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A Possibility Priority Degree Analyzing Process for Multiple Attributes Decision Making Problems

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Abstract: A multiple attributes decision making model is wildly used and studied. The goal of multiple attributes decision making problems is to select a perfect alternative. The existed methods pay attention to rank the alternatives and suggest a best alternative to decision makers. However, there is risk hiding on the priority order. When accepting the order, decision makers undertake the risk at the same time. It is unknown for decision makers. To show the advantages and disadvantages for each alternative, and the risk of a selection, we propose a possibility priority degree analyzing model. With this model, decision makers can be aware of the possibility of priority degree, similar degree and the priority risk, and then make decision. It will effectively reduce the decision risk and improve the decision efficiency.

Keywords: MADM, Possibility, Priority Degree, Alternatives, Attributes

1. Introduction

A multiple attribute decision making (MADM) [1] problem includes several alternatives. Each alternative can be described by an attribute system. The goal of solving MADM problem is to select a perfect one from alternatives. Since 1950s, it has been wildly used (Job selection [2]; Product design [3]; leisure time allocation [4]; making business investment decision [5]; Selecting military hardware [6]) and studied (Dominance method [7]; Satisficing method [8]; Maximin method [1]; Maximax method [1]; Lexicography method [1]; Additive weighting method [9]; Non-metric scaling method [10]). The dominance method shows that if some one alternative has higher attribute values for all attribute, we say that this alternative "dominates" the others [1]. The satisfying method shows that the decision maker supplies the minimal attribute values he can accept for each of the attributes. The alternatives whose attribute values are better than the minimal acceptable goal can be taken as feasible alternatives [1]. The maximin method is to note the lowest value of each alternative and select the alternative with the most acceptable value of its lowest attribute [1]. The maximax methods are to identify the highest attribute value of each alternative and select the alternative to the largest value [1]. Lexicography method is to consider the most important attribute to decision maker and select the alternative to the most important attribute value [1]. The additive weighting method is to weight each attribute value by a measure to get a weighted average of the contribution to each alternative and select the alternative to the highest weighted average [1]. The non-metric scaling method is to specify an ideal object (the most preferred values on each of the attributes) and determine the distance between each of the other alternatives and this ideal object. The alternative which is closest to the ideal object would be the chosen alternative [10].

These methods for MADM problems can be classified into three kinds. The first one is the dominance method. The decision is accurate. And the best choice is determined. It can't change into anyone in anyplace and anytime. But it is not practical. The second one is the satisfying method, Maximin method, Maximax method, lexicography method. These methods pay attentions to single attribute, and make decision largely depending on single attribute value. In this way, the untaken attribute value missed in the process of decision making. The third one is the additive weighting method and non-metric method. These two methods integrate all the attribute information. It is an average method. However, this kind of method neglects the worse attribute value and best attribute value. Thus, except for the first kind (one alternative has advantage for all attributes), the other two kinds of method comparing alternatives, generating that one alternative is better than another in 100% percentage and providing a best alternative for decision maker are not sensible. Each alternative has both advantages and disadvantages. Alternatives can't be ordered only by their advantage\disadvantage\weighted average. It would miss the information of other aspects. Take the advantages for example, when you select a basketball player, only takes the advantage of player into account, a man with very well skills and very poor cooperation may be selected. However, in playing basketball, cooperation is very important.

How to do multiple attributes decision making problem? A priority-possibility degree analyzing (PPDA) method will be proposed in this paper. The paper is organized as follows: Section 2 will introduce the MADM problem and existed MADM method. The priority-possibility degree analyzing method will be introduced in Section 3. In Section 4, the proposed analyzing method will be extended and the attribute weight will be considered. This paper will be concluded in Section 5.

2. The Existed MADM Method

A MADM problem is to select alternatives from a group of alternatives. Suppose there are m alternatives: $x_1, x_2, ..., x_m$; an quantitative attribute system ($u_1, u_2, ..., u_n$), in which each pair of attributes is independent, can be taken to express the characteristic of each alternatives. And all the attribute values (a_{ij}) are known uniquely. The MADM problem can be shown in Table 1.

Table 1. *A MADM problem.*

	u_1	u_2	...	u_n
x_1	a_{11}	a_{12}	...	a_{1n}
x_2	a_{21}	a_{22}	...	a_{2n}
...			...	...
x_m	a_{m1}	a_{m2}	...	a_{mn}

According to the numerical multiple attribute decision information ($A=(a_{ij})_{m\times n}$), which one should we choose?

There are several methods for this problem.

To make the numerical information of different attribute comparable, normalize [11-13] the attribute value $A=(a_{ij})_{m\times n}$ to $B=(b_{ij})_{m\times n}$ by

(i) If the j^{th} $(1\le j\le n)$ attribute is benefit attribute, then

$$b_{ij}=\frac{a_{ij}}{\max_{(1\le i\le m)} a_{ij}}, 1\le i\le m.$$

(ii) If the j^{th} $(1\le j\le n)$ attribute is cost attribute, then

$$b_{ij}=\frac{\min_{(1\le i\le m)} a_{ij}}{a_{ij}}, 1\le i\le m.$$

After that, the style of all attributes change to benefit. And the value of all attributes comparable.

a. Dominance method [7]

Denote one alternative by $x_i\sim(b_{i1},b_{i2},...,b_{in})$ and another by $x_k\sim(b_{k1},b_{k2},...,b_{kn})$. Then we say that the second alternative dominates the first $x_k\succ x_i$ if $b_{kj}\ge b_{ij}$ for all j, and further $b_{kj'}>b_{ij'}$ for some j'.

b. Satisficing method [8]

The decision maker supplies the minimal attribute values he will accept for each of the attribute $(b_1^0,b_2^0,...,b_n^0)$, the alternative x_i is taken as a feasible alternative if $b_{ij}\ge b_j^0$ for all j. After this process, we are still left with a number of feasible alternatives.

c. Maximin method [1]

This method takes the mean of an old saying that"the chain is only as strong as its weakest link". Select the weakest attribute value b_i^0 of x_i by

$$b_i^0=\min\{b_{i1},b_{i2},...,b_{in}\}, i=1,2,...m.$$

Then x_{i*} will be selected out as the best alternative if

$$b_{i*}^0=\max_{(1\le i\le m)} b_i^0.$$

d. Maximax method [1]

Select the strongest attribute value b_i^* of x_i by

$$b_i^*=\max\{b_{i1},b_{i2},...,b_{in}\}, i=1,2,...m.$$

x_{i*} will be selected out as the best alternative if

$$b_{i*}^*=\max_{(1\le i\le m)} b_i^*.$$

e. Lexicography method [1]

Suppose the attributes are ordered so that u_1 is the most important attribute to the decision maker, u_2 is the next most important, and so forth. Then take

$$\aleph^1=\{x_{i*}|b_{i*1}=\max_{(1\le i\le m)} b_{i1}, 1\le i^*\le m\}.$$

If $\aleph^1$ has a single element, then this one is the most preferred alternative. Else, consider

$$\aleph^2=\{x_{i*}|b_{i*2}=\max_{(i\in\{k|x_k\in\aleph^1\})} b_{i2}, i^*\in\{k|x_k\in\aleph^1\}\}$$

If $\aleph^2$ has a single element, then this one is the most preferred alternative. Else, continue this process until either (i) some $\aleph^*$ with only a single element is found, which is the most preferred alternative or (ii) all attributes have been considered, in which case if the remaining set contains more than one maximal element, they are considered to be equivalent [1].

f. Additive weighting method [9]

We can get the normalized weight $(w_1,w_2,...,w_n)$ for each attribute by subjective weights and objective weights (The subjective methods are to determine weights solely according to the preference or judgments of decision makers. Then apply some mathematic methods such as the eigenvector method, weighted least square method, and mathematical programming models to calculate overall evaluation of each decision maker. The objective methods determines weights by solving mathematical models automatically without any consideration of the decision maker's preferences, for example, the entropy method, multiple objective programming, etc.)[14], where

$$\sum_{j=1}^{n} w_j = 1, w_j \geq 0, j = 1, 2, ..., n.$$

Then, x_{i^*} will be selected out as the best alternative if

$$\sum_{j=1}^{n} w_j \cdot b_{i^*j} = \max_{(1 \leq i \leq m)} \sum_{j=1}^{n} w_j \cdot b_{ij}.$$

g. Non-metric scaling method [10]

Suppose $(w_1, w_2, ..., w_n)$ is the normalized attribute weights. The weighted attribute value $C = (c_{ij})_{m \times n}$ can be gotten by

$$c_{ij} = b_{ij} \cdot w_j, i = 1, 2, ...m; j = 1, 2, ..., n.$$

Denote one alternative by $x_i \sim (c_{i1}, c_{i2}, ..., c_{in})$ and another by $x_k \sim (c_{k1}, c_{k2}, ..., c_{kn})$. The distance between any two points x_i and x_j is defined to be

$$d(x_i, x_k) = \sqrt{\sum_{j=1}^{n} (c_{ij} - c_{kj})^2}.$$

Then, we locate an ideal object $x^* \sim (c_1^*, c_2^*, ..., c_n^*)$ in the alternative space, where

$$c_j^* = \min_{(1 \leq i \leq m)} c_{ij}, j = 1, 2, ..., n.$$

Thus, x_{i^*} will be selected out as the best alternative if

$$d(x_{i^*}, x^*) = \min_{(1 \leq i \leq m)} d(x_i, x^*).$$

Take an example from [1] to illustrate these methods.

Example 1 [1]. Suppose for a particular anticipated military requirement, say, within the general war mission, we must make a choice among designs for a future weapon system. Let us consider three possible types of system—call them X, Y, and Z. The attributes (Range (n mi) \ Delivery time (hr) \ Total yield (MT) \ Accuracy (high-low) \ Vulnerability (high-low) \ Payload delivery flexibility (high-low)) are generated by careful political-military consideration of this particular requirement within the overall mission and possibly also future uses of the proposed system. In this case, we can characterize each system uniquely by each set of attributes, which is shown in Table 2.

Table 2. *A Weapon System Decision Problem.*

	Range (n mi)	**Delivery time (hr)**	**Total yield (MT)**	**Accuracy (high-low)**	**Vulnerability (high-low)**	**Payload delivery flexibility (high-low)**
X	10,000	5	100	Average	Average	High
Y	8,000	0.5	50	Low	High	Low
Z	5,000	1	80	High	Very low	Average

The 1-9 scale [15] is taken for the corresponding qualitative ones in Table 2, which shows in Table 3.

Table 3. *1-9 numerical scale.*

Numerical Scale	**1**	**3**	**5**	**7**	**9**
Vulnerability	Very high	High	Average	Low	Very low
Payload delivery flexibility/Accuracy	Very low	Low	Average	High	Very high

Then, the weapon system decision problem in Table 2 will change to Table 4.

Table 4. *The Weapon System Decision Problem.*

	Range (n mi)	**Delivery time (hr)**	**Total yield (MT)**	**Accuracy**	**Vulnerability**	**Payload delivery flexibility**
X	10,000	5	100	5	5	7
Y	8,000	0.5	50	3	3	3
Z	5,000	1	80	7	9	5

To make the attributes comparable, normalize the information in Table 4, and we will get the decision information matrix.

Table 5. *Comparable Numerical Values for the Problem.*

	Range (n mi)	**Delivery time (hr)**	**Total yield (MT)**	**Accuracy**	**Vulnerability**	**Payload delivery flexibility**
X	1	1	1	0.7143	0.5556	1
Y	0.8	0.1	0.5	0.4286	0.3333	0.4286
Z	0.5	0.2	0.8	1	1	0.7143

The decision results of these existed methods are shown in Table 6.

Table 6. *The decision results of existed method.*

The existed method	**Parameter**	**Results**
Dominance method	**None**	**Invalid**
	(0.5, 0.1, 0.5, 0.4, 0.3, 0.4)	X, Y, Z
Satisficing method	(0.8, 0.1, 0.5, 0.4, 0.3, 0.4)	X, Y
	(0.5, 0.1, 0.8, 0.8, 0.3, 0.4)	Z

The existed method	Parameter	Results
Dominance method	**None**	**Invalid**
	(0.6, 0.1, 0.8, 0.8, 0.3, 0.4)	Empty
Maximin method	None	X≻Z≻Y
Maximax method	None	X~Z≻Y
Lexicography method	None	X≻Y≻Z
Additive weighting method	(0.05, 0.1, 0.1, 0.4, 0.15, 0.2)	X≻Z≻Y
	(0.04, 0.1, 0.1, 0.4, 0.16, 0.2)	Z≻X≻Y
Non-metric scaling method	(1, 1, 1, 1, 1, 1)	X≻Z≻Y

In this example, the Dominance method is unavailable. It can't be used to rank these alternatives. For the satisficing method, different decision maker provides different minimal attribute values, which will generate different ranking result of alternatives. Take four different groups of minimal attribute value in Table 6 for example. The minimal attribute values (0.5, 0.1, 0.5, 0.4, 0.3, 0.4) generate the result that all alternatives are feasible. If the minimal value of the first attribute changes to 0.8, then alternative Z will be infeasible and alternatives X and Y are feasible. Besides, if the minimal value group changes to (0.5, 0.1, 0.8, 0.8, 0.3, 0.4), then only alternative Z is feasible. Furthermore, if the minimal value group changes to (0.6, 0.1, 0.8, 0.8, 0.3, 0.4), then none of the alternatives is feasible.

For the Maximin method, the alternative is only as strong as its weakest attribute. So the weakest attribute for alternative X is Vulnerability, its value is 0.5556. That for alternative Y is Delivery time, its value is 0.1. That for alternative Z is also Delivery time, its value is 0.2. Thus, the order of alternatives is X≻Z≻Y. It means that X≻Z, Z≻Y and X≻Y. In Table 5, comparing the each attribute value of alternatives, X≻Y is obviously right. However, both the other two relations cannot stand up. Let us pay attention to alternative X and Z, in Table 5, if the attribute "Accuracy" or "Vulnerability" is very important in decision making, then the ranking may be reverse. In the same way, considering the alternative Y and Z, you will generate the same result. Thus, this method would neglect the importance of attributes.

For the Maximax method, the alternative is as good as its best attribute. So the best attributes for alternative X are Range\Delivery time\Total yield\Payload delivery flexibility, their value are all 1. That for alternative Y is Range, its value is 0.8. That for alternative Z are Accuracy/Vulnerability, their value are both 1. Thus, the order of alternatives is X~Z≻Y. It means that X~Z, Z≻Y and X≻Y. When you pay attention to each pair of alternatives, only the relation between X and Y is obviously right, the others may not when considering special attribute. So, this method neglects the same thing as the above method.

For the Lexicography method, if the first attribute Range is the most important attribute, then the value for alternative $X/Y/Z$ are 1, 0.8 and 0.5, respectively. So the order of alternatives is X≻Y≻Z. This method would neglect the weight of the attribute. For example, if the weight of the first attribute "Range" is 0.3, the weight of attributes "Accuracy" and "Vulnerability" are both 0.25. Then, alternative Z would exceed X and reverse to the best one.

For the additive weighting method, if we set the attribute weight be (0.05, 0.1, 0.1, 0.4, 0.15, 0.2), the order result would be X≻Z≻Y. If the attribute weight changes tiny to (0.04, 0.1, 0.1, 0.4, 0.16, 0.2), the order result will be different. This rank method largely relies on the attribute weight. If attribute weight changes little, you would make a different decision and take another alternative as the best one.

For the non-metric scaling method, when you locate an ideal object (1, 1, 1, 1, 1, 1, 1) in the attribute space, the distance between alternative $X/Y/Z$ and ideal object are 0.5283, 1.4824, 1.0058. Alternative X is close to the ideal object. And the order result is X≻Z≻Y. This method also neglect the advantage of alternative Z in "Accuracy" and "Vulnerability". If decision maker takes these two attribute as an important one, then alternative Z would be the optimal.

Each method has its special application environment. None of these methods can be taken as a universal method. To construct a universal method for MADM problem, in next section, an analyzing method different from every method introduced in this section will be proposed based on the Possibility degree. In order to understand the possibility-priority degree analyzing method easily, we consider the MADM problem from easy to general. Section 3 is to consider a MADM problem where the attributes are in same importance. Section 4 is to consider a MADM problem in general condition, considering different attribute importance.

3. Possibility Priority Degree for Special MADM Problem

In this section, we considering a special multiple attribute decision making problem, whose attributes take the same importance. Suppose there are m alternatives: $x_1, x_2, ..., x_m$, n attributes: $u_1, u_2, ..., u_n$. The MADM information can be shown in Table 7.

Table 7. *A MADM problem.*

	u_1	u_2	...	u_n
x_1	a_{11}	a_{12}	...	a_{1n}
x_2	a_{21}	a_{22}	...	a_{2n}
...			...	...
x_m	a_{m1}	a_{m2}	...	a_{mn}

Definition 1: For an attribute $u_j (j = 1, 2, ..., n)$ and any two alternatives x_i and x_k (Their attribute information for u_j is a_{ij} and a_{kj}, respectively; $i, k = 1, 2, ..., m$), if $a_{ij} > a_{kj}$, then alternative x_i is success to x_k for attribute

u_j, we call that the priority degree (P) of x_i success to x_k for u_j is 1, the similar degree (S) of x_i similar to x_k for u_j is 0, and the priority degree (P) of x_k success to x_i for u_j is 0. If $a_{ij} = a_{kj}$, then alternative x_i is similar to x_k for attribute u_j, we call that the priority degree of x_i success to x_k for u_j is 0, the similar degree of x_i similar to x_k for u_j is 1, and the priority degree of x_k success to x_i for u_j is 0. If $a_{ij} < a_{kj}$, then alternative x_k is success to x_i for attribute u_j, we call that the priority degree of x_i success to x_k for u_j is 0, the similar degree of x_i similar to x_k for u_j is 0, and the priority degree of x_k success to x_i for u_j is 1. We note that

(i) $Pr\{x_i^j \succ x_k^j\} = 1, S\{x_i^j \sim x_k^j\} = 0, Pr\{x_k^j \succ x_i^j\} = 0$, if $a_{ij} > a_{kj}$,

(ii) $Pr\{x_i^j \succ x_k^j\} = 0, S\{x_i^j \sim x_k^j\} = 1, Pr\{x_k^j \succ x_i^j\} = 0$, if $a_{ij} = a_{kj}$,

(iii) $Pr\{x_i^j \succ x_k^j\} = 0, S\{x_i^j \sim x_k^j\} = 0, Pr\{x_k^j \succ x_i^j\} = 1$, if $a_{ij} < a_{kj}$,

Where $'\succ', '\sim'$ are priority and similar, respectively (See more from "Multiple criteria decision making"[16]), $Pr\{\cdot\}$ is the priority degree and $S\{\cdot\}$ is the similar degree.

Property 1: In a MADM problem, for all $i, k, u = 1, 2, ..., m; j = 1, 2, ..., n$,

(i) $0 \le Pr\{x_i^j \succ x_k^j\} \le 1, 0 \le S\{x_i^j \sim x_k^j\} \le 1$,

(ii) $Pr\{x_i^j \succ x_k^j\} + S\{x_i^j \sim x_k^j\} + Pr\{x_k^j \succ x_i^j\} = 1$,

(iii) If $Pr\{x_i^j \succ x_k^j\} = 1, Pr\{x_k^j \succ x_u^j\} = 1$, then $Pr\{x_i^j \succ x_u^j\} = 1$,

(iv) If $Pr\{x_i^j \sim x_k^j\} = 1, Pr\{x_k^j \sim x_u^j\} = 1$, then $Pr\{x_i^j \sim x_u^j\} = 1$.

These properties in Property 1 can be illustrated as follows.

Property 1 (i) means the range of priority degree and similar degree. If $a_{ij} > a_{kj}$, then $Pr\{x_i^j \succ x_k^j\} = 1$. Else $Pr\{x_i^j \succ x_k^j\} = 0$. So, $0 \le Pr\{x_i^j \succ x_k^j\} \le 1$. In the same way, we can get that $0 \le S\{x_i^j \sim x_k^j\} \le 1$.

Property 1 (ii) shows that the relation between the attribute value of alternative x_i and that of x_k is certain. Whatever the relation is, only one of the three relations $Pr\{x_i^j \succ x_k^j\}, S\{x_i^j \sim x_k^j\}, Pr\{x_k^j \succ x_i^j\}$ takes 1, and the other two relations takes 0. So, these three sum to 1.

Property 1 (iii) shows the transitivity of priority degree. On a certain attribute u_j, if alternative x_i is success to x_k ($Pr\{x_i^j \succ x_k^j\} = 1$) and x_k is success to x_u ($Pr\{x_k^j \succ x_u^j\} = 1$), then alternative x_i is success to x_u ($Pr\{x_i^j \succ x_u^j\} = 1$).

Property 1 (iv) shows the transitivity of similar degree. On a certain attribute u_j, if alternative x_i is similar to x_k ($Pr\{x_i^j \sim x_k^j\} = 1$) and x_k is similar to x_u ($Pr\{x_k^j \sim x_u^j\} = 1$), then alternative x_i is similar to x_u ($Pr\{x_i^j \sim x_u^j\} = 1$).

For simply noting the Possibility, we simplify the note of $Pr\{x_i^j \succ x_k^j\} = 1, S\{x_i^j \sim x_k^j\} = 0$ as $x_i^j \succ_{100\%} x_k^j, x_i^j \sim_{0\%} x_k^j$.

Example 2 Take the example in Example 1 for example, the normalized decision information shows in Table 8.

Table 8. *Comparable Numerical Values for the Problem.*

	Range (n mi)	Delivery time (hr)	Total yield (MT)	Accuracy	Vulnerability	Payload delivery flexibility
X	1	1	1	0.7143	0.5556	1
Y	0.8	0.1	0.5	0.4286	0.3333	0.4286
Z	0.5	0.2	0.8	1	1	0.7143

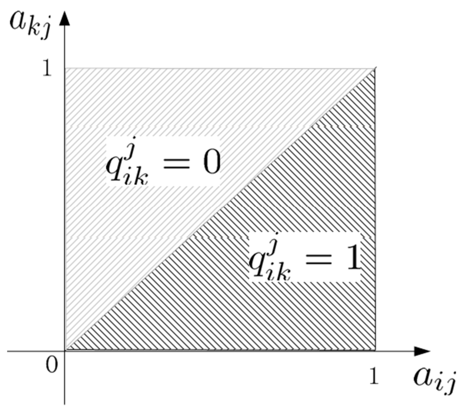

Figure 1. *Relations among a_{ij}, a_{kj} and q_{ik}^j.*

Random choose one attribute from Range\Delivery time\Total yield\Accuracy\Vulnerability\Payload delivery flexibility as an example. We take Accuracy. On this attribute, these relations can get.

$$Pr = \begin{bmatrix} X \succ_{0\%} X & X \succ_{100\%} Y & X \succ_{0\%} Z \\ Y \succ_{0\%} X & Y \succ_{0\%} Y & Y \succ_{0\%} Z \\ Z \succ_{100\%} X & Z \succ_{100\%} Y & Z \succ_{0\%} Z \end{bmatrix},$$

$$S = \begin{bmatrix} X \sim_{100\%} X & X \sim_{0\%} Y & X \sim_{0\%} Z \\ Y \sim_{0\%} X & Y \sim_{100\%} Y & Y \sim_{0\%} Z \\ Z \sim_{0\%} X & Z \sim_{0\%} Y & Z \sim_{100\%} Z \end{bmatrix}.$$

It means that on accuracy, alternative X is 100 percent success to Y. Z is 100 percent success to X. And Z is 100 percent success to Y. They are only 100 percent similar to themselves, separately.

To compare two alternatives, we take a binary variable $q_{ik}^j (i, k = 1, 2, ..., m; j = 1, 2, ..., n)$, which respect the relation between alternative x_i and x_k on attribute u_j, if alternative x_i on attribute u_j is 100 percent priority to x_k, then $q_{ik}^j = q^j(x_i, x_k) = 1$, else, $q_{ik}^j = q^j(x_i, x_k) = 0$, where

$$q_{ik}^{j}=q^{j}(x_i,x_k)=\begin{cases}1, & \text{if } x_i^j \succ_{100\%} x_k^j,\\ 0, & \text{else.}\end{cases}\quad i,k=1,2,...,m;j=1,2,...,n.$$

With the binary variable q_{ik}^{j}, the attribute, on which alternative x_i is 100 percent success to x_k, can be count that

$$Count(x_i \succ_{100\%} x_k)=\sum_{j=1}^{n} q_{ik}^{j}, i,j=1,2,...,m.$$

Then, the attribute, on which alternative x_i is 100 percent similar to x_k, can generate that

$$Count(x_i \sim_{100\%} x_k)=n-Count(x_i \succ_{100\%} x_k)-Count(x_k \succ_{100\%} x_i), i,j=1,2,...,m.$$

Definition 2: For any two alternatives x_i and x_k(Their attribute information for $u_j(j=1,2,...,n)$ is a_{ij} and a_{kj}, respectively; $i,k=1,2,...,m$),

(i) The Possibility of x_i is 100 percent success to x_k is that

$$P\{x_i \succ_{100\%} x_k\}=\frac{Count(x_i \succ_{100\%} x_k)}{n}, i,k=1,2,...,m.$$

(ii) The Possibility of x_i is 100 percent similar to x_k is that

$$P\{x_i \sim_{100\%} x_k\}=\frac{n-Count(x_i \succ_{100\%} x_k)-Count(x_k \succ_{100\%} x_i)}{n}, i,k=1,2,...,m.$$

where $P\{\cdot\}$ is Possibility degree. $P\{x_i \succ_{100\%} x_k\}$ is the Possibility Priority degree (PPD). $P\{x_i \sim_{100\%} x_k\}$ is the Possibility Similar degree.

Property 2: For any three alternatives x_i, x_k and x_u ($i,k,u=1,2,...,m$),

(i) $0 \le P\{x_i \succ_{100\%} x_k\} \le 1, 0 \le P\{x_i \sim_{100\%} x_k\} \le 1, i,k=1,2,...,m,$

(ii) $P\{x_i \succ_{100\%} x_k\}+P\{x_i \sim_{100\%} x_k\}+P\{x_k \succ_{100\%} x_i\}=1, i,k=1,2,...,m,$

(iii) If $P\{x_i \succ_{100\%} x_k\}=1, P\{x_k \succ_{100\%} x_u\}=1$, then $P\{x_i \succ_{100\%} x_u\}=1$,

(iv) If $P\{x_i \sim_{100\%} x_k\}=1, P\{x_k \sim_{100\%} x_u\}=1$, then $P\{x_i \sim_{100\%} x_u\}=1$.

Property 2 (i) means that the Possibility of alternative x_i 100 percent success to x_j is between 0 and 1. If every attribute value of alternative x_i is better than that of x_j, then the Possibility of alternative x_i 100 percent success to x_j is 1. If every attribute value of alternative x_i is not better than that of x_j, then the Possibility of alternative x_i 100 percent success to x_j is 0. If some attributes' value of alternative x_i is better than that of x_j while some attributes not, then the Possibility of alternative x_i 100 percent success to x_j is between 0 and 1. And the Possibility of alternative x_i 100 percent similar to x_j is between 0 and 1. If every attribute value of alternative x_i is equal to that of x_j, then the Possibility of alternative x_i 100 percent similar to x_j is 1. If every attribute value of alternative x_i is not equal to that of x_j, then the Possibility of alternative x_i 100 percent similar to x_j is 0. If some attributes' value of alternative x_i is equal to that of x_j while some attributes not, then the Possibility of alternative x_i 100 percent similar to x_j is between 0 and 1.

Property 2 (ii) means that for any two alternatives x_i and x_k, summation of the Possibility of alternative x_i 100 percent success to x_j, x_i 100 percent similar to x_j, and x_k 100 percent success to x_i is 1.

Property 2 (iii) means that for any three alternatives x_i, x_k, x_u, if alternative x_i is 100 percent success to x_k, and alternative x_k is 100 percent success to x_u, then alternative x_i is 100 percent success to x_u.

Property 2 (iv) means that for any three alternatives x_i, x_k, x_u, if alternative x_i is 100 percent similar to x_k, and alternative x_k is 100 percent similar to x_u, then alternative x_i is 100 percent similar to x_u.

Example 3 Take the example in Example 1 for example, the normalized decision information shows in Table 9.

Table 9. *Comparable Numerical Values for the Problem.*

	Range (n mi)	Delivery time (hr)	Total yield (MT)	Accuracy	Vulnerability	Payload delivery flexibility
X	1	1	1	0.7143	0.5556	1
Y	0.8	0.1	0.5	0.4286	0.3333	0.4286
Z	0.5	0.2	0.8	1	1	0.7143

For attribute Range, the priority and similar degree can generate that

$$P=\begin{bmatrix} X \succ_{0\%} X & X \succ_{100\%} Y & X \succ_{100\%} Z \\ Y \succ_{0\%} X & Y \succ_{0\%} Y & Y \succ_{100\%} Z \\ Z \succ_{0\%} X & Z \succ_{0\%} Y & Z \succ_{0\%} Z \end{bmatrix}, S=\begin{bmatrix} X \sim_{100\%} X & X \sim_{0\%} Y & X \sim_{0\%} Z \\ Y \sim_{0\%} X & Y \sim_{100\%} Y & Y \sim_{0\%} Z \\ Z \sim_{0\%} X & Z \sim_{0\%} Y & Z \sim_{100\%} Z \end{bmatrix}.$$

For attribute Delivery time, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{100\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{0\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}.$$

For attribute Total yield, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{100\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{0\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}.$$

For attribute Accuracy, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{0\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{100\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}$$

For attribute Vulnerability, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{0\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{100\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}$$

For attribute Payload delivery flexibility, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{100\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{0\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}.$$

Then, the Possibility of one alternative is 100 percent success to the other in each pair of alternative X, Y, Z can be calculated that

$$\begin{bmatrix} P\{X\succ_{100\%}X\} & P\{X\succ_{100\%}Y\} & P\{X\succ_{100\%}Z\} \\ P\{Y\succ_{100\%}X\} & P\{Y\succ_{100\%}Y\} & P\{Y\succ_{100\%}Z\} \\ P\{Z\succ_{100\%}X\} & P\{Z\succ_{100\%}Y\} & P\{Z\succ_{100\%}Z\} \end{bmatrix}=\begin{bmatrix} 0\% & 100\% & 67\% \\ 0\% & 0\% & 17\% \\ 33\% & 83\% & 0\% \end{bmatrix}$$

The possibility of one alternative is 100 percent similar to the other in each pair of alternative X, Y, Z can be calculated that

$$\begin{bmatrix} P\{X\sim_{100\%}X\} & P\{X\sim_{100\%}Y\} & P\{X\sim_{100\%}Z\} \\ P\{Y\sim_{100\%}X\} & P\{Y\sim_{100\%}Y\} & P\{Y\sim_{100\%}Z\} \\ P\{Z\sim_{100\%}X\} & P\{Z\sim_{100\%}Y\} & P\{Z\sim_{100\%}Z\} \end{bmatrix}=\begin{bmatrix} 100\% & 0\% & 0\% \\ 0\% & 100\% & 0\% \\ 0\% & 0\% & 100\% \end{bmatrix}$$

From the possibility of priority degree, we can see that the Possibility of alternative X 100 percent success to Y is 100%. It means that X is the best choice when you choose one from X and Y.

The Possibility of alternative X 100 percent success to Z is 67%. It means that the Possibility of X better than Z is 67%. It do not means that X is the best when you choose choice one from X and Z. While you choose X from X and Z, the risk of this decision exist. So when do decision making, decision maker should be aware of the risk.

The Possibility of alternative Y 100 percent success to Z is 17%. It means that the Possibility of Y better than Z is 17%. It does not mean that Z is the best choice. Alternative Y still has 17 percent chance better than Z.

The Possibility of alternative Z 100 percent success to X is 33%. It means that the Possibility of Z better than X is 33%. It does not mean that X is the best choice. Alternative Z still has 33 percent chance better than X.

The Possibility of alternative Z 100 percent success to Y is 83%. It means that the Possibility of Z better than Y is 83%. It means that Z is the best choice when choose one from Y and Z. But the risk still exists.

The Possibility of alternative X 100 percent success to X, Y 100 percent success to Y, Z 100 percent success to Z and Y 100 percent success to X are 0%. Alternative X, Y, Z success to itself is impossible. And Z success to Y is also impossible.

From the possibility of similar degree, we can see that only the possibility of alternative X, Y, Z similar to itself is 100%. The possibility degree of they similar to each other is 0.

So, a decision maker may take the alternative X as the best one. While choosing alternative X, decision maker should aware of the risk still existing.

4. Possibility Priority Degree for General MADM Problem

In this section, a general MADM problem with different attribute weight is taken into account.

With the traditional MADM method, there are a mount of methods to generate the attribute weight, including integrated weights method, objective weight method and subjective weight method [17]. According to the decision maker, each of them can be used to generate the attribute weight.

Suppose there are m alternatives: $x_1, x_2, ..., x_m$, n attributes: $u_1, u_2, ..., u_n$. And the attribute weight can get that $w_1, w_2, ..., w_n$, where

$$\sum_{i=1}^{n} w_j = 1, w_j > 0, j = 1, 2, ..., n.$$

Let

$$J_{ik}^{\succ} = \{u_j | x_i^j \succ_{100\%} x_k^j, j = 1, 2, ..., n.\}, i, k = 1, 2, ..., m.$$

be a subset of attributes, on which alternative x_i is 100 percent success to x_k and

$$J_{ik}^{\sim} = \{u_j | x_i^j \sim_{100\%} x_k^j, j = 1, 2, ..., n.\}, i, k = 1, 2, ..., m.$$

be a subset of attributes, on which alternative x_i is 100 percent similar to x_k.

Definition3: Suppose the attribute weights are $w_1, w_2, ..., w_n$. For any two alternatives x_i and x_k(Their attribute information for $u_j (j = 1, 2, ..., n)$ is a_{ij} and a_{kj}, respectively; $i, k = 1, 2, ..., m$),

(i) The Possibility of x_i is 100 percent success to x_k is that

$$P\{x_i \succ_{100\%} x_k\} = \sum_{u_j \in J_{ik}^{\succ}} w_j, i, k = 1, 2, ..., m.$$

(ii) The Possibility of x_i is 100 percent similar to x_k is that

$$P\{x_i \sim_{100\%} x_k\} = \sum_{u_j \in J_{ik}^{\sim}} w_j, i, k = 1, 2, ..., m.$$

Where $P\{\cdot\}$ is Possibility (Access more knowledge about Possibility form "Possibility: theory and examples" [18]). $P\{x_i \succ_{100\%} x_k\}$ is the Possibility Priority degree (PPD). $P\{x_i \sim_{100\%} x_k\}$ is the Possibility Similar degree.

Mark 2: The possibility of priority degree and similar degree in Definition 3 hold each property in Property 2.

Example 4 Take the example in Example 1 for example, the normalized decision information shows in Table 10.

Table 10. *Comparable Numerical Values for the Problem.*

	Range (n mi)	Delivery time (hr)	Total yield (MT)	Accuracy	Vulnerability	Payload delivery flexibility
X	1	1	1	0.7143	0.5556	1
Y	0.8	0.1	0.5	0.4286	0.3333	0.4286
Z	0.5	0.2	0.8	1	1	0.7143

Suppose the attribute weights are

$$w_1 = 0.15, w_2 = 0.05, w_3 = 0.1, w_4 = 0.4, w_5 = 0.2, w_6 = 0.1.$$

For attribute Range, the priority and similar degree can generate that

$$P = \begin{bmatrix} X \succ_{0\%} X & X \succ_{100\%} Y & X \succ_{100\%} Z \\ Y \succ_{0\%} X & Y \succ_{0\%} Y & Y \succ_{100\%} Z \\ Z \succ_{0\%} X & Z \succ_{0\%} Y & Z \succ_{0\%} Z \end{bmatrix}, S = \begin{bmatrix} X \sim_{100\%} X & X \sim_{0\%} Y & X \sim_{0\%} Z \\ Y \sim_{0\%} X & Y \sim_{100\%} Y & Y \sim_{0\%} Z \\ Z \sim_{0\%} X & Z \sim_{0\%} Y & Z \sim_{100\%} Z \end{bmatrix}.$$

For attribute Delivery time, the priority and similar degree can generate that

$$P = \begin{bmatrix} X \succ_{0\%} X & X \succ_{100\%} Y & X \succ_{100\%} Z \\ Y \succ_{0\%} X & Y \succ_{0\%} Y & Y \succ_{0\%} Z \\ Z \succ_{0\%} X & Z \succ_{100\%} Y & Z \succ_{0\%} Z \end{bmatrix}, S = \begin{bmatrix} X \sim_{100\%} X & X \sim_{0\%} Y & X \sim_{0\%} Z \\ Y \sim_{0\%} X & Y \sim_{100\%} Y & Y \sim_{0\%} Z \\ Z \sim_{0\%} X & Z \sim_{0\%} Y & Z \sim_{100\%} Z \end{bmatrix}.$$

For attribute Total yield, the priority and similar degree can generate that

$$P = \begin{bmatrix} X \succ_{0\%} X & X \succ_{100\%} Y & X \succ_{100\%} Z \\ Y \succ_{0\%} X & Y \succ_{0\%} Y & Y \succ_{0\%} Z \\ Z \succ_{0\%} X & Z \succ_{100\%} Y & Z \succ_{0\%} Z \end{bmatrix}, S = \begin{bmatrix} X \sim_{100\%} X & X \sim_{0\%} Y & X \sim_{0\%} Z \\ Y \sim_{0\%} X & Y \sim_{100\%} Y & Y \sim_{0\%} Z \\ Z \sim_{0\%} X & Z \sim_{0\%} Y & Z \sim_{100\%} Z \end{bmatrix}.$$

For attribute Accuracy, the priority and similar degree can generate that

$$P = \begin{bmatrix} X \succ_{0\%} X & X \succ_{100\%} Y & X \succ_{0\%} Z \\ Y \succ_{0\%} X & Y \succ_{0\%} Y & Y \succ_{0\%} Z \\ Z \succ_{100\%} X & Z \succ_{100\%} Y & Z \succ_{0\%} Z \end{bmatrix}, S = \begin{bmatrix} X \sim_{100\%} X & X \sim_{0\%} Y & X \sim_{0\%} Z \\ Y \sim_{0\%} X & Y \sim_{100\%} Y & Y \sim_{0\%} Z \\ Z \sim_{0\%} X & Z \sim_{0\%} Y & Z \sim_{100\%} Z \end{bmatrix}$$

For attribute Vulnerability, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{0\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{100\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}$$

For attribute Payload delivery flexibility, the priority and similar degree can generate that

$$P=\begin{bmatrix} X\succ_{0\%}X & X\succ_{100\%}Y & X\succ_{100\%}Z \\ Y\succ_{0\%}X & Y\succ_{0\%}Y & Y\succ_{0\%}Z \\ Z\succ_{0\%}X & Z\succ_{100\%}Y & Z\succ_{0\%}Z \end{bmatrix},\ S=\begin{bmatrix} X\sim_{100\%}X & X\sim_{0\%}Y & X\sim_{0\%}Z \\ Y\sim_{0\%}X & Y\sim_{100\%}Y & Y\sim_{0\%}Z \\ Z\sim_{0\%}X & Z\sim_{0\%}Y & Z\sim_{100\%}Z \end{bmatrix}.$$

Then, the Possibility of one alternative is 100 percent success to the other in each pair of alternative X, Y, Z can be calculated that

$$\begin{bmatrix} P\{X\succ_{100\%}X\} & P\{X\succ_{100\%}Y\} & P\{X\succ_{100\%}Z\} \\ P\{Y\succ_{100\%}X\} & P\{Y\succ_{100\%}Y\} & P\{Y\succ_{100\%}Z\} \\ P\{Z\succ_{100\%}X\} & P\{Z\succ_{100\%}Y\} & P\{Z\succ_{100\%}Z\} \end{bmatrix}=\begin{bmatrix} 0\% & 100\% & 40\% \\ 0\% & 0\% & 15\% \\ 60\% & 85\% & 0\% \end{bmatrix}$$

The possibility of one alternative is 100 percent similar to the other in each pair of alternative X, Y, Z can be calculated that

$$\begin{bmatrix} P\{X\sim_{100\%}X\} & P\{X\sim_{100\%}Y\} & P\{X\sim_{100\%}Z\} \\ P\{Y\sim_{100\%}X\} & P\{Y\sim_{100\%}Y\} & P\{Y\sim_{100\%}Z\} \\ P\{Z\sim_{100\%}X\} & P\{Z\sim_{100\%}Y\} & P\{Z\sim_{100\%}Z\} \end{bmatrix}=\begin{bmatrix} 100\% & 0\% & 0\% \\ 0\% & 100\% & 0\% \\ 0\% & 0\% & 100\% \end{bmatrix}$$

From the possibility of priority degree, we can see that the Possibility of alternative X 100 percent success to Y is 100%. It means that X is the best choice when you choose one from X and Y.

The Possibility of alternative X 100 percent success to Z is 40%. It means that the Possibility of X better than Z is 40%. It do not means that X is the worse when you choose choice one from X and Z. While you give up X, the advantages of X still exist. So when do decision making, decision maker should be aware of the risk.

The Possibility of alternative Y 100 percent success to Z is 15%. It means that the Possibility of Y better than Z is 15%. It does not mean that Z is the best choice. Alternative Y still has 15 percent chance better than Z.

The Possibility of alternative Z 100 percent success to X is 60%. It means that the Possibility of Z better than X is 60%. It does not mean that Z is the best choice. Alternative X still has 40 percent chance better than Z.

The Possibility of alternative Z 100 percent success to Y is 85%. It means that the Possibility of Z better than Y is 85%. It means that Z is the best choice when choose one from Y and Z. But the risk still exists.

The Possibility of alternative X 100 percent success to X, Y 100 percent success to Y, Z 100 percent success to Z and Y 100 percent success to X are 0%. Alternative X, Y, Z success to itself is impossible. And Z success to Y is also impossible.

From the possibility of similar degree, we can see that only the possibility of alternative X, Y, Z similar to itself is 100%. The possibility degree of they similar to each other is 0.

So, a decision maker may take the alternative Z as the best one. While choosing alternative Z, decision maker should aware of the risk still existing.

5. The PPDA Process

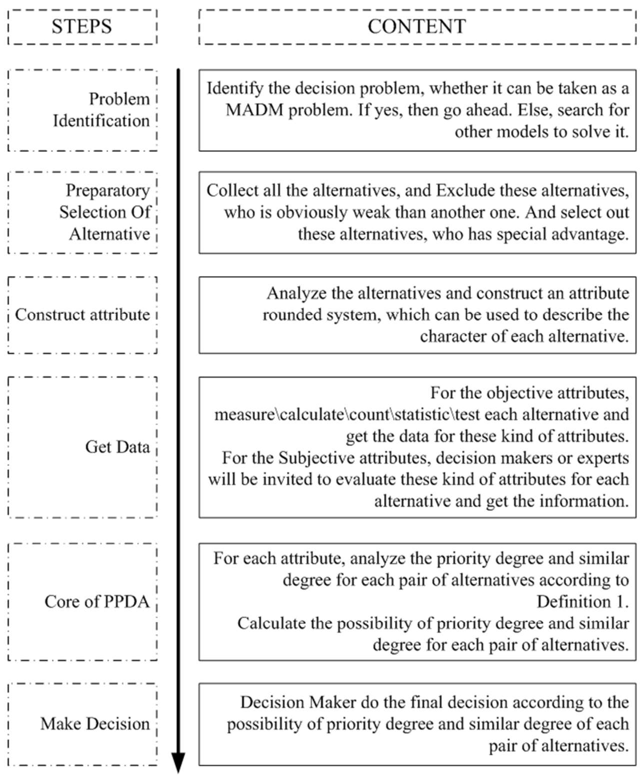

Figure 2. *The PPDA process.*

6. Conclusion

This paper is aim to propose a method for MADM problem, called possibility priority degree analyzing model. To overview the method for MADM problem, in Section 2, The existed methods, including Dominance method, Satisficing method, Maximin method, Maximax method, Lexicography method, Additive weighting method and Non-metric scaling method, are introduced and explained by an Example. The result shows that each method has its special application environment. None of the existed methods can be taken as a universal method.

To construct a universal method for MADM problem, in Section 3, a special MADM problem without considering the attribute importance is studied and a possibility priority degree model is proposed. We construct the priority degree and similar degree for two alternatives in a certain attribute. To summarize the relationship between two alternatives in all attributes, a possibility priority degree and possibility similar degree are proposed. With this possibility priority degree, decision maker can easily get the possibility degree of one alternative priority to another. At the same time, decision maker will be aware of the risk when he/she makes decision.

To generate a general MADM model, a MADM problem considering the attribute weights is studied in Section 4. With the integrated weights method, objective weight method or subjective weight method, decision makers can generate the attribute weights easily. In this condition, we propose the possibility priority degree and possibility similar degree model with different attribute weights. And the new possibility priority degree model satisfies the properties in Property 2. With this possibility priority degree model, the general PPDA process is generated.

Compare the possibility priority degree analyzing process with these existed, there are two main differences of this model:

(1) Return the decision right to decision maker

The host of a decision making is the decision maker. All we can discuss here is to analyze the problem. We have no right to do decision for decision maker. So, you can provide several feasible plans and their related information for decision maker to do final decision. If you provide a certain order of these alternatives, the decision maker is fired. From Table 6, we can see that each of these existed method excepting Dominance method and Satisficing method generates a ranking result finally. Decision maker have no chance to do a choice. With the possibility priority degree analyzing process, we provide a possibility priority degree matrix. Decision maker can get the advantages and disadvantages easily, and then do final decision.

(2) Remind the risk while do decision making

In real word MADM problem, when you decide to choose one and give up another one, there exists the risk hidden in the decision. Except for the group of alternatives which can make decision by the Dominance method, each one in the group of alternatives has both advantages and disadvantages. While choosing one of it, you give up the advantages of another one. It means that the other one may be better than the chosen one in some special condition. So, there are risks when you rank the alternatives. You should let the decision maker be aware of it. From Table 6, we can see that each of these existed method excepting Dominance method and Satisficing method generate a ranking result. It means that the order is the final decision. When suggest the order to decision maker, it means that the order is certain, the order is 100 percent right. Actually, the risk hides on each order. Conversely, the possibility of priority degree shows the possibility of one alternative better than another one. Decision maker can do finally decision with the possibility priority degree. When they do decision, they are aware of the risk of the priority relation.

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