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A Parametric Evaluation of Flexural and Tensile Strength Ratios, and Bundle Stresses of Axial Composites Using Weibull's Theory

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Abstract: Weibull theory works well for brittle materials. However, its application to composites is not very clear. The present study is a comparative parametric evaluation of flexural and tensile strength ratios, and fiber bundle stresses of axial composites with brittle fiber bundles. A composite based model that utilizes Weibull's theory is developed and compared with derived Weibull's theory for brittle fiber bundles. It was found that the predicted strength ratios and the stresses are of similar magnitude to that of Weibull's and the model converges to unity for composite materials with little or no variability.

Keywords: Axial Composites, Weibull Theory, Flexural Strength, Tensile Strength

1. Introduction

Carbon composites are used in many applications such as in beam design, aircraft, helicopter rotor blades and fishing rods [1, 2]. They have the ability to resist high amount of damage before failure [3]. According to Tsai [4] unidirectional composites with higher stiffness have lower strength in tension than bending and the stiffness is controlled by stress-strain relationship. Tension, flexure, and combination of both are of considerable practical interest in predicting the tensile and flexural strength of composites.

Whitney and Knight [5] calculated strength ratio within the range of 1.03 to 1.33 for tension and bending tests. Bullock [6] performed a three point test and obtained ratios within 1.35 to 1.50. As per Weibull statistical strength theory, strength is higher in bending than tension [7]. This can be attributed to the fact that in a bending test, a smaller value of material is subjected to stress as oppose to tension test. The probability of critical defect is lower and there-fore the strength is higher. The probability P of two parameter Weibull model is provided by:

$$P = e^{\left[-\int \left(\frac{S}{S_o}\right) w \, dv\right]} \tag{1}$$

Where P, S, S_o, w, e and dv are respectively survival probability, strength, specific strength, Weibull modulus, exponent and specific volume under stress [6]. Weibull theory works well for brittle materials, but its application to composite materials is less clear.

Recently [8] research work was carried out to evaluate the tensile, and flexural strength of hybrid composites using experimental work and Finite Element Analysis (FEM). The outcome indicated that the hybrid composites improved strength.

Development and characterization [9] of natural fiber based composites consisting of jute fiber as reinforcement and hybrid resin consisting of general purpose resin and cashew nut shell resin as matrix material. The tensile strength was studied using experiment-al and numerical analysis.

Tensile behavior of environment friendly jute epoxy laminated composite was experimentally studied [10]. It was found that the tensile properties of the developed composites were strongly dependent on the tensile strength of jute fibers. In addition, jute fibers are very much defect sensitive.

Unidirectional composites [11] were fabrica-fabricated in laboratory using compression molding. It was found that by increasing the molding temperature, the achievement ratio of tensile strength was decreased due to deterioration of jute fiber.

Strength prediction of composite components subjected to tension, flexure, or a combination of both are of considerable practical interest. However, most of the work has been either experimental, numerical or a combination of both. These tend to be expensive. Therefore,

this study makes a basic first attempt to parametrically study the flexural and tensile strength ratios, and bundle stresses of axial composites with brittle fiber bundles.

2. Description of Model

The individual elements are assumed to split under tension and act independently. Their strength is assumed to follow Weibull distribution. All the symbols used in the model are listed in the notation section.

3. Derivation of Proposed Model

Survival probability of fiber under strain is:

$$\mathrm{P} = \mathrm{e}^{[-\mathrm{L}(\varepsilon/\varepsilon_0)]\,\mathrm{w}} \tag{2}$$

Consider tension in the fiber direction of a unidirectional composite. It is assumed that tensile strain is responsible for failure in a bundle that carries the maximum amount of stress. The nominal stress of bundle is equivalent to individual element stress multiplied by fraction of remaining fibers. This provides:

$$\mathrm{S} = \varepsilon\, E\, P \tag{3}$$

Where, E is equal for all fibers. Substituting values from equation (2) into equation (3) provides:

$$\mathrm{S} = \varepsilon\,\mathrm{E}\,\mathrm{e}^{[-\mathrm{L}(\varepsilon/\varepsilon_0)]\mathrm{w}} \tag{4}$$

Maximum load carried by bundle is known as failure stress. Up to failure stress, a bundle can support a load while it suddenly fails in brittle materials. The maximum failure strain in tension of a bundle can be obtained by first derivative of nominal stress with respect to strain and equating it to zero. This provides:

$$\mathrm{dS/d\varepsilon} = \varepsilon\,\mathrm{E}\,\mathrm{e}^{[-\mathrm{L}(\varepsilon/\varepsilon_0)]\,\mathrm{w}}\,\mathrm{d\varepsilon} \tag{5}$$

Taking the derivative of equation (5) and equating it to zero yields:

$$\mathrm{Ee}^{-(\mathrm{L}\,\varepsilon/\varepsilon_0)\,\mathrm{w}} - \mathrm{wE}\,(\mathrm{L}\,\varepsilon/\varepsilon_0)^{\,\mathrm{w}}\,\mathrm{e}^{(-\mathrm{L}\,\varepsilon/\varepsilon_0)\,\mathrm{w}} = 0 \tag{6}$$

Further simplifying provides,

$$\left[\mathrm{Ee}^{-(\mathrm{L}\,\varepsilon/\varepsilon_0)\,\mathrm{w}}\right]\left[1 - \mathrm{w}(\mathrm{L}\ \varepsilon/\varepsilon_o)^{\mathrm{w}}\right] = 0 \tag{7}$$

But E is constant for model and $e^{o} = 1$ So, $\left[\mathrm{Ee}^{-(\mathrm{L}\,\varepsilon/\varepsilon_0)\,\mathrm{w}}\right]$ cannot be zero. Therefore,

$$\left[1 - \mathrm{w}(\mathrm{L}\,\varepsilon/\varepsilon_o)^{\mathrm{w}}\right] = 0 \tag{8}$$

$$1 = \mathrm{w}(\mathrm{L}\,\varepsilon/\varepsilon_o)^{\mathrm{w}} \tag{9}$$

$$1/\mathrm{w} = (\mathrm{L}\,\varepsilon/\varepsilon_o)^{\mathrm{w}}$$

Therefore maximum tensile strain is:

$$\varepsilon_t = \varepsilon_o(\mathrm{L\,w})^{-1/\mathrm{w}} \tag{10}$$

In the case of bending theory, the failure is taken as the value of strain that correspond to maximum moment. The flexural capacity is found by integrating the product of ε, E, εt, and n with respect to dn. Equation obtained from this assumption is:

$$\mathrm{F_c} = \mathrm{b}\int \varepsilon\ \mathrm{E}\varepsilon_t\,\mathrm{n\,dn} \tag{11}$$

Using strain profile we obtain

$$\varepsilon = (\varepsilon_t - \varepsilon_c)\,\mathrm{n/t} \tag{12}$$

Substituting the values of ε_t and ε from equation (10) and (12) respectively into equation (11) provides:

$$\mathrm{F_c} = \mathrm{b}\int \varepsilon\ \mathrm{E}\,\varepsilon_o(\mathrm{L\,w})^{-1/\mathrm{w}}\,\mathrm{n\,dn} \tag{13}$$

For comparison of tensile and bending failure strain, we can simplify equation (13) by assuming width, thickness, Young's modulus, and length as a unit. This yield,

$$\mathrm{F_c} = \int \varepsilon_0\,(\varepsilon_t - \varepsilon_c)(\mathrm{w})^{-1/\mathrm{w}}\,\mathrm{n^2\,dn} \tag{14}$$

Assuming that failure only occurs in tension part and specific strain remains constant for bundle. Integrating the whole bundle and taking the limits of integration yields:

$$\mathrm{F_c} = \int_{\varepsilon_c}^{0}(\mathrm{w})^{-1/\mathrm{w}}(\varepsilon_t - \varepsilon_c)\mathrm{n^2\,dn} + \int_0^{\varepsilon_t}(\mathrm{w})^{-1/\mathrm{w}}(\varepsilon_t - \varepsilon_c)\mathrm{n^2\,dn} \tag{15}$$

$$\mathrm{F_c} = (\mathrm{w})^{-1/\mathrm{w}}(\varepsilon_t - \varepsilon_c)\int_{\varepsilon_c}^{0}\mathrm{n^2\ dn} + (\mathrm{w})^{-1/\mathrm{w}}\,(\varepsilon_t - \varepsilon_c)\int_0^{\varepsilon_t}\mathrm{n^2\ dn} \tag{16}$$

$$\mathrm{F_c} = (\mathrm{w})^{-1/\mathrm{w}}(\varepsilon_t - \varepsilon_c)[\mathrm{n^3/3}]_{\varepsilon_c}^{0} + (\mathrm{w})^{-1/\mathrm{w}}\,(\varepsilon_t - \varepsilon_c)\int_0^{\varepsilon_t}\mathrm{n^2\ dn} \tag{17}$$

Failure occurs in tension part. For compression purposes the value of strain in compression part can be taken as one. Therefore,

$$(w)^{-1/w}(\varepsilon_t - \varepsilon_c) = 1 \tag{18}$$

$$\mathrm{F_c} = -\,\varepsilon_c{}^3/3 + (\mathrm{w})^{-1/\mathrm{w}}(\varepsilon_t - \varepsilon_c)\int_0^{\varepsilon_t}\mathrm{n^2\,dn} \tag{19}$$

This yields as:

$$\mathrm{F_c}\ = (\varepsilon_t - \varepsilon_c)(\mathrm{w})^{-1/\mathrm{w}}\int_0^{\varepsilon_t}\mathrm{n^2dn} - \varepsilon_c{}^3/3 \tag{20}$$

Axial force A_f is zero => pure bending moment. Therefore,

$$\mathrm{A_f} = \int \varepsilon\,\mathrm{E}\,\varepsilon_0(\mathrm{w})^{-1/\mathrm{w}}\,\mathrm{n\,dn} \tag{21}$$

For A_f we don't need to multiply strain with neutral axis distance, so strain produced is $(\varepsilon_t - \varepsilon_c)$. Therefore,

$$A_f = \int E\,\varepsilon_0(\varepsilon_t - \varepsilon_c)(w)^{-1/w}\, n\, dn \tag{22}$$

Limits are same as F_c, Young's modulus, and specific strain are taken as one. This yields,

$$A_f = \int_{\varepsilon_c}^{0}(w)^{-1/w}(\varepsilon_t - \varepsilon_c) n\, dn + \int_{0}^{\varepsilon_t}(w)^{-1/w}(\varepsilon_t - \varepsilon_c) n\, dn \tag{23}$$

Simplifying equation (23) yields:

$$A_f = (\varepsilon_t - \varepsilon_c)(w)^{-1/w}\int_0^{\varepsilon_t} n\, dn - \varepsilon_c^2/2 \tag{24}$$

In a bundle, the tension portion is considered to have the same size fibers. Figure 1 shows different segments and strain profile.

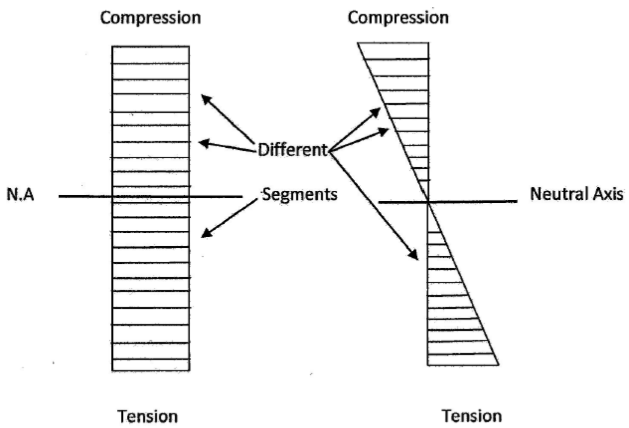

Figure 1. *Segments and Strain Profile.*

A solution from equation (24) is acquired as follows: Initially ε_c value is assumed. A trial and error method is adopted to calculate and iterate values of ε_t that provides zero axial force from equation (24). Strain values that provide maximum flexural capacity are considered failure strain in bending. The whole problem was repeated a number of times with increasing values of ε_c and the corresponding ε_t at which the maximum flexural capacity occurred is determined.

For values of $\varepsilon_t = \varepsilon_c$, we obtain A_f as approximately zero. This means the system is in pure bending. Taking the ratio of ε_t and equation (10) provides:

$$\frac{\varepsilon_t}{\varepsilon_0\,(w)^{-\frac{1}{w}}} \tag{25}$$

Considering ε_0 and ε_t as one provides:

$$\text{Bending to tension ratio} = (w)^{\frac{1}{w}} \tag{26}$$

4. Derivation of Weibull's Ratios for Brittle Fiber Bundles

Equation (2) provides the survival probability (P) of fibers under strain (ε) using Weibull's theory. Therefore, failure strain distribution can be (1-P). Using the survival probability, strength in brittle fiber bundle can be calculated. The average failure strain (S_f) in brittle fiber bundle is:

$$S_f = \int_0^{\infty} \varepsilon\left(-\frac{dP}{d\varepsilon}\right) d\varepsilon \tag{27}$$

This implies that

$$S_f = \varepsilon_0 L^{-1/w}\Gamma(1 + 1/w) \tag{28}$$

The average flexural stress σ_b in brittle fibers is:

$$\sigma_b = S_f E \tag{29}$$

$$\sigma_b = E\,\varepsilon_0 L^{-1/w}\Gamma(1 + 1/w) \tag{30}$$

The expression in equation (10) is the maximum failure strain in tension of a composite bundle. However, in terms of stress for a unit length the same relation holds for maximum failure stress in tension. Therefore, the ratio of maximum failure stress in bending equation (30) and tension is:

$$\frac{\sigma_b}{\sigma_t} = \frac{E\,\varepsilon_0 L^{-1/w}\,\Gamma(1+1/w)}{\varepsilon_0\,(Lw)^{-1/w}} \tag{31}$$

$$\frac{\sigma_b}{\sigma_t} = \left(\frac{w+1}{w}\right) w^{1/w} \tag{32}$$

5. Analytical Data

Table 1 shows a comparison of bending to tension ratio of the developed composite model to Weibull's ratio for brittle fiber bund-le using equations (26) and (32) respectively. Table 2 shows a comparison of average bundle stress in composite fibers with average bundle stress in brittle fibers. These stresses are obtained using equations (4) and (30) respectively. For comparison purposes, various parameters are assumed to be one.

Table 1. *Comparison of proposed model to Weibull's.*

W	Proposed Model (p)	Weibull Model (WM)	p/WM
13	1.21	1.31	0.923
18	1.15	1.24	0.927
23	1.14	1.19	0.957
28	1.12	1.16	0.966
33	1.11	1.14	0.973
38	1.09	1.12	0.973
50	1.08	1.10	0.981

Table 2. *Comparison of Bundle stresses.*

W	Bundle stress in composite fibers	Bundle stress in a brittle fibers
10	0.794	1.1
20	0.860	1.05
30	0.892	1.03
40	0.911	1.025
1000	0.99	1.00

6. Comparison of Ratios

The results show that the proposed model has a higher strength in bending than in tension. The magnitude of the effect seems to be decreasing for higher Weibull modulus.

The strength ratio for our model varies from 1.21 for W = 13 to 1.08 for W = 50. On the other hand, Weibull model predicts, 1.31 and 1.10 for respective W values. The ratios predicted by the proposed model are a bit conservative. In addition, for larger values of W the model converge to 1.0. This is indicative of a material with very little to no variation. A similar trend is observed for composite and brittle bundle fiber stresses.

7. Conclusions

Weibull theory works well with brittle materials. However, its application to composites is not clear.

The present study is a comparative parametric evaluation of flexural and tensile strength ratios, and fiber bundle stresses of axial composites with brittle fiber bundles. A composite fiber bundle model that applies Weibull theory is developed and compared with derived Weibull theory for brittle fiber bundles.

The following limited conclusions can be obtained from this work:

1. The predicted ratios and the stresses are of similar magnitude to that of Weibull's; and
2. At higher W values, the model converges to one.

Future Work

Good agreement is obtained between the models. However, more work is needed to further validate this approach. This may include:

1. Experimental work on materials of different variability; and
2. Comparison of experimental results with the model.

Notations

P	Survival probability of fibers
S	Nominal bundle stress
S_o	Specific strength
w	Weibull modulus
L	Length of fiber
E	Young's modulus of fiber
ε	Strain at a particular distance from neutral axis
ε_0	Specific strain
ε_t	Maximum tensile strain
ε_c	Compressive strain
t	Thickness
S_b	Strength of bundle in bending
F_c	Flexural capacity
n	Distance from neutral axis
A_f	Axial force
S_t	Strength of bundle in tension
S_f	Failure strain in bundle
Γ	Gamma function
b	Width of bundle
dv	First derivative with respect to volume
dS	First derivative with respect to stress
$d\varepsilon$	First derivative with respect to strain
dn	First derivative with respect to distance from neutral axis
σ_b	Maximum bundle stress in flexure
σ_t	Maximum bundle stress in tension
p	Proposed model
W	Weibull model

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