

# Positive Influence of Regulated Human Capital Accumulation on Economic Growth: A Theoretical Model

Kangping Wu, Mengtao Wu\*

School of Economics and Management, Tsinghua University, Beijing, P. R. China

## Email address:

wukp@sem.tsinghua.edu.cn (Kangping Wu), wumt.18@sem.tsinghua.edu.cn (Mengtao Wu)

\*Corresponding author

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**Abstract:** Human capital, as an important factor of production, directly promotes economic growth. On the other hand, it indirectly promotes economic growth by stimulating the accumulation of physical capital and improving the level of technological innovation. Therefore, human capital is the fundamental reason for economic growth. The key to make human capital play a long-term role is to correctly set the human capital accumulation equation. Different from the existing articles that set human capital accumulation equation from the perspective of human capital investment and physical capital investment, this paper focuses on the marginal output of physical capital. Based on the marginal output of physical capital, this paper makes human capital accumulation subject to a regulation that can completely offset the diminishing effect of physical capital marginal output and keep it at a reasonable level. This way of human capital accumulation is called “regulated” accumulation. The derivation of theoretical model proves that the regulated human capital accumulation, which is based on constant marginal output of physical capital, enables the growth rate of output to be constant and positive. Therefore, this paper finds the source of long-term economic growth from aspect of human capital. As the embodiment of a country’s comprehensive strength and core competitiveness, human capital has important strategic significance and great potential in China’s sustainable economic development. Regulated human capital accumulation can provide a new driving force for China’s economic growth and promote long-term development.

**Keywords:** Human Capital, Physical Capital, Regulated Accumulation, Long-Term Economic Growth

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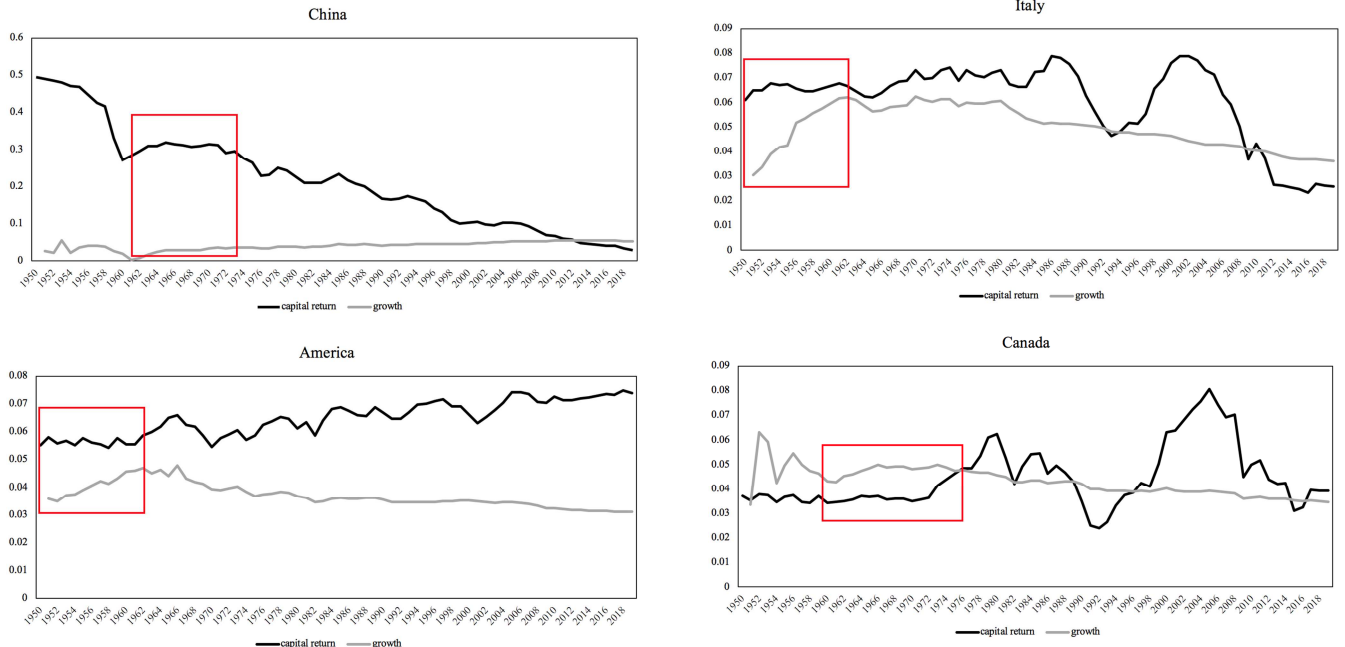
## 1. Introduction

In the theory of economic growth, physical capital, technological innovation and human capital are important driving factors of economic growth. However, due to diminishing effect of marginal output, physical capital fails to ensure sustainable economic growth. Technological innovation is a short-term discontinuous activity focusing on profits. It emphasizes commercial application and market value of new ideas and technologies, which means technological innovation cannot guarantee long-term economic growth either. By comparison, human capital is a vital element. Many studies have shown that human capital, as a production factor, can directly promote economic growth. On the other hand, it plays an indirect role by stimulating physical capital investment and improving technological innovation level. In conclusion, the ultimate driving force of

economic growth is human capital. Human capital refers to an integration of wisdom and physical fitness condensed in workers. Human capital accumulation is a long-term strategic process with lifelong benefits. To truly understand importance of human capital and let it play full role on promoting long-term economic growth is based on the premise that human capital accumulation mode is correctly set. Many literatures not only consider the way of human capital formation, but also consider the influence of physical capital when setting equation of human capital accumulation. It is found that physical capital and human capital accumulation affect each other, and economic growth is driven by these two kinds of capital accumulation. However, we can see that after derivation, the economy will eventually move towards a stable state with a growth rate of zero. In other words, accumulation of physical capital and human capital do not promote sustainable growth. How can human capital accumulation

successfully become the source of long-term economic growth?

This paper uses Penn World Table 10.0 database<sup>1</sup> to calculate marginal output of physical capital and economic growth rate of 10 representative countries from 1950 to 2019, as shown in Figure 1 below. It is shown that in a period of full employment, the marginal output of physical capital remains unchanged and has a significant correlation with economic growth, which means that physical capital with constant marginal output can guarantee sustainable growth. Therefore, starting from the marginal output of physical capital, this paper makes human capital subject to a regulation that can completely offset the diminishing effect of physical capital marginal output and keep it at a reasonable level. This way of human capital accumulation is called “regulated” accumulation. Corresponding to “regulated” accumulation is “free” accumulation – that is, human capital accumulation is not directly related to marginal output of physical capital. Free accumulation is completely endogenous, which will lead to inability of human capital to have long-term promotion effect. Compared with completely endogenous free accumulation, regulated accumulation is semi-endogenous, and regulation can only be carried out according to marginal output of physical capital. Theoretical derivation of this paper can prove that the regulated human capital accumulation keeps the marginal output of physical capital at a certain level that is able to become the source of long-term economic growth. To some extent, this paper will fill the gap in the field of human capital accumulation and provide a theoretical basis for the endogenous growth model of human capital, hence it has certain theoretical significance.



<sup>1</sup> <https://www.rug.nl/ggdc/productivity/pwt/>

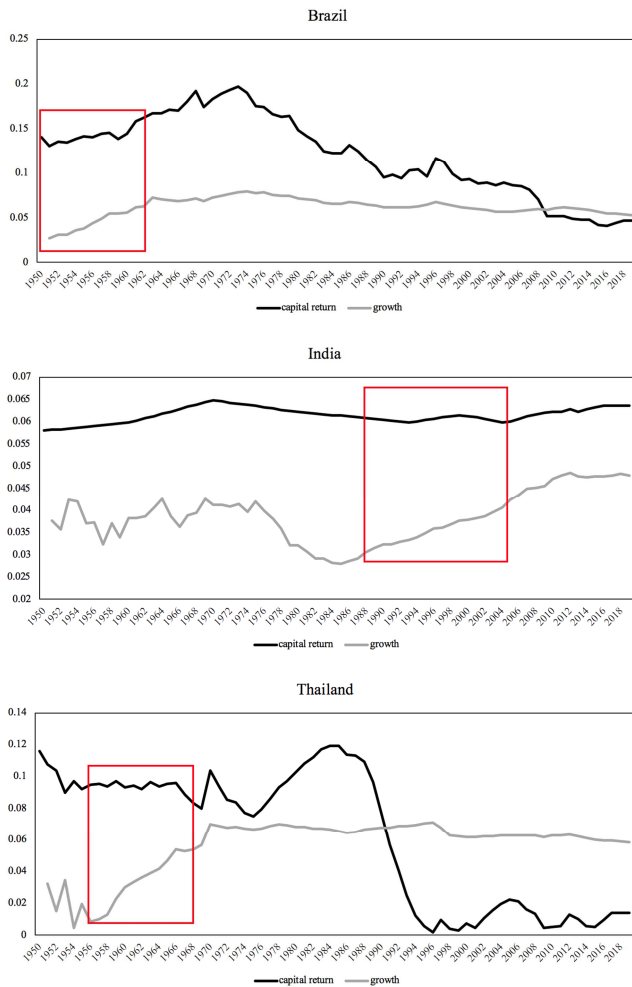


Figure 1. Marginal output of physical capital and economic growth in 10 representative countries from 1950 to 2019.

Since the reform and opening up, China's economy has grown rapidly at an annual average rate of 9.19%, and real GDP per capita has increased nearly 26 times, gaining the praise of "growth miracle". For a long time, however, China's economic growth has been characterized by high physical capital investment and energy consumption. Facing relative shortage of resources and pressure of ecological environment and population, this high-consuming growth model cannot meet the requirements of sustainable development. In addition, the contribution of technological innovation (or total factor productivity) to China's growth is relatively small. Therefore, the sustainability of economic growth must find a breakthrough from perspective of human capital. In 2002, the national strategy of strengthening human capital was proposed for the first time in China, which then became one of the three national developing strategies. Study on human capital accumulation in this paper has great practical significance for China's sustainable economic growth.

The rest of this paper is arranged as follows. The second section is literature review that mainly focuses on the relationship of human capital accumulation and physical capital as well as its marginal output. The third section is baseline theoretical model – that is, the regulated human

capital accumulation model which is divided into marginally-regulated and averagely-regulated accumulation. The fourth section is comparison between marginally-regulated and averagely-regulated accumulation. The last section is conclusion.

## 2. Literature Review

The study of human capital in economics began in the 17th century, but it was not until the late 1950s and early 1960s that the theory of human capital was gradually formed. In 1961, Schultz, the founder of human capital theory, put forward and discussed the complete concept of human capital. Human capital refers to the sum of knowledge, ability and health condensed on workers that can realize value proliferation. Education is the main way to form human capital [1]. Becker transformed human capital theory from concrete to abstract and established the human capital investment theory, marking the formal formation of modern human capital theory [2]. In the late 1980s and early 1990s, the new growth theory founded by Romer and Lucas greatly boosted the research of human capital. Human capital accumulation was introduced into growth model and became the source of economic growth [3, 4]. Subsequently, scholars carried out extensive discussion and research on accumulation mode of human capital and role of human capital in promoting economic growth. According to existing literature, the accumulation of human capital is inseparable from physical capital and is also related to marginal output of physical capital.

### 2.1. Human Capital Accumulation and Physical Capital

#### 2.1.1. Physical Capital Affecting Human Capital

It is pointed out that human capital accumulation is determined by human capital and physical capital, thus the impact of physical capital on human capital accumulation should not be ignored. Mankiw et al. established augmented Solow model  $Y(t) = K(t)^\alpha H(t)^\beta (A(t)L(t))^{1-\alpha-\beta}$ , and assumed that human capital was accumulated in the same way as physical capital, hence human capital accumulation was jointly determined by human capital and physical capital, i.e.  $\dot{h}(t) = s_h y(t) - (n + g + \delta)h(t) = s_h k(t)^\alpha h(t)^\beta - (n + g + \delta)h(t)$  [5]. In their economic growth model, however, if capital reaches the steady state, economy will also enter a stable state. This means human capital accumulated according to Perpetual Inventory Method cannot become the driving force of long-term economic growth. Graca et al. set the human capital accumulation equation as  $x_{t+1} = x_t[1 + \tau_t * \gamma(x_t, k_t)]$ , where  $\gamma(x_t, k_t)$  was marginal efficiency of human capital accumulation and assumed to be an increasing function of human capital and physical capital. After theoretical derivation, it was found that the main driving force in the initial stage of economic development was physical capital accumulation, which was called neoclassical growth. Before the accumulation of physical capital reached a sufficient level, the accumulation of human capital might be in a stagnant state, but when physical capital reached its critical level, human capital began to accumulate. At that time, the

driving force of economic growth changed from physical capital to human capital. To sum up, the accumulation of human capital can make the originally stagnant economy cross the threshold of long-term growth [6].

Bond et al. explored the impact of physical capital and human capital accumulation on economic growth by constructing an endogenous growth model, in which education sector needed both physical capital (such as library, laboratory) and human capital (such as teachers and students). The human capital accumulation equation was  $\dot{H} = (1 - u)Hg(k_y) - \eta H$ . Similarly, the equation of physical capital accumulation was  $\dot{K} = uHf(k_x) - \delta K - C$ . The results of general equilibrium analysis showed that there was a unique and stable balanced growth path. In other words, the economy eventually moved towards a stable state rather than sustained growth, and human capital did not actually play a role in promoting long-term economic growth [7]. Galor and Moav constructed an overlapping generation model and supposed that human capital was a strictly increasing and strictly concave function of individual education expenditure that depended on the capital-labor ratio, thus the accumulation of human capital depended on physical capital. They gained the same conclusion as Graca's that the driving force of economic growth was different in different stages. In the early stage, physical capital accumulation was the main driving force, and then human capital accumulation would gradually replace physical capital as the main engine of economic growth, and human capital accumulation would continue to promote physical capital accumulation. Finally, economic growth was jointly promoted by human capital and physical capital accumulation [8].

There is not much literature in China to explore the impact of physical capital on human capital accumulation. When exploring the impact of matching degree between human capital and physical capital on economic efficiency, Human Capital Research Group set the human capital accumulation equation referring to Mankiw et al., i.e.  $\dot{H}(t) = s_H Y(t) - \delta H(t) = s_H [A(t)L(t)]^{1-\alpha-\beta} K(t)^\alpha H(t)^\beta - \delta H(t)$ . It was stated that the matching degree of human capital and physical capital in total amount and structure was a key factor to improve economic efficiency [9]. Subsequently, Zhao and Zhao made a similar setting and derivation, and pointed out that the optimal proportion of human capital and physical capital accumulation was the ratio of capital-output elasticity. Further analysis showed that under this framework, the economy would move towards a balanced growth path – that is, the economy would eventually be in an equilibrium state with a growth rate of zero [10, 11]. In other words, this specific matching between human capital and physical capital, in the short term, can improve economic efficiency, but it cannot guarantee sustained economic growth in the long run.

### 2.1.2. Human Capital Affecting Physical Capital

Physical capital determines the accumulation of human capital, in turn, human capital affects physical capital investment. After investigating the impact of human capital on economic growth, Benhabib and Spiegel claimed that there was a significantly positive correlation between human capital and physical capital. Human capital could attract physical

capital, which was one of the indirect impacts of human capital on total output. The other way was that human capital improved technological innovation level [12]. Krueger and Lindahl pointed out that capital and skills were complementary. The improvement of education level will attract more physical capital investment [13]. Sianesi and Van Reenen studied the impact of human capital on economic growth on a basis of literature review and similarly concluded that human capital has both direct and indirect contributions to economic growth [14]. Moreover, Grier jointly modeled the accumulation of human capital and physical capital. His empirical study indicated that physical capital had a significantly positive impact on human capital, and human capital also had a significantly positive impact on physical capital [15]. Further, Amir-Ud-Din et al. established a system that included two equations simultaneously, in which physical capital and human capital were both independent variables and dependent variables. The empirical results of the three-stage least squares method and the fixed effect model showed that human capital accumulation had a significantly positive impact on physical capital. At the same time, physical capital had a significant impact on human capital accumulation [16]. In domestic research, the impact of human capital on economic development was not significant, while the impact of physical capital on economic development was quite significant. Further analysis showed that the role of human capital on economic growth was indirectly generated by affecting physical capital investment, hence human capital had played a key role in China's economic development [17]. On a basis of dynamic relationship between physical capital, human capital and economic growth, Wang stated that physical capital promoted economic growth in the short term, but human capital promoted sustained economic growth in the long term [18]. Similarly, Zhang and Wang found that human capital had a significant impact on physical capital investment. In the long term, human capital promoted economic growth through two channels [19].

### 2.2. Human Capital Accumulation and Marginal Output of Physical Capital

In addition to the interaction between human capital and physical capital, studies have found that there is a certain relationship between human capital and marginal output of physical capital. In fact, this is just the reason why scholars introduce human capital into economic growth model in the early stage. It was first pointed out by Hirschman that physical capital with diminishing marginal output could not become the driving force of sustainable economic growth [20]. Subsequently, Lucas used human capital accumulation to counteract the diminishing effect of physical capital marginal output, and drew the conclusion that human capital accumulation could promote sustained economic growth. However, Lucas's human capital accumulation equation is not constrained, which means that human capital per capita can grow indefinitely, which is obviously inconsistent with labor economics. In addition, the external effect of human capital in Lucas's model is uncertain and difficult to measure. The offset

of diminishing physical capital marginal output is feasible in theory, but not in practice. When López-Bazo and Moreno analyzed the impact of human capital on physical capital in Spain, they found that higher education level meant higher skill level, which would improve marginal output of physical capital so that decreasing trend would disappear [21]. Breton stated that human capital indirectly promoted economic growth by increasing rate of physical capital. The effect of human capital on physical capital output was more obvious in developing countries than that in developed countries. Therefore, in order to reduce the decline of physical capital marginal output, it was proposed that developing countries should increase financial expenditure on education, especially in the field of secondary and higher education [22].

In domestic research, Zheng and Yang established an economic growth model including both physical capital and human capital. Their mathematical derivation showed that human capital accumulation could overcome the diminishing effect of physical capital marginal output, which ensured a long-term economic growth [23]. Nevertheless, the results are based on the assumption that marginal output of physical capital remains unchanged, which means there is inevitably a problem of circular demonstration. Therefore, the mechanism of human capital on marginal output of physical capital needs to be further explored.

The above literature shows that human capital accumulation plays a significant role in promoting economic growth. Human capital accumulation is affected both by itself and physical capital. In the past literature, however, human capital accumulation can only promote short-term rather than long-term economic growth. Besides, scholars mainly focus on proving the relationship between human capital and economic growth, as well as its relationship with physical capital, while the relationship between human capital and marginal output of physical capital is less studied. Accordingly, starting from marginal output of physical capital, this paper explores a kind of regulated human capital accumulation and proves theoretically that the regulated human capital accumulation is the driving source of long-term economic growth.

### 3. Benchmark Theoretical Model

Based on the neoclassical growth theory, this paper constructs a benchmark model of regulated human capital accumulation that is divided into marginal regulation and average regulation. We analyze the long-term trend of economy after analyzing the motion of human capital and physical capital per capita. The total production function is set as  $Y = F(K, H, L)$ , where  $K$  is physical capital,  $H$  is human capital and  $L$  is labor force. The production function per capita is  $y = f(k, h) = F(k, h, 1)$ . The assumptions of the production function are as follows:

- (1) Constant return to scale:  $\forall \lambda > 0$ ,  $f(\lambda k, \lambda h) = \lambda f(k, h)$ .
- (2) Positive and diminishing marginal output:  $\forall k, h > 0$ ,  $f'_k(k, h) > 0$ ,  $f''_{kk}(k, h) < 0$ ,  $f'_h(k, h) > 0$ ,

$f''_{hh}(k, h) < 0$  (increasing and concave).

- (3) Inada conditions:  $\forall k, h \geq 0$ ,  $f'_k(0, h) = +\infty$ ,  $f'_k(\infty, h) = 0$ ,  $f'_h(k, 0) = +\infty$ ,  $f'_h(k, \infty) = 0$ .
- (4) Essentiality:  $\forall k, h \geq 0$ ,  $f(0, h) = f(k, 0) = 0$ .
- (5) Offsetting of marginal effects:  $\forall k, h > 0$ ,  $f''_{kh}(k, h) = f''_{hk}(k, h) > 0$ .

In addition, it is assumed that economy and society meet the following conditions:

- (1) No technological progress and population growth.
- (2) Full employment (working population is equal to total population).
- (3) The physical capital saving rate  $s_K$  is endogenous and constant, and depreciation rate  $\delta_K$  is constant.
- (4) The initial physical capital per capita is  $k_0$ , the initial human capital per capita is  $h_0$ , and the initial output per capita is  $y_0 = f(k_0, h_0)$ , where  $k_0 > 0$  and  $h_0 > 0$ .
- (5) The current capital stock determines the current output, i.e.  $y_t = f(k_t, h_t)$  ( $t = 0, 1, 2, \dots$ ). The current output determines the current savings, which is equal to the next investment, i.e.  $k_{t+1} - k_t = s_K y_t - \delta_K k_t$  ( $t = 0, 1, 2, \dots$ ).

#### 3.1. Marginal Regulation of Human Capital Accumulation

##### 3.1.1. Marginal Regulation

The requirement of marginal regulation is that the accumulation of human capital depends on the changes of physical capital stock, and the marginal output of physical capital should always be maintained at an appropriate level. The specific marginal regulation is making the marginal output of physical capital per capita be equal to a constant – that is,  $f'_k(k, h) = m$  and  $m$  is called the marginal index of human capital. It is assumed that the marginal index of human capital is not less than the ratio of physical capital depreciation rate and saving rate, i.e.  $m \geq \delta_K / s_K$ . The marginal regulation makes  $h$  become a function of  $k$ , which is recorded as  $h = \hat{h}(k)$  so that  $f'_k(k, \hat{h}(k)) = m$  holds for all  $k > 0$ . For  $k > 0$ , we can determine the corresponding human capital per capita  $\hat{h}(k)$  from  $f'_k(k, \hat{h}(k)) = m$ . Actually,  $\hat{h}(k)$  is another form of marginal regulation and has the following properties:

- (1) Following-up:

$$\forall k > 0, \hat{h}'(k) = d\hat{h}(k)/dk = -f''_{kh}(k, \hat{h}(k)) / f''_{kk}(k, \hat{h}(k)) > 0.$$

- (2)  $\hat{h}(0) = 0$ :

due to following-up property, there must be a limit  $\lim_{k \rightarrow 0} \hat{h}(k) = h^0 \geq 0$ . If  $h^0 > 0$ , then  $f'_k(0, h^0) = +\infty$  and  $m = \lim_{k \rightarrow 0} f'_k(k, \hat{h}(k)) = f'_k(0, \lim_{k \rightarrow 0} \hat{h}(k)) = f'_k(0, h^0)$ , but this will be a contradiction. Therefore, it can be deduced that  $\lim_{k \rightarrow 0} \hat{h}(k) = h^0 = 0$  and  $\hat{h}(0) = 0$  is proved.

There will be a new production function, i.e.  $g(k) \triangleq f(k, \hat{h}(k))$  and  $f(k_t, \hat{h}(k_t)) = g(k_t)$ , and output per capita depends on physical capital per capita. Net investment is the difference between capital savings and depreciation, thus the net investment function is  $I(k) = s_K g(k) - \delta_K k = s_K f(k, \hat{h}(k)) - \delta_K k$ .

Theorem A: for any  $k > 0$ , there is  $I'(k) = dI(k)/dk > s_K m - \delta_K \geq 0$ .

Proof: first, the derivative of new production function is  $g'(k) = f'_k(k, \hat{h}(k)) + f'_h(k, \hat{h}(k))\hat{h}'(k) = m + f'_h(k, \hat{h}(k))\hat{h}'(k)$  based on marginal regulation equation. The derivative of net investment function is  $I'(k) = dI(k)/dk = s_K g'(k) - \delta_K = s_K m + s_K f'_h(k, \hat{h}(k))\hat{h}'(k) - \delta_K$ . Combined with assumptions of production function,  $f'_h(k, \hat{h}(k))\hat{h}'(k)$  is positive, thus  $I'(k) > s_K m - \delta_K$ . Finally, the marginal index of human capital is not less than the ratio of physical capital depreciation rate and saving rate, i.e.  $s_K m - \delta_K \geq 0$ . Therefore,  $I'(k) = dI(k)/dk > s_K m - \delta_K \geq 0$  and Theorem A is proved.

### 3.1.2. Motion of Physical Capital

Economy starts from  $(k_0, h_0)$ , where  $k_0 > 0$ ,  $h_0 = \hat{h}(k_0)$  and  $y_0 = f(k_0, \hat{h}(k_0))$ . The motion equation of physical capital per capita is  $k_{t+1} - k_t = s_K f(k_t, \hat{h}(k_t)) - \delta_K k_t$  ( $t = 0, 1, 2, \dots$ ). There are two motion laws of physical capital per capita. The first motion law is that physical capital per capita is strictly increasing, i.e.  $k_{t+1} > k_t > 0$  ( $t = 0, 1, 2, \dots$ ). The physical capital per capita is freely accumulated based on Perpetual Inventory Method, i.e.  $k_{t+1} = k_t + I(k_t)$  ( $t = 0, 1, 2, \dots$ ). According to Theorem A,  $I'(k) > 0$  holds for all  $k > 0$ , so  $I(k)$  is a strictly increasing function of  $k$ . Combined with  $I(0) = 0$ , we have  $I(k) > 0$  for all  $k > 0$ . First, based on  $I(k_0) > 0$ ,  $k_1 = k_0 + I(k_0) > k_0 > 0$  can be obtained. Then based on  $I(k_1) > 0$ ,  $k_2 = k_1 + I(k_1) > k_1 > 0$  can be obtained. By analogy, finally it can be obtained that  $k_{t+1} > k_t > 0$  ( $t = 0, 1, 2, \dots$ ) and the first motion law is proved.

$$\mu_t = \frac{y_{t+1} - y_t}{y_t} = \frac{g(k_{t+1}) - g(k_t)}{g(k_t)} = \frac{g'(k'_t)(k_{t+1} - k_t)}{g(k_t)} > \frac{m(k_{t+1} - k_t)}{g(k_t)} = m \left[ s_K - \delta_K \frac{k_t}{g(k_t)} \right] > m \left[ s_K - \delta_K \frac{k_t}{mk_t} \right] \\ = s_K m - \delta_K \geq 0 \quad (t = 0, 1, 2, \dots)$$

Conclusion 1: marginally regulated human capital can ensure a sustainable economic growth. When marginal index of human capital satisfies the condition  $m \geq \delta_K/s_K$ , output per capita continues to grow at a positive rate, i.e.  $\mu_t = (y_{t+1} - y_t)/y_t > 0$ .

### 3.2. Average Regulation of Human Capital Accumulation

In last part, starting from marginal output of physical capital, we have made marginal regulation of human capital accumulation so that the marginal output of physical capital is equal to a constant. Subsequently, we regulate human capital accumulation by keeping the average output of physical capital unchanged. The average regulation is making the average output of physical capital per capita equal to a constant – that is,  $f(k, h)/k = a$  or  $f(k, h) = ak$ . The constant  $a$  is called the average index of human capital and satisfies the condition  $a > \delta_K/s_K$ . The average regulation makes  $h$  become a function of  $k$ , which is recorded as  $h = \bar{h}(k)$  so that  $f(k, \bar{h}(k)) = ak$  holds for all  $k > 0$ . It is obvious that  $\bar{h}(0)$  is equal to zero as  $k$  is zero.

Economy starts from  $(k_0, h_0)$  where  $k_0 > 0$ ,  $h_0 = \bar{h}(k_0)$  and  $y_0 = f(k_0, \bar{h}(k_0)) = ak_0 > 0$ . The motion equation of physical capital per capita is  $k_{t+1} - k_t = s_K f(k_t, \bar{h}(k_t)) -$

The second motion law is that increment of physical capital per capita is increasing, i.e.  $k_{t+2} - k_{t+1} > k_{t+1} - k_t$  ( $t = 0, 1, 2, \dots$ ). First, due to free accumulation of physical capital per capita, there are  $k_{t+2} = k_{t+1} + I(k_{t+1})$  and  $k_{t+1} = k_t + I(k_t)$ . If above two equations merged, we can get  $k_{t+2} - k_{t+1} = k_{t+1} - k_t + I(k_{t+1}) - I(k_t)$ . According to the mean value theorem for derivatives,  $(k_{t+1}) - I(k_t) = I'(k'_t)(k_{t+1} - k_t)$  holds, where  $k_t < k'_t < k_{t+1}$ . Combined with  $I'(k'_t) > 0$ , it can be deduced that  $k_{t+2} - k_{t+1} = k_{t+1} - k_t + I'(k'_t)(k_{t+1} - k_t) = [1 + I'(k'_t)](k_{t+1} - k_t) > k_{t+1} - k_t$  ( $t = 0, 1, 2, \dots$ ). The second motion law is proved.

### 3.1.3. Long-term Trend of Economy

Theorem B: due to  $g(0) = 0$ ,  $g(k) > mk$  always holds for all  $k > 0$ .

Proof: since  $f(0, h) = 0$  holds for all  $h \geq 0$ , it can be derived that  $g(0) = 0$ . Based on the proof of Theorem A,  $g'(k) > m$  and  $d(g(k) - mk)/dk > 0$  hold for all  $k > 0$ . Therefore,  $g(k) > mk$  and Theorem B is proved.

In the next step, we derive the growth rate of output per capita. Output per capita is  $y_t = f(k_t, \bar{h}(k_t)) = g(k_t)$  ( $t = 0, 1, 2, \dots$ ), whose growth rate is  $\mu_t = (y_{t+1} - y_t)/y_t$ . According to the mean value theorem for derivatives, there are  $g(k_{t+1}) - g(k_t) = g'(k'_t)(k_{t+1} - k_t)$  and  $k_t < k'_t < k_{t+1}$ . Combined with Theorem B,  $g'(k'_t) > m$  and  $g(k_t) > mk_t$ . Besides, accumulation of physical capital per capita is  $k_{t+1} - k_t = s_K g(k_t) - \delta_K k_t$ . The growth rate of output per capita is derived as follows:

$\delta_K k_t = (as_K - \delta_K)k_t$  ( $t = 0, 1, 2, \dots$ ). The motion equation of output per capita is  $y_{t+1} - y_t = f(k_{t+1}, \bar{h}(k_{t+1})) - f(k_t, \bar{h}(k_t)) = a(k_{t+1} - k_t)$  ( $t = 0, 1, 2, \dots$ ). The growth rate of output per capita is  $\mu'_t = (y_{t+1} - y_t)/y_t = a(k_{t+1} - k_t)/ak_t = (k_{t+1} - k_t)/k_t = as_K - \delta_K > 0$  ( $t = 0, 1, 2, \dots$ ). If  $A = a$ , then  $Y = F(K, \bar{h}(K/L)L, L) = AK$ . It can be seen that the economic growth under the average regulation of human capital accumulation is exactly the same as that of AK model.

Conclusion 2: the economy is kept stably growing at a positive rate by averagely-regulated human capital accumulation. This can provide theoretical support for the AK model.

## 4. Differences Between Marginal and Average Regulation

The differences between marginal regulation and average regulation can be revealed through a specific example. We suppose a Cobb-Douglas production function  $Y = F(K, H, L) = \sqrt[3]{KHL}$  so that output per capita is  $y = f(k, h) = F(k, h, 1) = \sqrt[3]{kh}$ . The marginal regulation is



$f'_k(k, h) = 1/3k^{-\frac{2}{3}}h^{\frac{1}{3}} = m$ . Subsequently, human capital per capita is  $h = \hat{h}(k) = (3m)^3 k^2$  and output per capita is  $y = f(k, \hat{h}(k)) = \sqrt[3]{k\hat{h}(k)} = \sqrt[3]{k(3m)^3 k^2} = 3mk$ . The corresponding AK model is  $Y = 3mK$ . The equation of physical capital accumulation is  $\Delta K = s_K Y - \delta_K K = (3s_K m - \delta_K)K$ . Therefore, the growth rate of output per capita is  $\mu = \frac{\Delta Y}{Y} = \frac{\Delta K}{K} = 3s_K m - \delta_K > 0$ . On the other hand, the average regulation is  $f(k, h) = k^{1/3}h^{1/3} = ak$ . Subsequently, human capital per capita is  $h = \bar{h}(k) = a^3 k^2$  and output per capita is  $y = f(k, \bar{h}(k)) = \sqrt[3]{k\bar{h}(k)} = \sqrt[3]{ka^3 k^2} = ak$ . The corresponding AK model is  $Y = aK$ , and the growth rate of output per capita is  $\bar{\mu} = \frac{\Delta Y}{Y} = \frac{\Delta K}{K} = s_K a - \delta_K > 0$ .

It can be seen that both marginal regulation and average regulation of human capital have the corresponding AK model, and the growth rates of output per capita under two regulations are positive. The economic growth rate of marginal regulation is  $\mu = 3s_K m - \delta_K > 0$  and that of average regulation is  $\bar{\mu} = s_K a - \delta_K > 0$ . The marginal index of human capital is equal to the marginal output of physical capital. The average index of human capital is referred to the marginal and average output of physical capital. Although they have different meanings, they are equal in value, i.e.  $m = a$ . Therefore,  $3s_K m - \delta_K > s_K a - \delta_K > 0$  and  $\mu > \bar{\mu}$ , which indicates that the economic growth rate under marginal regulation is higher than that under average regulation.

The advantages of marginal regulation are as follows. First of all, marginal regulation can achieve higher economic growth rate. The above comparison shows that the marginal regulation of human capital accumulation enables the economy to grow faster. Secondly, marginal regulation can achieve a higher level of physical capital marginal output. If human capital accumulation adopts Perpetual Inventory Method, the marginal output of physical capital will gradually decline and finally fall into a low-level trap. Under marginal regulation, however, the marginal output of physical capital is no longer decreasing but increasing and finally remains at a high level. Thirdly, the rule of marginal regulation is simple. In previous papers, human capital was assumed to have externality that was difficult to measure or human capital accumulation was assumed to be completely endogenous that was complicated to study. In our human capital regulated model, human capital accumulation is semi-endogenous depending on the marginal output of physical capital. Last but not least, the biggest advantage of human capital marginal regulation is that one can subjectively set an expected value for marginal output of physical capital, based on which human capital is accumulated. This can provide a broader platform for government to implement policies and is more suitable for China.

## 5. Conclusion

Human capital is the embodiment of a country's comprehensive strength and core competitiveness. It has important strategic significance and great potential in

sustainable economic development. The key to transforming human capital potential into real power, turning demographic dividend into human capital dividend, strengthening human capital investment with directions, realizing the optimal combination of human capital and physical capital, making human capital promote growth with maximum utility and become the source of sustainable economic growth lies in a correct setting for mode of human capital accumulation. Starting from marginal output of physical capital, this paper introduces human capital into the neoclassical growth model and makes it subject to a predetermined regulation in process of accumulation. The regulation is that human capital accumulation can completely offset the diminishing effect of marginal output of physical capital and keep it at a reasonable level. This method is called the regulated accumulation of human capital, which is divided into marginal regulation and average regulation. Marginal regulation makes the marginal output of physical capital equal to a constant, and average regulation makes the average output of physical capital equal to a constant. After theoretical derivation, we find that even without technological progress and population growth, the economy continues to grow at a positive growth rate under either marginal regulation or average regulation. Therefore, human capital accumulated by regulation becomes the source of long-term economic growth. In addition, from the perspective of promoting effects and regulating rules, marginal regulation has more advantages than average regulation. Regulation indicates that one can subjectively set the final expected level of physical capital marginal output and accumulate human capital according to this level, which provides more policy implications for government in China. This is the practical significance of our paper. In theory, the regulated accumulation of human capital can provide a new driving force for long-term economic growth and enhance the sustainable development. Subsequently, future research can choose an enterprise for case study. In order to obtain the practical application effect of regulated human capital accumulation and provide empirical support for the theoretical model, we can implement regulated accumulation of human capital within an enterprise and then explore the relationship between human capital accumulation and marginal output of physical capital.

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