

Cellular Automata in the Economy: Application to the Behavior Theory of the Consumers

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To cite this article:

Martin Pomares Calero. Cellular Automata in the Economy: Application to the Behavior Theory of the Consumers. *Economics*. Vol. 11, No. 1, 2022, pp. 43-48. doi: 10.11648/j.econ.20221101.16

Received: January 20, 2022; **Accepted:** February 16, 2022; **Published:** February 25, 2022

Abstract: The application of cellular automata is a technique of modelling complex systems with broad uses into the economy, especially into the approach of game theory. In this research the author pretends to use a model of cellular automata related to a probabilistic model to estimate the entropy in an economical system with the objective of analyze satisfaction or non-satisfaction in the behavior of consumers, and understand how cellular automata allow visualize the dynamical of consumption. The method used in this research was based in the idea to describe the dynamics of consumption considering a cellular automata 2D which represents prices versus goods, and the dynamics of it shows how consumption behaves. For that purpose a Schelling's model was used including a model of interchange game and Gibbs distribution to understand the trends of entropy of the economic system. There were analyzed four possibilities in the evolution of the system regarding previously the entropy into the evolution of the system. The results reflect preliminary data to regard improvements in the model as a preliminary model in order to consider it as an application of the Econophysics to studies of theory of consume through the utilization of cellular automata class four.

Keywords: Cellular Automata, Rent, Entropy, Satisfaction, Non-Satisfaction

1. Introduction

The origins of *cellular automata* are coming from Von Neumann (between 1940 and 1970) with the purpose of study automata or machines that can self reproduces (Figure 1), the with J. H. Conway (between 1970 and 1983) in order to study the game of life. After, since 1983 S. Wolfram developed a concept to study that he denominates "The statistical mechanics of cellular automata". And it is from Wolfram that have been studied the mathematical concept of cellular automata with several applications to the science [1].

The current mathematical background to understand a cellular automata comes by definition from a graphical picture, which a cellular automata is composed by states, rules and surroundings or neighborhoods. As a consequence, the states can be plotted by cells in a diagram 2D, which could be binaries states and we can set up the colors black and white for such states. The white color can represent the binary state 1, and the black color can represent the binary state 0, or vice versa.

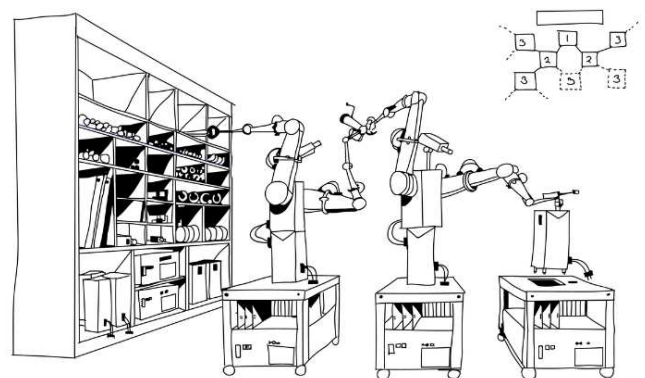


Figure 1. Fundamental sketch of Von Neumann about the conception of cellular automata (Source: Wikimedia).

The neighborhood corresponds to mathematical form which a state dynamical pass from one state to other. And the rules corresponds to the mathematical way in which an state pass dynamically from one state to other nearly or far away depending on how is defined the interaction between states.

In the Figure 2 is represented a cellular automata simple which is regarded only two states. Those states are interacting according to the rule number 30.

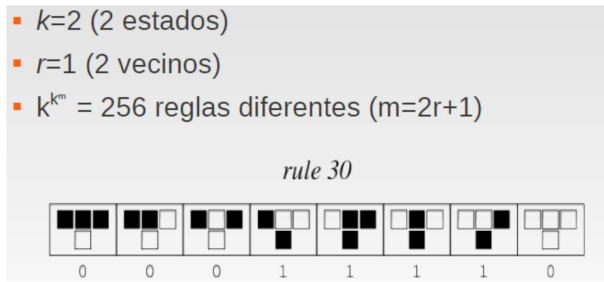


Figure 2. Representation of an elemental cellular automata (Source: E. W. Weisstein, Mathworld).

There are several utilities or applications of the theory of cellular automata, which the most important point of view are the qualitative analysis by Wolfram and the structure of the elementary cellular automata by Li et al [2, 3], quantitative analysis by Weunsche using the concept of entropy of entrance [4], and the study of artificial life by Langton and Hooker who use the exponents of Lyapunov [5, 6]. Other contributions from the thermodynamics in the analysis of irreversibility in systems as it is proposed in thermodynamics by Heiko [7]. In addition, with the theory of cellular automata are studied emergent behavior like self-regulation, hierarchical and non-hierarchical structures. In the economy are used mostly in systems of marketing. In the Figure 3 are represented some examples of cellular automata studied.

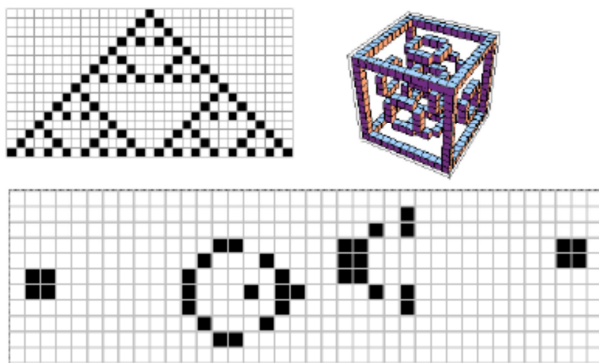


Figure 3. Rule 90 by Wolfram (up left), AC in 3D (up right) and Game of life by John Conway (down). Source:[8].

Moreover, some models of cellular automata are used in the study of complex systems, which according to [9], in 1989, Santa Fe, a group of physicist and economist were studying about the connection in between both disciplines. As a consequence the memory of the conference was titled *The Economy as an Evolving Complex System* which contains papers about the scope of technics that includes chaos theory, neural networks, and cellular automata.

The scope of this research is to make a broad application about the mathematical concepts of cellular automata into the economy which belongs to the Econophysics, and so to implement new models in order to understand the dynamics of consumption based in the

theory of preferences. As a consequence the primordial objectives are: (1) to develop a mathematical model in microeconomics which vinctuate concepts from physics and economy in models of consumption, such as the phase space (continuous and discrete) related to the configurations space for products versus goods, (2) to apply the cellular automata to the statistical mechanics in order to represent a dynamic which will reflect the behavior of consumption in the space of configurations for product versus goods, (3) to describe through three elemental scenarios the dynamic behavior based on the theory of preferences and describe this dynamics in a graphical way using cellular automata.

2. Mathematical Model

In economy even there is not so much emphasis in the modelling of the theory of preferences revealed. As a consequence, the present research considers to study the dynamics of the theory of consumption through a graphic of prices (p_i) versus goods (x_i) which in the vectorial space is represented a state which corresponds to the rent (w_i) (Ver Figure 4). Such rent will be represented in a quantized way by a state of cellular automata in an array of 2D. The dynamics of the rent correspond to the same dynamic which the cellular automata will change of state in a configuration space. The area of each state corresponds to the minimal rent in that a consumer select spend into a good. Regarding the fact that the selection will be optimal, when the dynamic of rent increase also increase the possibility to obtain adequate preference or satisfaction. As a consequence, and in a simultaneity to the model of cellular automata will be simulated a *Gibbs distribution with measurement of entropy*. In our case the entropy will be measurement of nonsatisfaction as a consequence of the dynamic. To study the dynamic of cellular automata that will represent the rent was selected the segregation model of Schelling in which will be modulated two agents or goods.

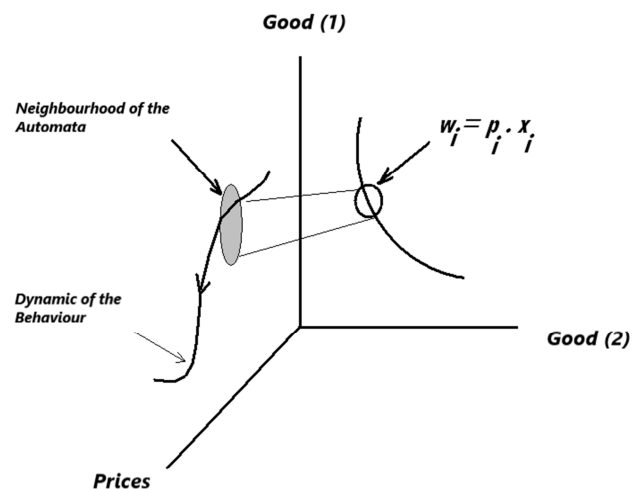


Figure 4. Pictorial idea of the use of cellular automata for this research. The configuration space of Prices versus Goods will be useful to understand the dynamic of consumption.

2.1. Schelling's Segregation Model

This simulation model consist of N agents that develop local interactions, it is to say with his nearly neighbors that let us to consider the area of local interactions as a neighborhood of ratio r . We start from the assumption that the agents are represented by "D cells located in a rectangular reticula, where each one has a neighbor to watch, and each one has neighbor of the same type in his neighborhood. A restriction is that each agent has eight neighbors to watch. In our case each agent represents a good (B_i), and for commodity of the study two different agents will be simulated. We regards a tolerance represented by a positive number $m \in \{1, \dots, 2r + 1\}$, which determine the maximal number of neighbors non fancy that the agent is able to admit. The tolerance can be understood as a threshold of nonsatisfaction that each agent admits in his neighbor. In our model, if the agent poses les neighbors that the parameter THRESH (threshold), then those are no satisfied, and as a consequence interchange his positions in order to restart their observations [10].

The individual utility, which measures in a binary way the level of satisfaction in individual way is generated by its neighborhood. The Set of information of each agent i , its neighborhood of ratio r in the state t denoted by $V^t(i, r)$ is equal to an element of $\{0,1\}^{2r+1}$ focused in i and the utility of the agent i in the state t is represented as bellow [10]:

$$U^t(i) = \begin{cases} 1 & \text{si } |\{j \in V^t(i, r) \text{ tal que } j \neq i\}| \leq m \\ 0 & \text{si } |\{j \in V^t(i, r) \text{ tal que } j \neq i\}| > m \end{cases}$$

The utility function set that each member corresponds only the number of neighbors as much as prefer as are not preferred, that implies that to the agents are substantial more the composition than the configuration. The dynamic is an iterative process where the agents selects the best answer given by the set of local information [10].

2.1.1. Schelling's Code in Matlab According to [11]

As follow bellow it is presented the code designed in Matlab about the Schelling model, which the net is defined by a random variable U , which distribute the values of one and zero with the same probability. It is set up a function to measure satisfaction: *measures_satisfaction(one, x, y)*. Such function count the number of neighbors of the same color (type) of an agnet. When both agents has the same amount of neighbors according to his preference under the critical umbral, randomly is established the permutation of places in the net. This model corresponds to a cellular automata class 4.

2.1.2. Schelling's Code

```
function nn = schelling(N0, thresh)
global U N;
step = 150;
rand('state',0);

N = N0;
% title of main window
titulo = ['Schelling Game : N=' , num2str(N*N)];
```

```
U = round( rand(N,N) ); % sets random field

% sets main figure
film = imagesc(U, [ 0 1 ] );
axis off; axis square; axis on;
title ( titulo );
xlabel ('x','fontsize',16);
ylabel ('y','fontsize',16);
axis ( [ 0 N+1 0 N+1 ] );
i=1;
while i <=90000,
i
% S e l e c t s one agent
one = round( rand( 1, 2 )*(N-1))+1;
x1 = one ( 1 ); y1 = one ( 2 );

% s e l e c t s second agent and measures s a t i s f a c t i o
n
two = round( rand( 1, 2 )*(N-1))+1;
x2 = two ( 1 ); y2 = two ( 2 );

% computes s a t i s f a c t i o n
if U( x1, y1 ) ~= U( x2, y2 )
sat1 = measures_satisfaction ( one, x1, y1 );
sat2 = measures_satisfaction ( two, x2, y2 );

if sat1 < thresh & sat2 < thresh
temp = U( x1, y1 );
U( x1, y1)=U( x2, y2 );
U( x2, y2)=temp;
end;
end;
if (mod( i,step )==0)
set (film,'cdata',U);
drawnow
end;
i=i +1;
end

function sat = measures_satisfaction ( one, x1, y1 )

global U N;
x1 = mod( x1-2,N)+1; % inde x l e f t
xr = mod( x1,N)+1; % inde x r i g h t
yb = mod( y1-2,N)+1; % inde x b o t t o m
yt = mod( y1,N)+1; % inde x t o p

% measures the number of 1 s in neighborhood
neig = U( x1, y1 ) + U( xr, y1 ) + U( x1, yb ) + U( x1,
yt );
neig = neig + U( x1, yb ) + U( x1, yt ) + U( xr, yb ) +
U( x1, yt );

% d e f i n e s s a t i s f a c t i o n
if U( x1, y1 ) == 0
sat = 8-neig;
```

```

else
sat = neig ;
end;

```

2.2. Application to the Statistical Mechanics to the Case of the Agents Modelled with Cellular Automata

According to Dragulesku and Yakovenko [12, 13], it is possible to develop a statistical analysis to a system with multiple agents composed by N players, each one of them with an amount of money m_i .

The characteristic of the model is that the N players are committed in a lottery where draw a pair i, j of them and then randomly is decided who of them win an amount m , and who of them loss the same amount. If the agent that loss cannot afford the payment dm , the play is canceled. Now it is necessary to estimate which will be the probability that we can find agents with the trend of amount of money m that results stationary against this kind of transactions. Such probability is calculated with the Gibbs distribution used in the statistical mechanics described by $P(m) = Ae^{-\beta m}$, where $\beta = 1/T = N/M$ represents the temperature that for an economical system is the amount of money average by agent.

However, it is needed to know the entropy of the system, in other words, the amount of disorder that the statistical process defined by $S = \log(M)$.

Code of the Game with Random Interchange and Gibbs Distribution

```

function [m g hx s] = gibbs(m_ini,N,tim_total)

m_step = 1 ; % money exchanged per
interaction
m = m_ini*ones(1,N) ; % initial money for all agents

out_steps = N;
num_bins = 50 ;
hx = 1:num_bins ;
% calc el valor asintotico
g = exp(-hx/m_ini)/sum(exp(-hx/m_ini));
sg = -sum( g.*log(g));

figure(1)
s = [] ;
tim = 0;
while tim < N*tim_total,

coin = randint(1,2,[1 N]);
up = coin(1);
down = coin(2);
dm = m_step*rand(1,1);

if m(down)< dm,
continue
end
m(up) = m(up) + dm;

```

```

m(down) = m(down)-dm;

```

```

if mod(tim,out_steps) == 0,
[tim/N,sum(m)]

```

```

% compute entropy at this step
h = hist(m,hx);
p = h/sum(h);
xx = find(p>0);
s = [s;tim/N,-sum(p(xx).*log(p(xx))),sg];

```

```

if mod(tim,100*out_steps) == 0,
drawplot(p,s,h,hx,m_ini);
end
end
tim = tim + 1 ;
end

```

```

% normalized prob distribution
h = hist(m,hx);
p = h/sum(h);

```

```

g = drawplot(p,s,h,hx,m_ini)

```

```

end

```

```

function [g,sg] = drawplot(p,s,h,hx,m_ini)

```

```

% theoretical function
g = exp(-hx/m_ini)/sum(exp(-hx/m_ini));

```

```

subplot(2,1,1)
plot(hx,p,'r+',hx,g,'b')
%semilogy (xx,h/sum(h),'r+',xx,exp(-xx/m_agent)/sum(
exp(-xx/m_agent)), 'b')
xlabel('m'); ylabel('p(m)');
legend('p(x)', 'gibbs');
subplot(2,1,2)
plot(s(:,1),s(:,2),'b',s(:,1),log(length(p)), 'r',s(:,1),s(:,3), 'g')
xlabel('t'); ylabel('s');
drawnow;

```

```

end

```

3. Methodology

The proposal of the author to study the dynamic of the consumption theory is through a graphic of prices (p_i) versus goods (x_i) which in the cartesian vectorial space it will be represented by a cellular automata 2D where each state or cell represents the rent (w_i) which area describe the minimal value of the rent connected to the good. It is used the Segregation Schelling's model of cellular automata in order that fulfil form the mathematical point of view with the adequate characteristics to study the dynamics of states. In our case, two agents or goods. In the measurement of nonsatisfaction, the model needs a statistical estimation of

nonsatisfaction caused by the dynamics itself when it is necessary to reach the option more adequate possible, then not any system is optimum or of maximum efficiency. This fulfil with the dynamical characteristics of system according to the second law of the thermodynamics, which always there is a certain degree of entropy. For this case, it was used the model of *random interchange game considering a Gibbs distribution*, as a part of the selection process. Therefore, it is regarded two players ($N = 2$) which start with the same dotation $m_{ini} = 2$, then for each step of time is selected randomly a pair of agents, and then the amount interchanged that goes in between 0 and m_{step} . In our case to have a sequence with the modelling of cellular automata 150 steps are regarded. In order to coordinate the states of the automata with the Gibbs probability, the Schelling model was run for several steps: 57, 70, 91, 135, which corresponds to times in

the diagram of entropy (Figure 5), and with the parameters of 7 states, and an umbral value “threshold” equal to 10. For this reason the automata will be arrayed in 49 states bicolored.

4. Results and Discussions

In the Figure 5 and Figure 6 are represented the results obtained from the simulation. The Figure 5 presents the evolution of entropy considering the selection of states of satisfaction and nonsatisfaction for a good or other of the agents evaluated. Besides, according to the methodology are presented time that corresponds to the steps of simulation of automata over the Schelling model which results are presented in the Figure 6. Such picture, presents the evolution of the states of decision of preferences in between two agents considering pre-established conditions for the model.

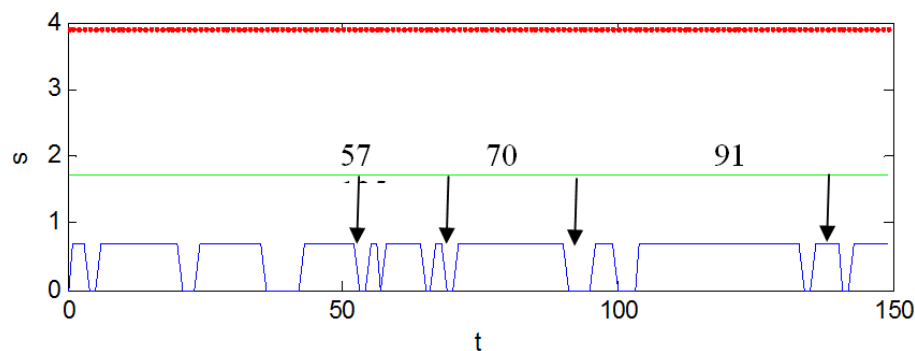


Figure 5. Diagram of Entropy (S) versus time of the probabilistic estimation.

The valid states or of preferred decision are presented in red color, regarding the initial state in the upper left corner for all the items of the Figure 6. The blue color represents an empty state it is to say without value. All the red reticula represents the dynamic followed by the automata. The assumption is that for all the states presented in the Figure 6 represent more stable states in comparison with the states

presented in the Figure 5 but in which there is more satisfaction. In the graphic of the Figure 5 it is possible to observe the evolution of the entropy, it is to say in between satisfaction (entropy equals to zero) and nonsatisfaction (maximal entropy equals to 0.63 units). Furthermore, the pathways of the red cells indicates the places of the dynamics of the rent where there are optimal decisions of the consumer.

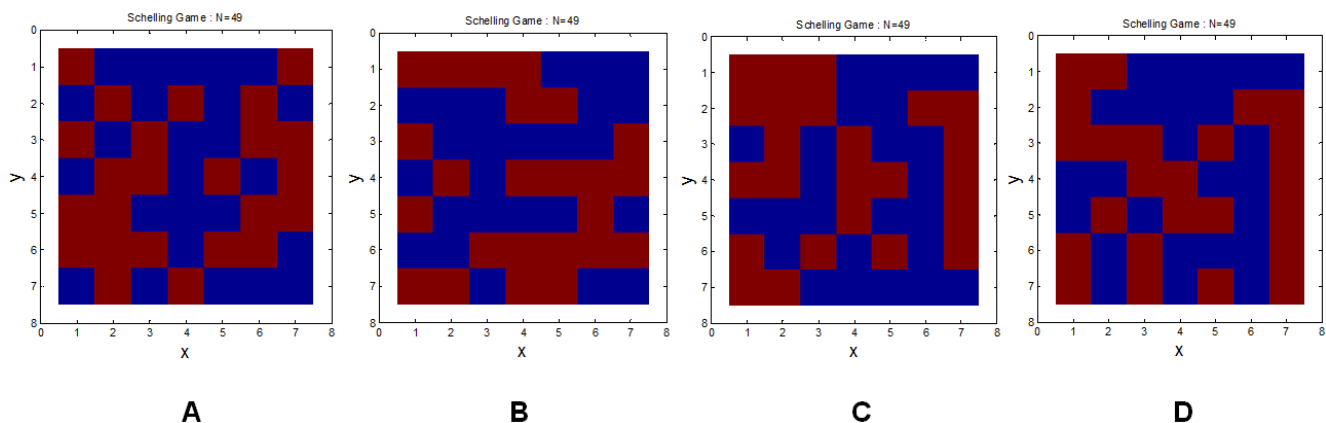


Figure 6. Evolving of the states of the cellular automata for several steps: $A = 57$, $B = 70$, $C = 91$, $D = 135$.

There were not found prior researches on cellular automata applied to economy in the analysis of consumption as it is pretended in this study. Some of the applications of cellular automata to the economy are more related to: Economic

structure and dynamic such as is shown Venkatachalam et al [14] where is studied a morphogenetic framework; landscape use and urban growth are discussed by Haase et al [15]; and Housing Privatization Decision is studied by Plotnikova et al

[16]. Furthermore, there is a research where cellular automata is applied for population growth is presented by Eide [17].

Otherwise, it is non discussible that the model need several improvements: (i) simultaneous calculus of entropy when the Schelling model is running, (ii) to reduce the size of the reticula 2D for the phases diagram independent of the selection of the enter data, (iii) to establish as a rule of the dynamic of the cellular automata the pathway defined of demand that generally is defined for a space “good” versus “good”. This last one let us to that the results of the model to be more independent of data generated to model the dynamic of population. The only detriment of these model mentioned before is that are not adequate data to study the dynamic of decision according to the Schelling model.

5. Conclusions

In the present research work was studied the dynamic of preferences reveled in a new space of configurations of prices versus goods with the purpose of to obtain a graphical idea of the dynamics of how evolves the state of satisfaction and nonsatisfaction. For that purpose, it is regarded the measurement of entropy of the system over predefined considerations showing a maximal entropy stable. Since the point of view of the micro-economy it is needed a detailed study in this new approach especially in the interconnection of the physics and the economy. In that way, it is possible to argue that it is one of the few works in Econophysics where is studied the preference reveled from the point of view of the cellular automata, that makes the present study as a first approximation. Of course, other models can be used to study satisfaction regarding the initial ideas of this research document. However, since the point of view of the mathematics there will be so much to implement in studies of dynamics related to time series that it is needed to be regarded for the future.

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