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On Recurrence Relations and Application in Predicting Price Dynamics in the Presence of Economic Recession

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Abstract: Recurrence relations is one of the fundamental Mathematical tools of computation as most computational tasks rely on recursive techniques at one time or the other. In this paper, we present some important theorems on recurrence relations and give more simplified approach of determining an explicit formula for a given recurrence relation subject to specified boundary values (initial conditions). We recursively apply Recurrence Relation technique to model Economic wealth decay as a result of recession. We show both numerical computation and graphical representation of our simple model and analysis of market price dynamics due to Economic recession.

Keywords: Recurrence Relations (RR), Price Dynamics, Economic Recession

1. Introduction

Recurrence Relations (RR) is an aspect of discrete mathematics which its usage cannot be overemphasized. It is an equation that defines a sequence based on a rule that gives the next term as a function of the previous term(s) [1, 3-4]. Recurrence relations are one of the fundamental mathematical tools of computation as most computational tasks rely on recursive techniques, at one time or another [5]. The extensive use of recurrence relations can be attributed to their fundamental constructive quality, and the great simplicity with which they are agreeable to mechanization. Some researchers have used recurrence relation to model varieties of problems such as determining the number of moves in the Tower of Hanoi puzzle with n-number of disks, closed form formula for finding compound interest, Lucas number, Fibonacci number [7-9] etc. Many papers have contributed to solving of Recurrence Relations and applications but to our best knowledge, we have not come across applications of Recurrence Relations to Economic Recession. The newer textbooks on Discrete Mathematics, such as [13, 18] discussed some methods of solving RR. Among others is Differential Operator approach [22-24].

1.1. The Focus of This Paper

In this article, we review recurrence relations and show some useful results. We apply the knowledge of Recurrence Relation (RR) to model Economic decay as affected by Economic recession. We gave some analysis of the behavior of market price (price dynamics) in the presence of Economic Recession. In addition, this paper contributes to the existing literatures on recurrence relations by presenting a more simplified approach of conceptualizing the solution of recurrence Relation, obtaining an explicit formula for RR, traders predictable trading strategy and self-financing strategy in discrete time setting while deciding on price determination of their goods, and power series approach of obtaining a general solution of a differential equation associated with the pricing process.

The paper is organized with section 1 Introduction, subsection 1.1 The focus of this paper, 1.2 Relation with other papers, section 2 Preliminaries, subsection 2.1 Recurrence relations (RR), 2.2 Economic Recession, section 3 Main Results, subsection 3.1 First order non-linear Economic decay model, 3.2 Numerical computation and graphical representation of the Economic decay model, 3.3

Price dynamics due to Economic recession, 3.4 Calibration of price dynamics, 3.5 The solution to the dynamical model, section 4 Result Discussions, and section 5 Conclusion.

1.2. Relation with Other Papers

Most papers consulted on price dynamics with their different associated features (parameters) has some links with the one created here in terms of dynamics in investment, analysis base on price determination, traders decision making but differs in method of presentation. Also, some papers such as [19-21] laid emphasis on US recession but we are silent on US recession data as our direction is not really based on a case study but on mathematical presentation via Recurrence relation technique and Economic analysis.

2. Preliminaries

2.1. Recurrence Relations (RR)

Some papers have contributed to methods of solving Recurrence relations such as [11, 12, 14]. We classify Recurrence relations by the number of previous terms needed to find the new term. In a first *order recurrence* the value of each term depends only on one initial term. For example, the Recurrence Relation

$$f_{n+1} = f_n + 6 \tag{1}$$

is an example of a *first order recurrence*. If a new term depends on two previous terms, then one has a *second order recurrence* and an example of this is the Fibonacci sequence, $\{f_n\}$, $n \in N$

$$f_n = f_{n-1} + f_{n-2}, n \geq 2 \tag{2}$$

with initial values $f_0 = 0$, $f_1 = f_2 = 1$.

In this case we need two starting values to fully define the sequence. Higher order recurrences are defined in the similar way. Recurrence with finite history depends on fixed number of earlier values say

$$a_n = f(a_{n-1}, a_{n-2}, a_{n-3}, \ldots, a_{n-m}), n \geq m \tag{3}$$

Recurrence relation (RR) is either linear or n*onlinear*. Recurrence Relation is called *linear*, if it expresses sequence $\{a_n\}$ as a linear function of fixed previous terms, otherwise it is called *nonlinear* [6]. Equation (1) and (2) are example of linear recurrence relation.

Definition 1: An m th order recurrence relation with constant coefficients is an equation of the form

$$a_n + c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_m a_{n-m} = f(n), \ n \geq m, \tag{4}$$

where $c_1, c_2, \cdots, c_m \neq 0 \in IR$ are all constants and $f(n)$ is a function of n. We have homogeneous recurrence relations if $f(n) = 0$ but if $f(n) \neq 0$, then it is called nonhomogeneous. When the initial conditions (boundary values) of recurrence relations are given, then it is easy to determine a general solution that will generate every other terms of the sequence $\{a_n\}$. To solve a linear homogeneous recurrence relations, it is expedient to seek for a solution of the form $a_n = x^n$ where x has a constant value.

Proposition 1: Let $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_m a_{n-m}$. Then $a_n = x^n$ is a solution of the linear homogeneous recurrence relation if and only if

$$x^n = c_1 x^{n-1} + c_2 x^{n-2} + \cdots + c_m x^{n-m}. \tag{5}$$

It is very easy to see that we can rewrite equation (5) as

$$x^n - c_1 x^{n-1} - c_2 x^{n-2} - \cdots - c_{n-1} x - c_n = 0 \tag{6}$$

and by dividing the both sides by x^{n-m}, we have

$$x^m - c_1 x^{m-1} - c_2 x^{m-2} - \cdots - c_{m-1} x - c_m = 0. \tag{7}$$

In [8] the consequence of the sequence $\{a_n\}$ having $a_n = x^n$ as solution if and only if x (called the characteristics roots) is a solution of the characteristics equation. Characteristics roots enable us to determine the general solution to our recurrence relations.

Theorem 1: Let c_1 and c_2 be an arbitrary real numbers and suppose the characteristics equation $x^2 - c_1 x - c_2 = 0$, has two distinct roots x_1 and x_2. Then the sequence $\{a_n\}$ is a solution of a recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if and only if $\beta_1 x_1^n + \beta_2 x_2^n$ for $n = 0, 1, 2, \cdots$ where β_1 and β_2 are constants.

Proof: Since the statement of the theorem consists of if and only if statement, we will do the proof in two ways as follows. Firstly we show that if $a_n = c_1 a_{n-1} + c_2 a_{n-2}$, then the sequence $\{a_n\}$ is a solution of the recurrence relation. Taking x_1 and x_2 as the characteristic roots of $x^2 - c_1 x - c_2 = 0$, implies that we can write

$$x_1^2 = c_1 x_1 + c_2, x_2^2 = c_1 x_2 + c_2 \tag{8}$$

From all these equations, we have:

$$c_1 a_{n-1} + c_2 a_{n-2} = c_1(\beta_1 x_1^{n-1} + \beta_2 x_2^{n-1}) + c_2(\beta_1 x_1^{n-2} + \beta_2 x_2^{n-2})$$

$$c_1 a_{n-1} + c_2 a_{n-2} = \beta_1 x_1^{n-2}(c_1 x_1 + c_2) + \beta_2 x_2^{n-2}(c_1 x_1 + c_2) \tag{9}$$

$$c_1 a_{n-1} + c_2 a_{n-2} = \beta_1 x_1^{n-2} x_1^2 + \beta_2 x_2^{n-2} x_2^2 \tag{10}$$

$$c_1 a_{n-1} + c_2 a_{n-2} = \beta_1 x_1^n + \beta_2 x_2^n \tag{11}$$

$$c_1 a_{n-1} + c_2 a_{n-2} = a_n. \tag{12}$$

This shows that the sequence $\{a_n\}$ is a solution of the recurrence relation.

Secondly, suppose that a_n is a solution of the recurrence relation and that the initial conditions $a_0 = C_0$ and $a_1 = C_1$ hold. We need to solve for the values for constants β_1 and

β_2. This we will do by substituting into the equation

$$a_n = \beta_1 x_1^n + \beta_2 x_2^n \tag{13}$$

as follows.

$$a_0 = C_0 = \beta_1 x_1^0 + \beta_2 x_2^0 \equiv \beta_1 + \beta_2 \tag{14}$$

$$a_1 = C_1 = \beta_1 x_1 + \beta_2 x_2 \tag{15}$$

From Equation (14),

$$\beta_1 = C_0 - \beta_2 \tag{16}$$

Substituting Equation (16) into (15), we have

$$C_1 = (C_0 - \beta_2)x_1 + \beta_2 x_2 \tag{17}$$

$$C_1 = C_0 x_1 - \beta_2 x_1 + \beta_2 x_2 \tag{18}$$

$$C_1 = C_0 x_1 - (x_1 - x_2)\beta_2 \tag{19}$$

$$C_1 - C_0 x_1 = (x_2 - x_1)\beta_2, \tag{20}$$

$$C_0 x_1 - C_1 = (x_1 - x_2)\beta_2,$$

$$\text{Hence, } \beta_2 = \frac{C_0 x_1 - C_1}{x_1 - x_2}. \tag{21}$$

Similarly, from Equation (16)

$$\beta_1 = C_0 - \beta_2$$

$$\beta_1 = C_0 - \left(\frac{C_0 x_1 - C_1}{x_1 - x_2}\right) \tag{22}$$

$$\beta_1 = \frac{C_0(x_1 - x_2) - (C_0 x_1 - C_1)}{x_1 - x_2} \tag{23}$$

$$\beta_1 = \frac{C_0 x_1 - C_0 x_2 - C_0 x_1 + C_1}{x_1 - x_2} \tag{24}$$

$$\beta_1 = \frac{C_1 - C_0 x_2}{x_1 - x_2} \tag{25}$$

The results are valid for $x_1 \neq x_2$ indicating that the two roots are distinct. Hence, we have obtained the formula $\beta_1 x_1^n + \beta_2 x_2^n$ which solves the recurrence equation and satisfies the two boundary values referred to as initial conditions. By extension of uniqueness of solutions, it follows that this formula is a generator of the sequence $\{a_n\}$.

Example 1: Determine an explicit formula for the Fibonacci sequence: $0,1,1,2,3,5,8,13,\cdot\ldots$

Solution: For Fibonacci numbers above, the sequence $\{f_n\}$ satisfies the recurrence relations $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$ with the initial values $f_0 = 0, f_1 = f_2 = 1$. The characteristics roots of the characteristics equation are:

$$x_1 = \frac{(1+\sqrt{5})}{2} \text{ and } x_2 = \frac{(1-\sqrt{5})}{2}. \tag{26}$$

Since the roots are distinct, we seek for the solution in the form

$$f_n = \beta_1\left(\frac{1+\sqrt{5}}{2}\right)^n + \beta_2\left(\frac{1-\sqrt{5}}{2}\right)^n. \tag{27}$$

To solve for the constants β_1 and β_2, we use the initial conditions $f_0 = 0$ and $f_1 = 1$, in the equation (27) as follows.

$$f_0 = \beta_1\left(\frac{1+\sqrt{5}}{2}\right)^0 + \beta_2\left(\frac{1-\sqrt{5}}{2}\right)^0.$$

$$f_0 = 0 = \beta_1 + \beta_2 \tag{28}$$

$$\text{Therefore } \beta_1 = -\beta_2. \tag{29}$$

Similarly,

$$f_1 = 1 = \beta_1\left(\frac{1+\sqrt{5}}{2}\right) + \beta_2\left(\frac{1-\sqrt{5}}{2}\right). \tag{30}$$

Using equation (29) and (30), we have the following.

$$1 = -\beta_2\left(\frac{1+\sqrt{5}}{2}\right) + \beta_2\left(\frac{1-\sqrt{5}}{2}\right) \tag{31}$$

$$1 = -\frac{\beta_2}{2} - \frac{\sqrt{5}}{2}\beta_2 + \frac{\beta_2}{2} - \frac{\sqrt{5}}{2}\beta_2 \tag{32}$$

$$1 = \frac{-2\sqrt{5}}{2}\beta_2 \tag{33}$$

$$1 = -\sqrt{5}\beta_2 \tag{34}$$

$$\beta_2 = -\frac{1}{\sqrt{5}}. \tag{35}$$

Substituting for β_2 in equation (30) gives $\beta_1 = \dfrac{1}{\sqrt{5}}$.

Hence, the explicit formula for the Fibonacci numbers is

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{-1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n. \tag{36}$$

Example 2. Solve the homogeneous recurrence relation $f_n = 5f_{n-1} - 6f_{n-2}$ where the initial conditions are $f_0 = 1, f_1 = 4$.

Solution: The characteristics equation of the recurrence relation is $x^2 - 5x + 6 = 0$. By factorization, we have $(x-3)(x-2) = 0$. The roots are $x = 3$ and $x = 2$. Since the two roots atre distinct, the solution to the RR will be in the form

$$f_n = \beta_1 x_1^n + \beta_2 x_2^n. \tag{37}$$

$$f_n = \beta_1 3^n + \beta_2 2^n. \quad (38)$$

Substituting the initial conditions, we have:

$$f_0 = 1 = \beta_1 + \beta_2 \quad (39)$$

$$\beta_1 = -\beta_2. \quad (40)$$

Similarly,

$$f_1 = 4 = 3\beta_1 + 2\beta_2 \quad (41)$$

Solving (40) and (41) simultaneously gives $\beta_1 = 4, \beta_2 = -4.$

Hence the Explicit solution for the recurrence relation is given as

$$f_n = 4(3^n) - 4(2^n). \quad (42)$$

$$f_n = 4(3^n - 2^n). \quad (43)$$

Theorem 2: [3,4,7] (For the case of repeated roots $x_1 = x_2$). Let c_1 and c_2 be real numbers with $c_2 \neq 0$. Suppose that the characteristics equation $x^2 - c_1 x - c_2 = 0$ has only one root x. A sequence a_n is a solution if and only if $a_n = \beta_1 x^n + \beta_2 n x^n$ where β_1 and β_2 are constants.

Example 3: Determine the solution of a recurrence relation $f_n = 8f_{n-1} - 16f_{n-2}$ with initial conditions $f_0 = 1, f_1 = 3$.

Solution: The characteristic equation of the recurrence relation is $x^2 - 8x + 16 = 0$. The only root of the characteristic equation is $x = 4$. Hence, the explicit solution is of the form

$$f_n = \beta_1 4^n + \beta_2 n 4^n. \quad (44)$$

Using the initial conditions to determine the values of the constants β_1 and β_2, we have $f_0 = 1 = \beta_1$. Similarly, $f_1 = 3 = 4\beta_1 + 4\beta_2$. when solved, we have $\beta_2 = -\frac{1}{4}$ and substituting for the constants β_1, β_2, the solution becomes

$$f_n = 4^n - \frac{1}{4} n 4^n. \quad (45)$$

Remark: In this paper, the case of complex roots and non-homogeneous was skipped. The reader can see [7] for complex roots and nonhomogeneuos case.

2.2. Economic Recession

Economic recession is a period of general economy decline and is typically accompanied by a drop in the stock market, an increase in unemployment rate, high inflation rate and a decline in the housing market [15-17]. It is a downturn in economic activities. Recession does not just start a day but as an accumulation of wrong doings against Economic wellbeing or negligence of all the stakeholders (Government policies makers, Economic policies makers and private individual player in the market) involved in Economic activities. The blame for a recession generally falls on the federal leadership, often either the President, the Head of the Federal Reserve, or the entire administration.

First order recurrence relations can be used to model situations that involve repeated percentage increases such as savings accounts or debt repayment plans. We present our applied result in what follows.

3. Main Results

3.1. First Order Non-Linear Economic Decay Model

Consider a nation with an economy decay given by a recurrence relations of the form

$$E_{n+1} = rE_n, E_0 = C, \quad (46)$$

where E_0 is the initial state (wealth) of the Economy before recession takes place and $r = (\sum_{i=1}^{k} r_i)/k, k \neq 0$

as the mean rate of the various factors $r_i, i = 1, 2, \cdots, k$ that could lead to economy recession such as high inflation rate, poor economy strategies, rate of poor decision making, high foreign exchange rate etc such that $r \in (0,1)$. E_n is the economy growth at n-number of years or months as the case may be. We intend to recursively model the economy decay as follows:

Assume the initial economy wealth $E_0 = C = \$20 Billion, n \in Z^+$ with

$$E_1 = rE_0 \quad (47)$$

$$E_2 = rE_1 = r.rE_0 = r^2 E_0 \quad (48)$$

$$E_3 = rE_2 = r.r^2 E_1 = r^3 E_0 \quad (49)$$

$$E_4 = rE_3 = r.rE_2 = r^3 E_0 \quad (50)$$

$$E_n = r^n E_0 \quad (51)$$

$$E_{n+1} = r^{n+1} E_0 \equiv r.r^n E_0 = rE_n \quad (52)$$

To this end, suppose the mean rate, r, as defined above is assigned value 60%, then we can write $E_{n+1} = 0.6E_n$. To solve this, we can rewrite the equation as $E_{n+1} - 0.6E_n = 0$. The characteristic equation is given as $x - 0.6 = 0 \Rightarrow x = 0.6$.

Hence, the general solution is now given as

$$E_n = C \times 0.6^n. \quad (53)$$

Suppose we want to obtain the value for the constant C, then we use the initial condition given earlier that $E_0 = 20$.

This we show in sequel:

When $n = 0 \Rightarrow E_0 = C \times 0.6^0 \Rightarrow 20 = C \times 0.6^0 \Rightarrow C = 20.$

Whence the general solution is now

$$E_n = 20 \times 0.6^n. \tag{54}$$

This is the result we obtained for our First Order Non-linear Economic Decay model based on our own assumption in this paper.

3.2. Numerical Computation and Graphical Representation of the Economic Decay Model

Recall that we assumed the initial Economy wealth is $E_0 = \$20 Billion$, Economy decay mean rate $r = 60\% = 0.6$. Using the general solution of our model above, we obtain the table below.

Table 1. Numerical values of Economic Decay Model.

N (Months)	Economy Decay Level	Values, E_n
0	E_0	20
1	E_1	12
2	E_2	7.2
3	E_3	4.32
4	E_4	2.592
5	E_5	1.5552
6	E_6	0.93312
7	E_7	0.559872
8	E_8	0.3359232
9	E_9	0.20155392
10	E_{10}	0.120932352
11	E_{11}	0.072559411

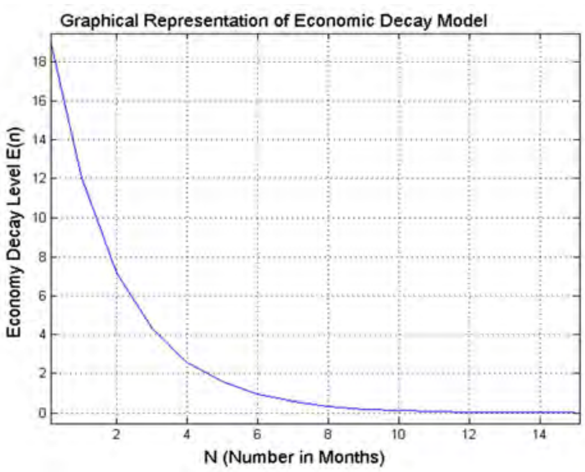

Figure 1. Economic Wealth Decay.

3.3. Price Dynamics due to Economic Recession

The set of customers buying from a firm at a given point in time is an important determinant of firm performance [21]. Many literatures emphasized that price is a crucial instrument to attract and retains customers, and firms seek to maintain and grow their customers base through their pricing decisions [2, 19, 21]. This present paper investigate into how recession can trigger price dynamics of an investment even when traders or service providers bear it in mind not to overblow the prices of their products and services so that patronage will not be too low but at the same time look forward to make profit.

3.4. Calibration of Price Dynamics

Let $P(t)$ be pricing process at time t which is $C^n(\mathbb{R})$ n-times continuously differentiable on set of real numbers. Define

$$P(t) = \sum_{n=0}^{\infty} a_n t^n$$

where a_n are constant coefficients to be determined recursively. Further assumed that $P(t)$ is a solution of a dynamical system (homogeneous differential equation) given as

$$(t^3 + \alpha)P''(t) - \beta p'(t) + \gamma P(t) = 0. \tag{55}$$

where $\alpha, \beta, \gamma \neq 0 \in \mathbb{R}$ are all nonzero real numbers which the trader can choose to manipulate at any time. We shall obtained an expression for a_n and try to write a general solution for the differential equation above. The reference [22] has done justice to solving differential equation involving power series. For better understanding of the solution steps of the dynamical system described in this paper, readers should check [22]. Before we delve into the solution of the dynamical system in (55), any trader that thought of adopting a trading strategy to determine his/her price movement must ensure that the trading strategy is self-financing and predictable.

Definition 2: A discrete type trading strategy ψ is self-financing if for $t = 1, \cdots, T-1$,

$$(\psi(t).P(t)) = \sum_{k=0}^{\infty} \psi_k(t) P_k(t) = \sum_{k=0}^{\infty} \psi_k(t+1) P_k(t) \equiv (\psi(t+1)P(t)).$$

with initial investment $V(0) = (\psi(1).P(0))$. It is wise for traders to choose a predictable trading strategy in deciding on their move in price changing without the knowledge of what the market payoff will be some days or weeks or months or years later as the case may be. Let consider discrete time setting here. Letting $\hat{P} = (1, \frac{P_1}{P_0}, \frac{P_2}{P_0}, \cdots, \frac{P_n}{P_0}) = (1, \hat{P}_1, \hat{P}_2, \cdots, \hat{P}_n)$ represent discounted prices of goods and services such that the portfolio (investment) $\hat{V}(t)$ is given as

$$\hat{V}(t) = \frac{V}{P_0}(t) = \psi_0(t) + (\psi(t).\hat{P}(t)). \tag{56}$$

The change in the portfolio (investment) is given as

$$\Delta\hat{V}(t+1) = (\psi(t+1).\Delta\hat{P}(t+1)) \tag{57}$$

The prove is shown in what follows.

Proof:

$$\Delta\hat{V}(t+1) = \hat{V}(t+1) - \hat{V}(t)$$

$$= (\psi_0(t+1) + (\psi(t+1)\cdot\hat{P}(t+1))) - (\psi_0(t) + (\psi(t)\cdot\hat{P}(t)))$$

$$=\psi_0(t+1)+(\psi(t+1)\cdot\hat{P}(t+1))-\psi_0(t+1)-(\psi(t+1)\cdot\hat{P}(t))$$

$$=\psi(t+1)(\hat{P}(t+1)-\hat{P}(t))$$

$$\Delta\hat{V}(t+1)\ =\psi(t+1)\cdot\Delta\hat{P}(t+1).$$

The left hand side of the three equations above is $\Delta\hat{V}(t+1)$. One can write the final value of the investment as

$$\hat{V}_T=\hat{V}_0+\sum_{t=1}^{n}(\psi(t)\cdot\Delta\hat{P}(t)) \tag{58}$$

with V_0 being the initial value of the investment.

3.5. The Solution to the Dynamical Model

Taking $P(t)=\sum_{n=0}^{\infty}a_n t^n$,as defined earlier as a pricing process given in power series which is n-times continuously differentiable over R and assumed to satisfy the dynamical system

$$(t^3+\alpha)P''(t)-\beta p'(t)+\gamma P(t)=0. \tag{59}$$

where $\alpha,\beta,\gamma\neq 0\in$ R are all nonzero real numbers which the trader can choose to manipulate at any time.

$$P'(t)=\sum_{n=1}^{\infty}na_n t^{n-1},\ \ P''(t)=\sum_{n=2}^{\infty}n(n-1)a_n t^{n-2}$$

We stop at second derivative because we are dealing with second order differential equation. Substituting into the dynamical system gives

$$(t^3+\alpha)\sum_{n=2}^{\infty}n(n-1)a_n t^{n-2}-\beta t\sum_{n=1}^{\infty}na_n t^{n-1}+\gamma\sum_{n=0}^{\infty}a_n t^n=0 \tag{60}$$

By expansion, we have

$$\sum_{n=2}^{\infty}n(n-1)a_n t^n+\alpha\sum_{n=2}^{\infty}n(n-1)a_n t^{n-2}-\beta\sum_{n=0}^{\infty}na_n t^n+\gamma\sum_{n=0}^{\infty}a_n t^n=0 \tag{61}$$

Shifting the index in the second summation by +2 such that t^n is common to all the terms, and $n\to n+2$, gives

$$\sum_{n=2}^{\infty}n(n-1)a_n t^n+\alpha\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}t^n-\beta\sum_{n=0}^{\infty}na_n t^n+\gamma\sum_{n=0}^{\infty}a_n t^n=0. \tag{62}$$

We can start the first summation from zero without changing anything, so we have:

$$\sum_{n=0}^{\infty}[n(n-1)a_n+\alpha(n+2)(n+1)a_{n+2}-\beta na_n+\gamma a_n]t^n=0. \tag{63}$$

but $t^n\neq 0$, therefore by identity principle (see page 199 Theorem 2 in [22]), we have

$$\alpha(n+2)(n+1)a_{n+2}=-n(n-1)a_n-\gamma a_n+\beta na_n \tag{64}$$

$$a_{n+2}=\frac{\beta na_n-n(n-1)a_n-\gamma a_n}{\alpha(n+2)(n+1)} \tag{65}$$

$$a_{n+2}=\frac{(\beta n-\gamma-n(n-1))a_n}{\alpha(n+2)(n+1)},\ \ n\geq 0. \tag{66}$$

Equation (66) is the recursive relation for the constant coefficients $\{a_n\}$ in the pricing process $P(t)$. For a specified values of α,β and γ and initial conditions, one can obtain real values for the constants.

The general solution will be difficult to be obtained without specifying values of α,β and γ. Supposing $\alpha=3,\beta=7,\gamma=16$ in the differential equation, then one can simplify the RR of the coefficient terms a_{n+2} further and thus obtain general solution of the form

$$P(t)=a_0(1-\frac{8t^3}{3}+\frac{8t^4}{27})+a_1(t-\frac{t^3}{2}+\frac{t^5}{120}+9\sum_{n=3}^{\infty}\frac{(-1)^n(2n-5)!!\,t^{2n+1}}{(2n+1)!\,3^n}). \tag{67}$$

A very similar differential equation is in [22] left as exercise there. Other results can be obtained from the model (59) above if the values of α,β and γ change.

4. Result Discussions

Economic Decay: Investment experiences depression in output as a result of recession. Market price goes up geometrically in the case of Nigeria recession which occurs as at year 2016 till this year 2017. Market survey report shows that price of goods and services spontaneously change in the market which threatens the economy the more. Every trader is on look out to change the price of goods and services. As observed in our simple Economic decay model in this paper, as ‘n’ increases, the value of the Economic wealth (E_n) decreases as shown in Table 1. The implication is that the market price of goods and services increases as the Economic decay deepens. The caution here is that all stakeholders of an Economy need to do all they could to revive the Economy from decaying before recession deepens. It is easy to see that the general solution,

$$E_n=20\times 0.6^n\to 0 \text{ as } n\to\infty \tag{68}$$

converges to zero as n tends to infinity in the model. The equation (68) above shows the future of an Economy wealth affected by recession if nothing is done to revive the economy at the right time. Infact, it is evident in the *Figure 1* that there is possibility for Economic wealth declining to zero level after sequence of months if measures to revive the economy are not put in place. The curve (nonlinear graph) indicates that Economic downturn can follow nonlinear pattern.

The results on price dynamics discussed in this paper emphasizes on traders or firms carefully selecting a predictable and self-financing strategy in price determination most especially during the period of Economic recession. The dynamical system presented here gives room for varying the values of some underlying factors while changing the prices of stocks in the period of recession.

5. Conclusion

In this paper, we have been able to show simplified way of forming recurrence relation, understanding the solution steps of recurrence relation, obtaining an explicit formula for a given recurrence relation. The application of Recurrence relation in modeling Economic wealth decay due to recession was shown. The numerical computation and graphical representation of the simple model of Economic decay was presented in the *Table 1* and *Figure 1* respectively. The Matlab simulation of the general solution of the model showed in the *Figure 1* above is a well representation of the numerical computation of the model in *Table 1*.

This paper further investigates how economic recession can trigger price dynamics of an investment. We suggested that a trader, service provider, firm in general, should incorporate predictable and self-financing strategy while taking decision in price determination of their stocks during economic recession period. The parameters α, β, γ in the dynamical system (60) of a pricing process can be manipulated by any firm to obtain another result other than the one showed here. Therefore, the general solution of the dynamical system showed here is not unique as other solutions can emanate depending on variation in the values of α, β, γ.

For further research, one can extend the Economic wealth decay model beyond first order by considering various factors leading to recession separate instead of finding the mean rate of the factors mentioned here. Also, the study of price dynamics if viewed as a stochastic process will required diffusion term (Wiener Process) to capture unpredictable fluctuations in price.

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