4.10%
Prop. Corr. Channel "(56)" and "(64)"	19.0611 bps/Hz	

Table 4. *Average Channel Capacity in [bps/Hz] with 4x8 MIMO System at 30 [dBs] SNR (When $M_T \gg M_R$ and Receiver Side Correlation).*

Channel Model	Receiver Side Correlation only	Percentage (%) at Correlated Channel
i.i.d. Channel "(36)"	19.5300 bps/Hz	
Conv. Corr. Channel "(50)" and "(65)"	14.3781 bps/Hz	2.03%
Prop. Corr. Channel "(56)" and "(67)"	14.6710 bps/Hz	

The line graph which has been generated by Mat-Lab program compares the channel capacity in [bps/Hz] at 20 [dBs] and 30 [dBs] SNR values in the three different channel models in Figure 1, Figure2, Figure 3 and Figure 4, respectively. The Figure 1 exhibits the channel capacity at 20 [dBs] SNR values in the case of transmitter side correlation. When the transmitter and receiver side correlation is applied at 20 [dBs] in two different correlated channel models, then the proposed channel gain performs 1.02 [bps/Hz] over than ordinary correlated channel has been shown in Figure 1 and Figure 2. Overall, it can be seen that in Figure 3 and Figure 4 represents channel gain at least 1 [bps/Hz] than general correlated channel model.

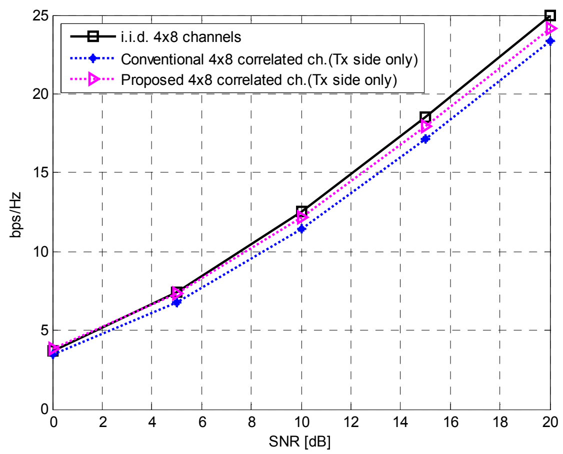

Figure 1. *Channel capacity at transmitter side correlations and 20 [dB] SNR with 4x8 MIMO systems.*

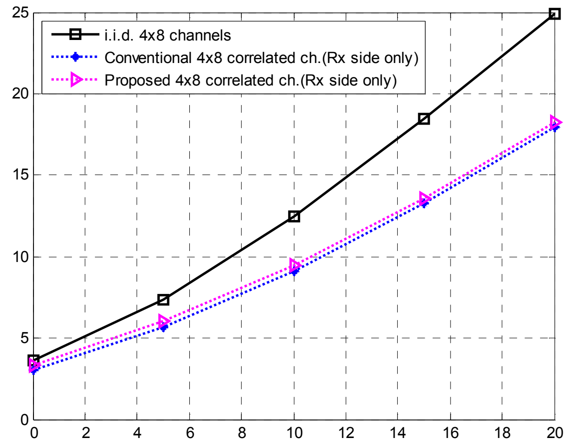

Figure 2. *Channel capacity at receiver side correlations and 20 [dB] SNR with 4x8 MIMO systems.*

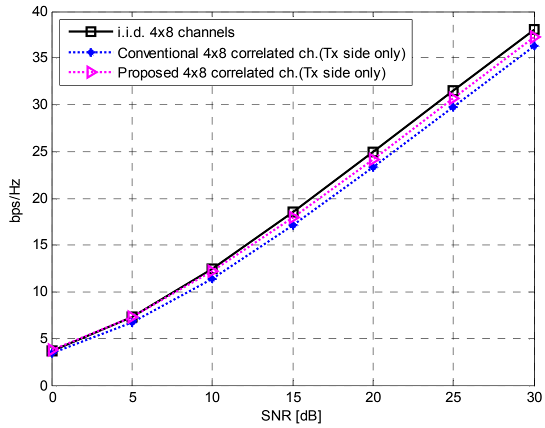

Figure 3. *Channel capacity at transmitter side correlations and 30 [dB] SNR with 4x8 MIMO systems.*

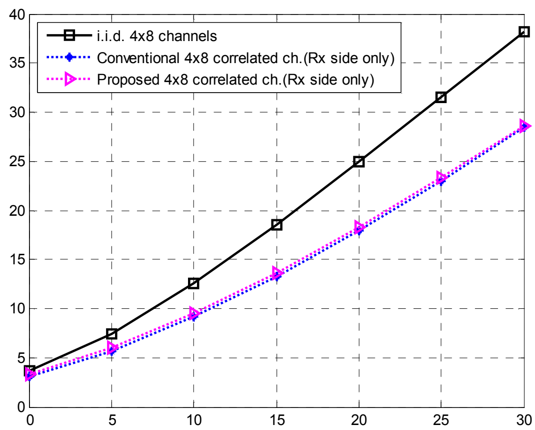

Figure 4. *Average channel capacity at receiver side correlations and 30 [dB] SNR with 4x8 MIMO systems.*

8. Conclusions

In this paper, we have mainly investigated the ordinary Toeplitz structure and the proposed Toeplitz-Jacket structure of transmit and receive correlation matrices over the MIMO correlated channel. There are two prospects in this paper. In one is lower order channel matrix and the other is higher order of channel matrix. When a lower order channel matrix is applied to MIMO system, the capacity and performance of an ordinary Toeplitz structure are better than proposed method. In contrast, a higher order of channel matrix is applied to MIMO system, the performance of proposed Toeplitz-Jacket structure is much better than ordinary method. Thus, we can say the proposed Toeplitz-Jacket structure of transmit and receive correlation matrices is suitable for the case of the high dimensional MIMO system. Future research works will more universal cases such as more than recent general scenarios.

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