



Assessing the Potential of Shallow Groundwater for Agriculture Adapting to Drought and Saltwater Intrusion, Ben Tre Province, Mekong River Delta, Vietnam

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Abstract: Ben Tre province is located in the lower delta-plain of the Mekong River Delta, southern Vietnam surrounded by the Tien River system and 65 km long coastline on East Sea. Tropical monsoon climate characterized by high annual rainfall of 1,250 - 1,500 mm and high average annual temperature which are favorable for agricultural cultivation. In the dry season, due to the long coastline and large river mouths, saltwater intrusion appears seriously, greatly affecting cultivation and people's lives in rural areas, especially drought and saltwater intrusion are more serious in the recent years due to the effect of climate change. Research on the use of shallow groundwater for living use and crop irrigation, contributing to agricultural development in drought and salinity conditions. On the base of available research data, 8 geological boreholes and 6 hydrogeological boreholes were carried out in this study. The research results have determined characteristics of sediment from the cores and distributions of shallow groundwater consisting of layers and lenses that interbedded with silty layers. Fresh water is found at the depths of 10 - 14 m belonging to Holocene aquifer and the 40 - 60 m deep in upper Pleistocene aquifer, and brackish- salt water is at 24-35 m deep. Analytical results show low level of chloride, sulphate, Fe total, and As total, concentrations, excepting Fe total is high - very high in many places. Two models of Fe treatment from shallow groundwater were established showing good efficiency in Cho Lach and Chau Thanh districts.

Keywords: Groundwater, Shallow Aquifer, Saltwater Intrusion, Agriculture, Irrigation, Ben Tre Province, Mekong River Delta

1. Introduction

Recently, issue of using clean water for daily life in rural areas is more difficult than in urban areas in the Mekong River Delta, the proportion of rural population with clean water supply is only about 30- 35%. People mostly use rain water, ponds, canals, rivers, or drilled wells, open wells are often contaminated with iron, alum, microorganisms... so their health is significantly affected. Along with the socio-economic development, the demand for living water cultivation is increasing and the problem of water pollution is becoming more

and more serious. Fresh water sources from surface water and groundwater supply for daily life and production are now significantly reduced and will be seriously lacking in the near future, especially in the context of climate change, sea level rising, hence saltwater intrusion and severe droughts are becoming more serious in the coastal area. The issue of clean water for residents in rural areas has been invested and developed by the authorities. Due to the urgent need for living water and agricultural production, many projects such as water channels, lakes and dams prevent salt water and store freshwater, reclamation of water marshes, exploitation of

shallow groundwater aquifer have been studied and applied. This has brought a significant effect, contributing to socio-economic development and improving people's living standards. However, due to the uncontrolled exploitation of groundwater, lack of planning and management by local authorities, some negative impacts often occur such as lowering the groundwater level, pollution of aquifers, land subsidence [1]. The exploitation of groundwater exceeds the natural replenishment capacity and serious land subsidence has occurred in urban areas such as Tokyo, Shanghai and Bangkok [2]; in rural areas in Bangladesh, Nepal and India, where groundwater has been used for agricultural cultivation [3, 4]. Researching and clarifying distribution characteristics and hydrogeochemistry of shallow groundwater play important role assessing irrigation capacity for agriculture [5-7], and urban park trees [8]. The quality of groundwater for agricultural irrigation and the problem of replenishment increases the amount of exploitation in the present and future scarcity of irrigation water [9]. Solutions to artificially supplement groundwater to limit land subsidence and/or increase groundwater reserves to ensure sustainable exploitation of

water resources are being researched and applied [10, 11]. In addition, in coastal area, surface water is contaminated with salt water due to tidal influence, bringing sea water into rivers and canals, especially in the dry season and prolonged drought, so the solution to artificially recharge groundwater, limiting land subsidence, serving agricultural development are interested in many researches [1, 12]. Together with the recharge well system, changes in water level and groundwater quality are collected on a predetermined well system, the results show that rainwater replenishment for shallow groundwater is effective, increasing the amount of shallow water to serve local people's life [13]. Due to the effect of climate change, drought and saltwater intrusion, groundwater in the sand dune system is severely deficient in the dry season in the coastal area in Ba Tri district Ben Tre province [14, 15].

Ben Tre province is located in the lower delta plain in the Mekong River Delta, with 2,360 km² in area, surrounded by 4 rivers, namely Tien, Ba Lai, Ha Luong and Co Chien Rivers with the lengths of 83, 59, 71 and 82 km, respectively, and facing the East Sea in the southeast by a 65 km long coastline [Figure 1].

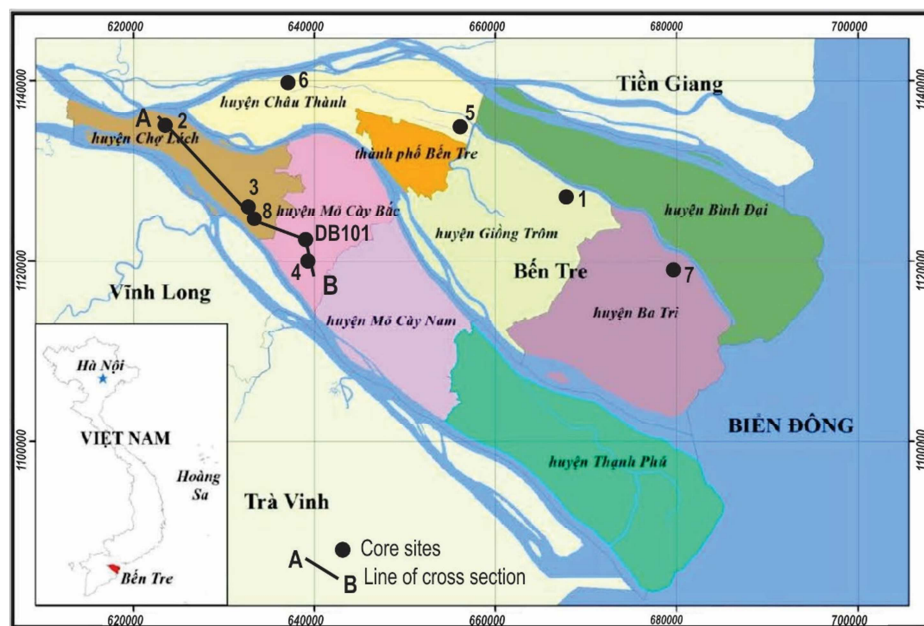


Figure 1. Study area and locations of boring cores and line of cross section.

The climate is tropical monsoon, the rainy season is from May to October, the remaining months are the dry season. The average annual temperature is about 26 to 27°C, and average annual rainfall is 1,250 to 1,500 mm. In the rainy season, Ben Tre receives a significant amount of fresh water coming from the Mekong River upstream and local rainwater, so it is convenient for production and daily life. On the contrary, in the dry season, due to the long coastline and large river mouths, saltwater intrusion occurs seriously, significantly affecting production and people's daily life. Living water of people in rural areas is mainly groundwater, rain water and surface water (rivers and canals) are relatively stable in the rainy season but very limited in the dry season, especially intrusive droughts. Saltwater intrusion has become more

serious in recent years due to the effects of climate change. In the recent years, in the dry season, salt water penetrates early upstream with high concentration, affecting the supply water and production in rural areas: orchards, seedlings, ornamental flowers... are crops that require large amounts of irrigation fresh water and are very sensitive to saltwater. Especially in the dry season of 2019-2020, salinity intrusion appeared earlier, reaching than the average of many years and about 2 months earlier than the recorded salinity drought in 2016. Results of monitoring of saline intrusion in the province's main rivers in February and March 2020, on Ham Luong and Co Chien rivers, salinity of 4‰ penetrates as deep as 60-78 km away from the river mouth and upstream forward into 3- 6 km more than in 2016, directly affecting river water and water

supply from water plants that cannot serve people's daily life. In addition, local people store fresh water in ponds, lakes, etc., which can be used for about 2 months, so the problem of domestic water shortage is serious, especially in the hot season of 2020 in many localities in Ben Tre province.

On the basis of the characteristics of depth, distribution and exploitation using the groundwater can be distinguished into 3 portions (1) The Holocene aquifer is largely exposed above ground to depths of 10.0 m and 40.5 m, averaging 24.0 m. The thickness of the layer of fine sand and mixed silt containing water ranges from 0.0 to 39.9m, with an average of 16.3m. The aquifer tends to sink to the western part of the province. (2) The Middle - Upper Pleistocene aquifer has a depth of less than 120 m and is widely distributed throughout the province, in which the aquifer with a depth of 15 - 60 m is often contaminated with alum and some places are saline water. The water layer at a depth of 60- 120 m and deeper is more limited and is exploited for rural activities. Aquifers with a depth of more than 60 m are not within the scope of the study; (3) Aquifers with a depth of 120- 395 m (belonging to the complex of lower Pleistocene aquifers, Middle Pliocene, Lower Pliocene and Upper Miocene) are good quality [11, 16]. The aquifer with a depth of 120-395

m (belonging to the complex of lower Pleistocene aquifers, middle Pliocene, lower Pliocene and upper Miocene) is a valuable water resource for exploitation, living use, and industrial production services. Therefore, this study assesses the shallow groundwater from 0- 60 m, exploitation and use for agricultural development.

On the basis of the research results obtained from Ben Tre province's natural resources as follows: Depending on the distribution characteristics and sediment structure, the water storage capacity and the quality of the groundwater will be different, in addition, the water quality and water volume will change according to the rainy-sunny season and the amount of exploitation and use of the sediment. local people. Therefore, the study focuses on surveying and evaluating the distribution range, sediment structure including thickness, width, grain level composition, and related sediments studied through geological boreholes, allowing to determine determining the distribution characteristics and potential of groundwater in the sediments. In addition, the actual situation of exploiting and using wetlands, vegetation cover, etc. to build a model of adding fresh water to the wetlands for agricultural development to adapt to the risk of drought and salinity.

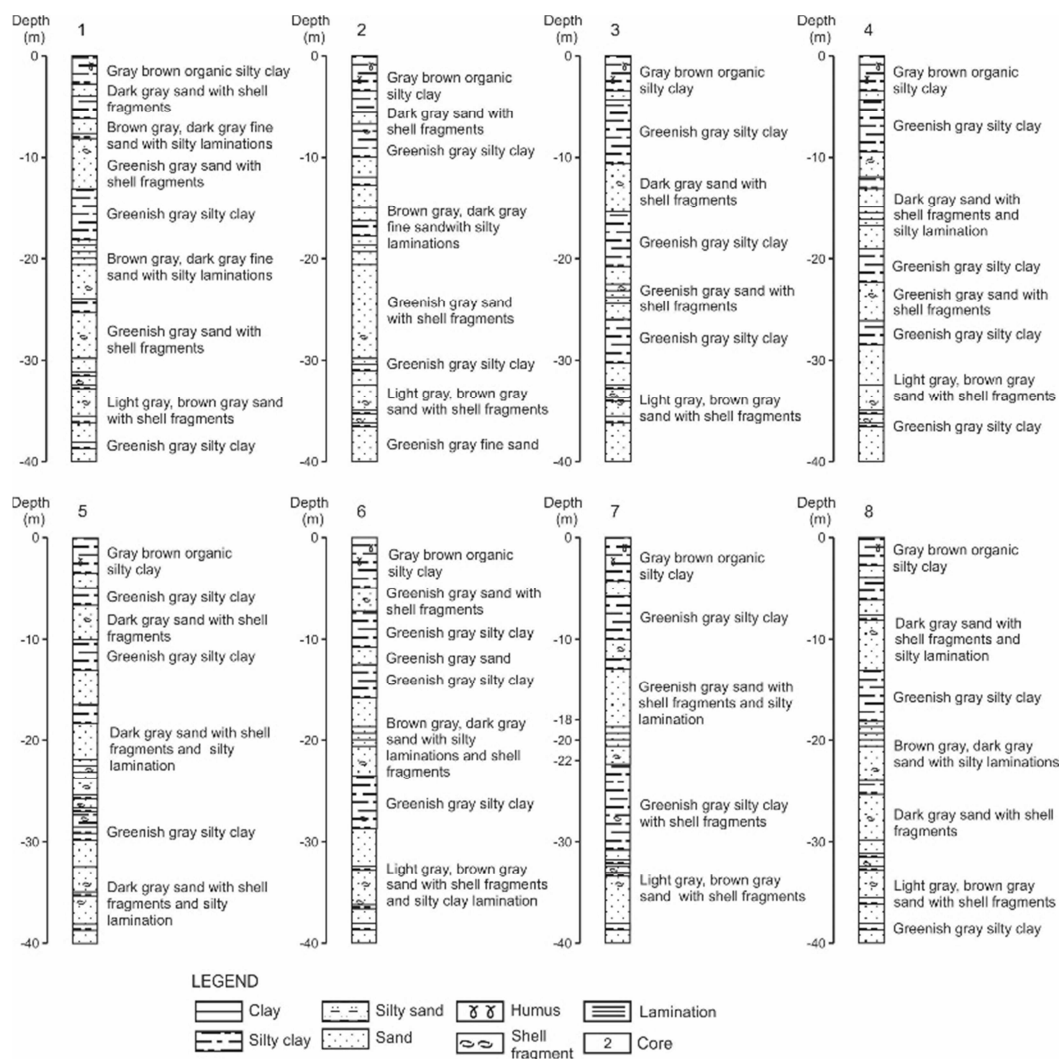


Figure 2. Column sections of boring cores indicating characteristic of sediments (see locations at Figure 1).

2. Method

On the basis of the available research results, the study carried out eight geological drilling holes, namely 1, 2, 3, 4, 5, 6, 7 and 8, and six hydrogeological drilling holes reaching 40 m deep in each borehole [Figure 1]. Analysis and determination of the sedimentary environment, distribution characteristics of shallow groundwater [Figure 2]. Analysis of chemical indicators of underground water for Cl^- , total Fe, SO_4^{2-} , total As and Coliform to determine water quality for living use and crop irrigation, comparing result iron treatment model in 2 positions. In addition, the current situation of exploitation and use of underground water, vegetative cover... to build a model of supplementing fresh water for shallow groundwater for agricultural development to adapt to the risk of drought and salinity.

3. Results and Discussion

3.1. Saltwater Intrusion and Water Use for Agricultural Development

Saltwater intrusion is a phenomenon in which salt water with a salinity concentration above 4‰ (salinity standards of water used for agricultural cultivation in Vietnam) penetrates deep into the field during high tides, sea level rise and/or depletion of fresh water come from the upstream (Ministry of Agriculture and Rural Development, 2016).

In the period of 2015- 2016, salinity of 4‰ on the main rivers has penetrated deep into 45-65 km long from the coastline, and 1‰ salinity reaching 50 -70 km long, 155/ 164 communes and towns of Ben Tre province are affected by saline intrusion. Hence, crop cultivation and daily life of people have been affected. Saline drought causes damage to

winter-spring rice in many localities in Ben Tre province, and impact of groundwater depletion on agricultural production was assessed in Vinh Chau, Soc Trang province [17].

In the period of 2019-2020, according to the General Department of Natural Disaster Prevention and Control, 2019 rainy season appears quite late in the Mekong River basin, the rainy season duration is short, the total annual flow is only low. The flow to the Mekong Delta is seriously deficient compared to the average watershed, even lower than the whole year 2015-2016. This is the main cause of early, deep and prolonged saltwater intrusion in the 2019 - 2020 dry season. The result showing that saltwater intrusion has appeared at a sudden high level from December 12, 2019 (December 12 to 15, 2019), the salinity line of 4g/l in the river mouths is as high as 57 km (Ham Luong river), which is longer distance than the annual average of 24 km, in 2015 is around 17 km.

3.2. Characteristics of Groundwater Distribution in Shallow Aquifers

On the basis of analyzing and synthesizing collected documents on geological characteristics, geomorphology, hydrogeology and new data from 8 boring cores (Figure 2), the distribution of shallow aquifers and silty layer containing poor-water are clarified reaching 50 m deep. The shallow groundwater belongs to Holocene (qh) and upper Pleistocene (qp3) aquifers, distributed at the depth of 8 - 60m that is consistent with the previous research [16]. The AB section shows in detail the uneven distribution of shallow aquifers at depths of 12-14 and 40- 60 m in Cho Lach and Mo Cay Bac districts (Figure 3). These shallow aquifers are quite abundant and are exploited to irrigate gardens and ornamental plants most commonly in Ben Tre province.

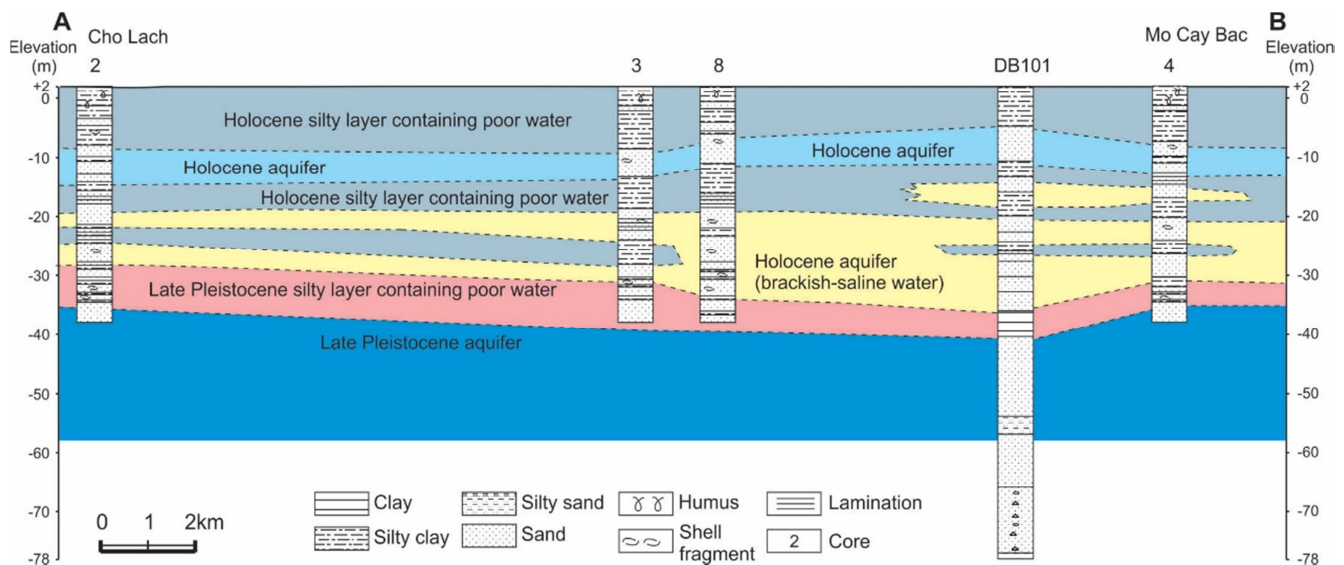


Figure 3. AB cross section showing shallow groundwater of Holocene and Late Pleistocene aquifers.

3.2.1. Holocene Aquifers (qh)

This Holocene pore-bearing stratum is formed by

coarse-grained sediments of various origins consisting of channel, delta front, sandy tidal facies ... mostly exposed on the ground or covered by a silty layer about 8–12 m thick,

distributed mainly in the northern area of Ben Tre province, including Cho Lach, Mo Cay Bac and Mo Cay districts. The petrographic composition of the formation is mud, clay, sandy clay, gray-green, light yellow-gray, light-black gray, and sticky. The bottom of the aquifer is at a depth of 10.0 to 40.5m, with an average of 24.0m. The aquifer thickness ranges from 0.0 to 16.3m on average, tends to sink to the west of the province. The main composition is fine sand, silt, clay mud, interspersed with layers of water-saturated clay powder, the aquifer tends to sink to the west of the province. This groundwater aquifer has water levels ranging from rich, moderate to poor. In which, poor water reservoirs are distributed almost throughout the province, occupying most of the area of about 1,987 km², rich and medium water reservoirs occupy a limited area of about 121.1 km², distributed discontinuously.

Exploiting flow of small wells is from 0.01 to 2.22 l/s, average 0.25 l/s, Static water level changes seasonally from 0.30m to 3.00m, average 1.19m and tidal fluctuations of rivers and seas [16 10]. In the dry season, the water level fluctuates less, tends to decrease gradually and reaches a minimum at the end of the season around May to June. In the rainy season, the water level changes strongly and fluctuates with many extremes and reaches the maximum value in the end of rainy season in October because the aquifer receives its supply directly from rainwater. The number and amplitude of water level fluctuations depend on the intensity and duration of the rains. According to monitoring results, the highest water level is -0.70m, the lowest -2.25m. The annual amplitude fluctuates from 0.87m to 1.36m; average 1.11m, and the water level tends to rise gradually over time. The results could compare to characteristics of distribution and quality of shallow aquifers in related sediments, exploitation and use groundwater [7, 9], and the problem of groundwater replenishment to adapt to drought and salinity situations [18, 19].

Result of chemical analysis of ground water in 12- 15 m deep showing as follows:

Chloride (Cl⁻): 30- 883 mg/l, Sulphate (SO₄²⁻): 41 - 266 mg/l; Fe total (Fe): 0 - 53.40 mg/l, As total: 0.0- 0.05 mg/l; Coliform CFU/100ml: 0 – 240.

The Holocene shallow aquifer has a small thickness, mostly poor water, uneven water quality, and is unable to provide concentrated water. However, the water source is significant in the supply of household water, single, for daily life and eating in areas with limited freshwater such as Ben Tre.

3.2.2. Upper Pleistocene Aquifer (qp3)

This aquifer is made up of upper Pleistocene sediments, composed mainly of fine to medium-grained sand and little gravel. This aquifer is distributed almost throughout Ben Tre province, not exposed on the ground. The aquifer roof was detected at a depth of 25.0 to 62.5m, with an average of 42.8m; The bottom of the aquifer is at depths from 31.0 to 115.0m, with an average of 86.7m. The aquifer thickness varies from 0.0 to 72.0m, with an average of 38.9m. The aquifer tends to sink to the east and southeast and covers the middle and upper Pleistocene formations, which are very poor in water. The

aquifer capacity of aquifers varies from rich to poor, in which rich aquifers occupy a small area in the eastern part of the province with a total area of about 34.40km²; Areas with average water storage are distributed from the central part to the east of the province, the rest are areas with poor water storage. The flow rate of the experimental boreholes was from 0.08 to 6.24 l/s (Q219020), with an average of 0.58 l/s. The static water level of this layer varies from 1.71 to 4.80m, with an average of 3.76m. The monthly mean value of water level ranges from -3.47m to -5.61m. The annual amplitude is from 0.29 to 0.91m. Middle - Upper Pleistocene aquifer The aquifer is made up of coarse-grained sediments of the Middle - Upper Pleistocene, widely distributed throughout Ben Tre province, not exposed on the surface. The roof of the aquifer was detected at depths from 46.0 to 126.0m, with an average of 89.7m; the bottom of the aquifer is from 88.0 to 166.0m, the average is 140.9m. The aquifer thickness is from 13.0 to 113.0m, the average is 47.5m. The aquifer tends to sink gradually from the northwest to the east, from the south to the north and northeast of the province. The aquifer is located on the very water-poor lower Pleistocene formations. The aquifer roof is located directly below the upper Pleistocene aquifer, except for the eastern part and a small part of the northern area under very poor formations of middle-upper Pleistocene sediment. Soil composition includes: gray sand gravel gravel; fine to medium sand mixed with gravel and small pebbles of gray, yellow-gray color; gravel gravel, containing seashells; clay powder sand. The stratum has medium water richness. Suctioning experimental water from borehole 22-I-NB gives a static water level of 2.58m, a flow rate of 3.49 l/s, and a lower water level of 18.37m [16 10].

Result of chemical analysis of ground water in 40 m deep showing as follows:

Chloride (Cl⁻): 30- 268 mg/l, Sulphate (SO₄²⁻): 16 - 41 mg/l; Fe total (Fe): 0 – 4,24 mg/l, As total: 0.0- 0.05 mg/l; Coliform CFU/100ml: 0 – 23.

In summary, the middle - upper Pleistocene aquifer has a wide distribution area, quite large thickness, medium water capacity.

3.3. Treatment of Iron Alum in Shallow Groundwater

The analysis results show that groundwater has very low levels of SO₄²⁻, Cl⁻, and total As, in generally meeting the water quality requirements for living use and crop irrigation, excepting for iron content ranges are low to high in many locations. The medium - high iron content is usually found in wells of 10-14 m deep in Cho Lach and Mo Cay Bac districts and less common in Mo Cay Nam district. Wells of 40 m deep have low- moderate iron content ranging 0- 0.3 mg/l.

Treatment of iron alum concentration in groundwater for crop irrigation.

Pumping shallow water from 40m deep well (a) bringing into pond 1 with carbonate (b) then transferring into pond 2 (c) finally reserving in pond 3 for use (d).

The treatment of iron alum concentration is carried out in 2 locations, Cho Lach town, Cho Lach district and Phu Duc commune, Chau Thanh district, Ben Tre province as follows:

Well water at a depth of 40 m is pumped up, spray water (increasing oxygen exposure) from a height of about 0.6-1.0 m to a wooden panel containing $\text{Ca}(\text{OH})_2$ and falls to pond 1, which has aquatic plants (biological treatment), about 1-2 days after the water is transferred to pond 2 continue to self-treat for about 1 day, then moves to pond 3 for use. The samples analyzed for comparison showed that the iron content decreased significantly and met the requirements of water quality for domestic use and crop irrigation.

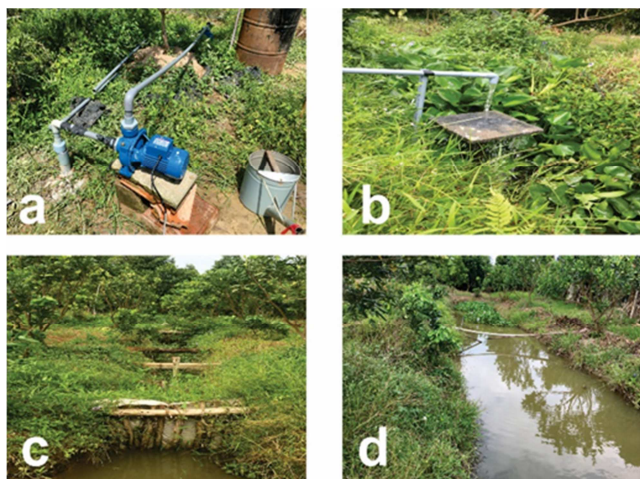


Figure 4. Treatment of iron alum model in Cho Lach District.

4. Conclusion

The shallow groundwater belongs to Holocene (qh) and upper Pleistocene (qp3) aquifers, distributed at the depth of 8 - 60m. The Holocene shallow groundwater is formed by coarse-grained sediments of various origins consisting of channel, delta front, sandy tidal facies ... mostly found in 8–14 m deep being fresh water in the northern area of Ben Tre province. Meanwhile groundwater from 25- 35 m deep is brackish and saline water formed in subaqueous delta-plain sediments. Fresh water is found at the depth of 38-40 to 50 m forming in the upper Pleistocene sediments, commonly occurring in Chau Thanh and Cho Lach districts.

Treatment of iron alum concentration in groundwater is carried out in 2 locations on Cho Lach town, Cho Lach district and Phu Duc commune, Chau Thanh district, Ben Tre province for crop irrigation. It is a simple treatment using locally available materials, the result is a reduction of iron content that meets the requirements for using water to irrigate crops for local agricultural development. This is the initial result of experimenting with the iron treatment model in underground water, so continue to experiment on a larger scale in different locations for other aquifers, making an important contribution to exploitation and use.

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