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Mathematical Model of Root Crop Digging with Longitudinal Vibrations

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Abstract: The problem how to reduce damage to tubers when they are dug up is urgent. For the new design of a vibrating digging working body for root crops the mathematical model of longitudinal vibrations of a root crop in the soil is developed as an elastic body in an elastically damped medium. The Ostrogradsky-Hamilton variational principle is applied for the analytical description of the process. The Ritz method was applied to find the frequencies of natural vibrations, the amplitudes of forced vibrations of a root crop as a solid elastic body when it is captured by a vibrating digging body. The frequency equation for the discussed vibrational process was obtained. The values of the first proper frequency of longitudinal vibrations of the considered elastic body of the root crop with specific geometric physical parameters are found. Graphs of the dependence of the first natural frequency upon the elastic deformation coefficient, the damping coefficient of the soil as an elastic damping medium are obtained. When the soil damping coefficient changes within 0 to 10 $N{\cdot}s^2{\cdot}m^{-3}$, the first proper frequency changes within 500 to 750 s^{-1} (80 to 119 Hz) at soil elastic deformation coefficient $2{\cdot}10^5$ $N{\cdot}m^{-3}$. Dependence of the elastic body forced vibration amplitude upon the change in the amplitude of the disturbing force have been obtained. When the amplitude of the disturbing force changes within 100 to 600 N, the amplitude of forced vibrations of the root crop body changes within 0.30 to 0.68 mm.

Keywords: Root Crop, Longitudinal Vibrations, Amplitude, Damper

1. Introduction

Sugar beet growing is an important agricultural sector in many European countries [1-3]. Of all production operations, sugar beet harvesting is the most laborious costly technological process [4]. In order to ensure the storage of sugar-containing root crops for a certain period to ensure further high quality processing, it is necessary to separate the soil, tops other impurities from the root crops during harvesting [5-7].

When digging out root crops from the soil, there is a problem of reducing the degree of breaking of the tail parts of their bodies during extraction from the soil, ruptures of the outer surface, chips other damage [8-10]. The existing digging bodies for root crops, operating in a wide variety of conditions (different hardness moisture of the soil surrounding the root crops, non-straightness of the crop rows, etc.), are not always able to fulfil the specified requirements, which leads to significant losses of the grown crop [11].

The more advanced widely used vibratory digging tools for root crops do not exclude the loss damage of root crops either, especially when they are dug from the hard soil [4].

However, the main disadvantages of the vibratory digging working bodies are low reliability (this applies, first of all, to the vibration drive), which occurs when working especially on heavy hard soils, increased metal consumption energy consumption of this technological process, in general [12]. The presence of digging working bodies of the vibratory type in the of design of a multi-row root harvester which have a significant mass perform vibratory movements with a frequency of 20 Hz, generally contributes to the creation of large loads and a decrease in the reliability of work [4, 12].

Therefore there is a need for further development of new provisions of the theory of vibrational digging of root crops, which can be efficiently used to substantiate the design parameters of more advanced digging working bodies of the root harvesting machines [14-16]. In particular, when simulating vibrational digging, a root crop in the soil (actually fixed in it) can, with a certain degree of accuracy, be presented as an elastic rod with one end fixed in an elastic damping medium, which is subject to vibrations, transmitted from a vibrating digging working body.

Fundamental analytical studies of transverse vibrations of the root crop body were carried out published in [17], where the root crop was modelled as a cone-shaped body with one point fixed in the lower part, which has elastic properties. Besides, the transverse vibrations of the body of the root crop are described using a partial differential equation of the fourth order. The solution of this equation made it possible to determine the proper frequencies of the free transverse vibrations of the body of the root crop. The process of extracting a root crop from the soil in this study is directly investigated according to the additional equations of kinetostatics, which made it possible, with a certain degree of accuracy, to find the conditions for its complete extraction from the soil.

However, from a structural technological point of view, high-quality efficient digging of root crops from the soil by creating lateral vibrations for the latter turned out to be practically unfeasible, which necessitated the use of devices that ensure the transfer of the vibration forces to the bodies of the root crops in the longitudinal-vertical plane.

The theory of vibrational digging of root crops from the soil with the application of disturbing forces in the longitudinal-vertical plane in a fundamentally new formulation was published in [18, 19]. The case of transverse free forced vibrations of a root crop as an elastic body with the coinciding directions of the disturbing forces with the direction of the forward movement of the vibrating digging working body was published in [11, 20], it is of interest from both theoretical practical points of view. Therefore, there is a need to consider the general problem of longitudinal vibrations of a root crop as a solid elastic cone-shaped body, fixed in the soil, taking into account its elastic damping properties. This problem can be solved using the general theory of vibrations of straight rods of variable cross-section, presented in [17].

Purpose of the study – analytical determination of the optimal parameters of the process of the root crop vibrations during their vibrational digging from the soil based on the theory of longitudinal vibrations of an elastic cone-shaped body, located in an elastic damping medium.

2. Mathematical Description

The research was carried out using the methods of the theory of agricultural machines, analytical mechanics, higher mathematics, in particular the theory of oscillations, variational methods, methods for studying holonomic systems, as well as compiling programs carrying out numerical calculations of systems of differential equations, using a PC [21–28].

We have created a design of a new vibration-type digging working body for root crops, which is protected by the Patent of Ukraine [29]. The structural technological diagram of this digging working body and its general view is shown in Figure 1.

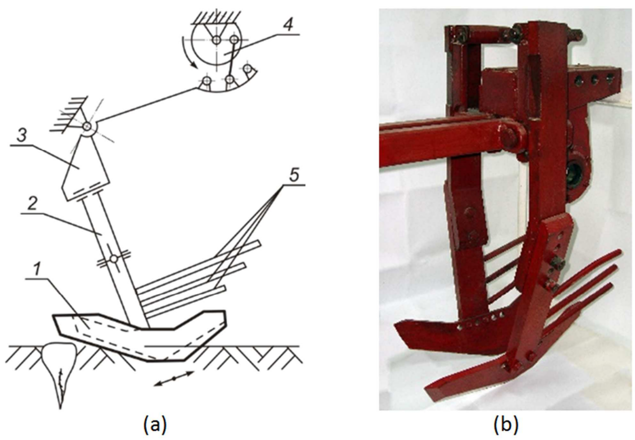


Figure 1. *A vibrating-type digging tool for root crops: (a) – constructive technological scheme; (b) – general view 1 – a digging blade; 2 – an inclined post; 3 – a bracket for fastening the inclined posts; 4 – the drive of the vibratory movement; 5 – fingers for guiding the dug up roots.*

The vibration-type digging tool for the root crops creates longitudinal vibrations of the root crops, it consists of two digging blades installed at an angle to each other, which move at a pre-set depth of travel in the soil cover the row of root crops from both sides. Each digging blade is installed at the end of an individual inclined post, which by means of brackets for fixing the individual inclined posts are connected to the drive of the said digging blades in a vibratory movement. The drive has a screw mechanism that can set various frequency amplitude of vibratory movements of the digging blades in the following ranges: frequency – 8 to 30 Hz, amplitude – 8 to 24 mm. To exclude the loss of root crops at the final stage of their extraction from the soil, fingers are used, which provide the direction of the movement of the dug root crops, which form two additional lattice planes are installed at the rear ends of each digging blade.

The technological process of digging out root crops from the soil with this vibrating-type digging working body is carried out in such a way that the digging blades, covering the root crop from both sides during the forward movement along the row of crops, contact them, transfer the longitudinal vibrations to the root crops, destroy their bonds with the soil, loosen, thanks to the tapering working bed during the forward movement, is lifted up. At the same time, the bulk of the soil is not captured together with the root crops but remains in the extraction zone.

In order to construct a mathematical model of vibrations of a root crop as an elastic body during its vibrational digging, taking into account the elastic damping properties of the soil in which it is actually fixed, it is first necessary to construct an equivalent scheme. Figure 2 shows this equivalent scheme in which the root crop is presented in the form of a conical body with an apex angle 2γ. The root crop is actually fixed in the soil, its upper part is slightly above the soil surface.

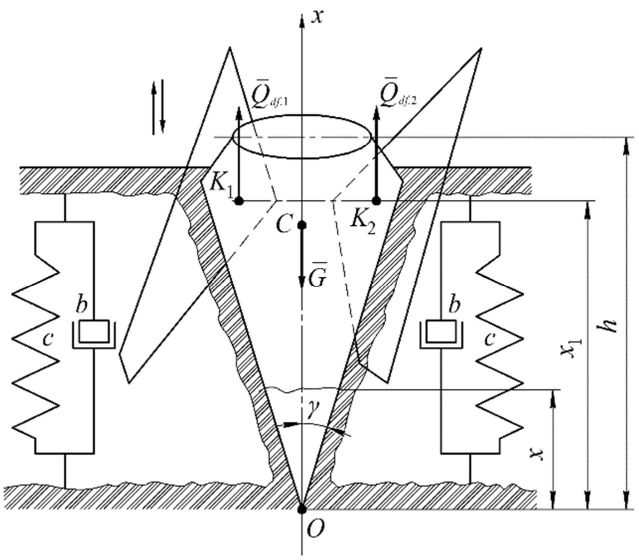


Figure 2. *An equivalent scheme of longitudinal vibrations of a root crop as an elastic body in an elastic damping medium.*

Thus there is every reason to model the root crop in the soil as a rod of variable cross-section with a fixed lower end (point O is the lower anchor point). In the soil at a preset depth of movement of the vibrating-type digging working body is conditionally presented by two planes, located at an angle (blades 1 in Figure 1), which at the same time cover the root crop from its two sides, as shown in the equivalent scheme, contacting it. The contact of the planes of the digging working body of the vibration type with the body of the root crop occurs at a pre-set depth in the soil is carried out at two points K_1 K_2. The length of the root crop is indicated by h.

Let us show the forces applied to the cone-shaped body of the root crop. At the previously indicated points of contact K_1 K_2, the disturbing forces $\bar{Q}_{df.1}$ $\bar{Q}_{df.2}$ are transmitted to the root crop from the digging working body of the vibration type, the vectors of which are directed upwards are parallel to each other. These are the forces that cause longitudinal vertical vibrations of the root crop. These forces are applied at distance x_1 from the conditional point O of anchoring the root crop in the soil. At the centre of mass of the root crop, which is indicated by point C, the force of its weight $\bar{G}$ is applied. The soil surrounding the cone-shaped body of the root crop is presented in the form of two elastic damping models located in the longitudinal-vertical plane having the same elastic coefficients and damping coefficients *c* and *b* respectively.

Next we denote on the equivalent scheme a coordinate system Ox in which the origin is at a point O, axis Ox coincides with the longitudinal symmetry axis of the cone-shaped body of the root crop is directed vertically upward. In the equivalent scheme, the current coordinate (the distance from point O to an arbitrary cross-section of the root crop body, which moves along axis Ox during the longitudinal deformation of the root crop elastic body) is denoted by x. The direction of vibrations of both planes of the vibrating digging working body is shown in the equivalent scheme by arrows.

It should also be remarked that the dynamic system "root crop – soil – working body" that we are considering is a system with holonomic connections since the connections do not depend on the velocities of the system elements.

To study the vibrations of holonomic systems with an infinite number of degrees of freedom, the Ostrogradskiy-Hamilton principle of stationary action or, as it is often called, the principle of the least action, is most applicable (Bulgakov et al. 2015). In addition, in the theory of longitudinal, torsional transverse vibrations of straight rods, the Ostrogradsky-Hamilton equations of the following form are most widely used [30]:

$$S=\int_{t_1}^{t_2}\int_{o}^{l} L\left(t,x,y,\frac{\partial y}{\partial t},\frac{\partial y}{\partial x},\frac{\partial^2 y}{\partial t^2},\frac{\partial^2 y}{\partial t\,\partial x},\frac{\partial^2 y}{\partial x^2}\right)dx\,dt \qquad (1)$$

where $L=T-P$ – the Lagrange function; T – the kinetic energy of the system; P – the potential energy of the system.

The function *S* has dimensions of the product of work by time (dJ•s). It is a special kind of action called "Ostrogradsky-Hamilton action". The essence of the Ostrogradsky-Hamilton principle is that the action of *S* on the actual movement has a stationary value compared to its values along any roundabout paths that transfer the system from one initial position to the same final position in the same time interval t_2 – t_1. In addition, on the actual movement the variation of action *S* is always equal to zero, that is $\delta S = 0$. As we see from expression (1), the subintegral function of *S* depends not only on the first-order derivatives but also on the second-order derivatives, although it can also depend on higher-order derivatives. For such function the necessary conditions for an extremum will be expressed by partial differential equations of the fourth higher orders. To study the longitudinal vibrations of a root crop as an elastic body in an elastic damping medium, using the Ostrogradsky-Hamilton principle, it is necessary to specify an analytical expression for the disturbing force. In this case, the longitudinal vibrations of the root crop will occur under the action of the above-mentioned vertical disturbing force $\bar{Q}_{df}$, which can be taken as varying according to the harmonic law of the following analytical form:

$$Q_{df}(t) = H\sin(\omega t), \tag{2}$$

where H – the amplitude of the disturbing force, N; ω – the disturbing force frequency, s^{-1}, t – the current time, s.

Next, we will compose the Ostrogradsky-Hamilton function S for the considered vibration process. For this, we will introduce the necessary notation. So we will assume that $F(x)$ is the cross-sectional area of the root crop at any point, located at a distance x from the lower end; (m^2); E is Young's modulus for the root crop material, ($N{\cdot}m^{-2}$); $y(x, t)$ is the longitudinal displacement of any cross-section of the root crop at time t, (m); $Q(x, t)$ is the intensity of the longitudinal external load, directed along the axis of the root crop, ($N{\cdot}m^{-1}$); $\mu(x)$ is the running weight of the root crop, ($kg{\cdot}m^{-1}$).

Let us first write the Ostrogradskiy-Hamilton function S that can be used to describe the longitudinal vibrations of a root crop as an elastic body, using the well-known expression for the case of longitudinal vibrations of straight rods, in the following form [17]:

$$S = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left[\mu(x)\left(\frac{\partial y}{\partial t}\right)^2 - E\cdot F(x)\left(\frac{\partial y}{\partial x}\right)^2 + Q(x,t)\,y\right]dx\,dt \tag{3}$$

where $\frac{\partial y}{\partial t}$ – the rate of longitudinal deformation of the elastic body of the root crop (the rate of longitudinal displacement of any cross-section of the root crop), ($m{\cdot}s^{-1}$); $\frac{\partial y}{\partial x}$ – a change in the longitudinal deformation of the elastic body of the root crop per unit of its length (dimensionless value, more precisely, $m{\cdot}m^{-1}$); $\frac{1}{2}\int_0^h \mu(x)\cdot\left(\frac{\partial y}{\partial t}\right)^2 dx$ – the kinetic energy of longitudinal displacements of the elements of the root crop body (longitudinal vibrations of the root crop body), (dJ); $\frac{1}{2}\int_0^h E\cdot F(x)\cdot\left(\frac{\partial y}{\partial x}\right)^2 dx$ – the potential energy of elastic deformation of the root crop body (work of the restoring elastic forces), (dJ); $\frac{1}{2}\int_0^h Q(x,t)\cdot y\,dx$ – the potential energy of stretching of the root crop body from the longitudinal load $Q(x,t)$, (dJ).

Let us find the values of all quantities included in expression (3). Considering that the root crop has the shape of a cone, we find that the area of its cross-section $F(x)$ at a point located at an arbitrary distance x from point O will be equal to:

$$F(x) = \pi\cdot x^2\cdot\tan^2\gamma \tag{4}$$

Clearly, the linear mass of the root crop can be determined using the following expression:

$$\mu(x) = \rho\cdot F(x) = \rho\cdot\pi\cdot x^2\cdot\tan^2\gamma \tag{5}$$

where ρ – the density of the root crop ($kg{\cdot}m^{-3}$).

Since value $Q(x, t)$ included in function (3) is the intensity of the distributed load which is measured in ($N{\cdot}m^{-1}$), the disturbing force must have the dimension of the load intensity. By means of the impulsive function of the first order $\sigma_1(x)$ (the Dirac function) in [30] it is possible to determine the intensity of the concentrated load thus to include the concentrated forces into the load distributed along the length.

So, if Q_{df} a is a concentrated disturbing force applied at the point x_1 measured in Newtons, then the function:

$$Q_{df}(x, t) = Q_{df}(t)\cdot\sigma_1(x - x_1) \tag{6}$$

has dimension ($N{\cdot}m^{-1}$) expresses the intensity of the concentrated load at point x_1.

The function $\sigma_1(x - x_1)$ will be equal to zero for all x, except for $x=x_1$, where it turns into infinity.

If the disturbing force Q_{df} changes according to law (2), then, according to (6), we can write:

$$Q_{df}(x,t) = H\cdot\sin(\omega t)\cdot\sigma_1(x - x_1) \tag{7}$$

Since the solid elastic body of the root crop is connected to the soil which is an elastic damping medium, then, when a disturbing force (2) acts upon it, a force of resistance of the soil to the movement of the body of the root crop arises during its vibration. This force also affects the process of proper vibrations of the body of the root crop in the soil, especially at the beginning of the vibratory process, when its bonds with the soil have not yet been broken.

It is obvious that the force of elastic soil resistance (for the entire root crop) is a distributed load over the area of its contact with the soil, therefore we define its intensity as the force of soil resistance to the displacement of a unit of root crop length.

Let c be the coefficient of elastic deformation of the soil (the ratio of the first Winkler coefficient to the contact area, $N{\cdot}m^{-3}$. Then intensity $P(x, t)$ of the soil resistance to the movement of the body at point x will be equal to:

$$P(x,t) = 2\pi\cdot c\cdot x\cdot\tan\gamma\cdot y(x,t) \tag{8}$$

Indeed, the length of the circle covering the cross-section of the root crop body, located at distance x from the conditional point O of fixing the root crop in the soil, will be equal to $2\pi\cdot x\cdot\tan\gamma$. Then the contact area of the surface of the soil surrounding the root crop as a result of longitudinal deformation of the body of the root crop will be equal to $y(x,t)$ will be $2\pi\cdot x\cdot\tan\gamma\cdot y(x,t)$ then the intensity $P(x,t)$ of the soil resistance will be determined according to expression (8).

Next let us take into account the damping properties of the soil. Let b be the soil damping coefficient, measured in $(N{\cdot}s^2){\cdot}m^{-3}$.

Since the deformation rate of the root crop body is sufficiently high, hence the high deformation rate of the soil surrounding the root crop, we will assume that the soil damping force will have a quadratic dependence on the deformation rate $\frac{\partial y(x,t)}{\partial t}$ of the root crop body itself. Then the damping force of the soil to the movement of the root crop at point x, taking into account its conical shape, will be determined using such an expression:

$$R(x,t) = 2\pi \cdot b \cdot x \cdot \tan\gamma \cdot \left[\frac{\partial y(x,t)}{\partial t}\right]^2 \tag{9}$$

Taking into account expressions (4), (5), (7) – (9), the Ostrogradsky-Hamilton functional (3) will have the following form: following form:

$$S = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left\{\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\left(\frac{\partial y}{\partial t}\right)^2 - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\left(\frac{\partial y}{\partial x}\right)^2 + H\cdot\sin(\omega t)\cdot\sigma_1(x-x_1)\cdot y(x,t) - \right.$$
$$\left. -2\pi\cdot c\cdot x\cdot\tan\gamma\cdot y^2(x,t) - 2\pi\cdot b\cdot x\cdot\tan\gamma\left(\frac{\partial y}{\partial t}\right)^2\right\}dx\,dt. \tag{10}$$

Since the proper frequencies are associated with free vibrations of the system, it is necessary to select in function (10) that part which describes precisely the free vibrations of the system. Clearly, this will be a function S_1 of this kind:

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left[\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\left(\frac{\partial y}{\partial t}\right)^2 - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\left(\frac{\partial y}{\partial x}\right)^2 - \right.$$
$$\left. -2\pi\cdot c\cdot x\cdot\tan\gamma\cdot y^2(x,t) - 2\pi\cdot b\cdot x\cdot\tan\gamma\cdot\left(\frac{\partial y}{\partial t}\right)^2\right]. \tag{11}$$

When finding the proper form frequencies of longitudinal vibrations of a root crop in the soil, we will rely on the general principle of the linear theory of vibrations – the principle of superposition of small vibrations, that is, we will assume that the small vibrations of a system with an infinite number of degrees of freedom represent the superposition of the main harmonic vibrations. Guided by this principle, we will look for harmonic longitudinal vibrations of the root crop in the following form:

$$y(x,t) = \varphi(x)\cdot\sin(pt+\alpha), \tag{12}$$

where $\varphi(x)$ – the proper form of the main vibrations; p – the proper frequency of the main vibrations.

By substituting expression (12) into functional (11), we obtain:

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\left\{\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\varphi^2(x)\cdot p^2\cdot\cos^2(pt+\alpha) - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\left[\varphi'(x)\right]^2\cdot\sin^2(pt+\alpha) - \right.$$
$$\left. -2\pi\cdot c\cdot x\cdot\tan\gamma\cdot\varphi^2(x)\cdot\sin^2(pt+\alpha) - 2\pi\cdot b\cdot x\cdot\tan\gamma\cdot\varphi^2(x)\cdot p^2\cdot\cos^2(pt+\alpha)\right\}dx\,dt. \tag{13}$$

Integrating expression (13) over t within one period $T=2\pi\cdot p^{-1}$, we will have:

$$S_2 = \frac{\pi}{2p}\int_{o}^{h}\left\{\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\varphi^2(x)\cdot p^2 - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\left[\varphi'(x)\right]^2 - \right.$$
$$\left. -2\pi\cdot c\cdot x\cdot\tan\gamma\cdot\varphi^2(x) - 2\pi\cdot b\cdot x\cdot\tan\gamma\cdot\varphi^2(x)\cdot p^2\right\}dx. \tag{14}$$

To find the proper form frequencies of longitudinal vibrations of the body of a root crop in the soil, the Ritz method is used [30].

The essence of the Ritz method is to reduce the variational problem to the problem of finding the extremum of a function of many independent variables. Such a reduction is performed by selecting from all possible admissible functions on which the values of the function are considered, some special class of functions that depend on a finite number of initially undefined parameters $\alpha_1, \alpha_2, ..., \alpha_n$. Substitution of such functions in the expression of the function turns it into a function of these parameters, for which the extremum can be found by the known elementary methods.

According to the Ritz method the values of function (14) are considered on a set of linear combinations of functions, that is, expressions that have the following form:

$$\varphi(x) = \sum_{i=1}^{n}\alpha_i\cdot\psi_i(x), \tag{15}$$

where α_i – parameters to be determined; $\psi_i\ (x)$ – the basic functions that are specially chosen are known functions that satisfy the geometric boundary conditions of the problem.

Thus, substituting expression (15) into expression (14) taking into account that

$$\left[\sum_{i=1}^{n}\alpha_i\cdot\psi_i(x)\right]^2=\sum_{i,\,k=1}^{n}\psi_i(x)\cdot\psi_k(x)\cdot\alpha_i\cdot\alpha_k\,,$$

$$\left[\sum_{i=1}^{n}\alpha_i\cdot\psi_i'(x)\right]^2=\sum_{i,\,k=1}^{n}\psi_i'(x)\cdot\psi_k'(x)\cdot\alpha_i\cdot\alpha_k\,,$$

after the appropriate transformations we obtain:

$$\begin{aligned}S_1=\frac{\pi}{2p}\int_0^h\Bigg[&\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot p^2\cdot\sum_{i,\,k=1}^{n}\psi_i(x)\cdot\psi_k(x)\cdot\alpha_i\cdot\alpha_k-E\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\sum_{i,\,k=1}^{n}\psi_i'\cdot(x)\cdot\psi_k'(x)\cdot\alpha_i\cdot\alpha_k-\\&-2\pi\cdot c\cdot x\cdot\tan\gamma\cdot\sum_{i,\,k=1}^{n}\psi_i(x)\cdot\psi_k(x)\alpha_i\cdot\alpha_k-2\pi\cdot b\cdot x\cdot\tan\gamma\cdot p^2\cdot\sum_{i,\,k=1}^{n}\psi_i(x)\cdot\psi_k(x)\alpha_i\cdot\alpha_k\Bigg]dx.\end{aligned}\tag{16}$$

To solve the problem further, it is necessary to introduce the following designations:

$$\begin{gathered}\int_o^h\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\psi_i(x)\cdot\psi_k(x)\,dx=T_{ik},\\\int_o^h E\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\psi_i'(x)\cdot\psi_k'(x)\,dx=U_{ik},\\\int_o^h 2\pi\cdot c\cdot x\cdot\tan\gamma\cdot\psi_i(x)\cdot\psi_k(x)\,dx=C_{ik},\\\int_o^h 2\pi\cdot b\cdot x\cdot\tan\gamma\cdot\psi_i(x)\cdot\psi_k(x)\,dx=R_{ik},\\(i,\,k=1,2,...,n).\end{gathered}\tag{17}$$

Substituting (17) into (16), we obtain the function as a function of the parameters $\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_n$:

$$\begin{aligned}S_1(\alpha_1,\alpha_2,...,\alpha_n)=&\frac{\pi}{2p}\cdot p^2\cdot\sum_{i,\,k=1}^{n}T_{i\,k}\cdot\alpha_i\cdot\alpha_k-\frac{\pi}{2p}\cdot\sum_{i,\,k=1}^{n}U_{i\,k}\cdot\alpha_i\cdot\alpha_k-\\&-\frac{\pi}{2p}\cdot\sum_{i,\,k=1}^{n}C_{i\,k}\cdot\alpha_i\cdot\alpha_k-\frac{\pi}{2p}\cdot p^2\cdot\sum_{i,\,k=1}^{n}R_{i\,k}\cdot\alpha_i\cdot\alpha_k\,.\end{aligned}\tag{18}$$

Let us investigate function (18) for an extremum. For this, we differentiate expression (18) with respect to the parameters αi (i=1, 2, …, n) equate the resulting partial derivatives to zero. As a result, we obtain a system of linear homogeneous equations for the unknowns $\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_n$, from which, in turn, we find the Ritz frequency equation for longitudinal vibrations of the root crop, as a solid elastic body fixed in the soil:

$$\begin{vmatrix}p^2(T_{11}-R_{11})-U_{11}-C_{11} & p^2(T_{12}-R_{12})-U_{12}-C_{12} & .... & p^2(T_{1n}-R_{1n})-U_{1n}-C_{1n}\\p^2(T_{21}-R_{21})-U_{21}-C_{21} & p^2(T_{22}-R_{22})-U_{22}-C_{22} & .... & p^2(T_{2n}-R_{2n})-U_{2n}-C_{2n}\\\cdots & \cdots & \cdots & \cdots\\p^2(T_{n1}-R_{n1})-U_{n1}-C_{n1} & p^2(T_{n2}-R_{n2})-U_{n2}-C_{n2} & .... & p^2(T_{nn}-R_{nn})-U_{nn}-C_{nn}\end{vmatrix}=0\tag{19}$$

Since this technological process of vibrational digging of root crops from the soil practically occurs at the lowest vibration frequencies, in the future they must be determined. Therefore, to determine the first (main) frequency of the proper vibrations, equation (19) can be presented in such a form:

$$p_1^2(T_{11}-R_{11})-U_{11}-C_{11}=0\,.\tag{20}$$

From expression (20) we find:

$$p_1^2 = \frac{U_{11} + C_{11}}{T_{11} - R_{11}}, \quad (21)$$

where U_{11}, C_{11}, T_{11} R_{11} can be determined from expressions (17) at $n = 1$.

In this case, we have:

$$U_{11} = \int_0^h E \cdot \pi \cdot x^2 \cdot \tan^2 \gamma \cdot \left[\psi_1'(x)\right]^2 dx \quad (22)$$

$$C_{11} = \int_0^h 2\pi \cdot c \cdot x \cdot \tan \gamma \cdot \psi_1^2(x)\, dx, \quad (23)$$

$$T_{11} = \int_0^h \rho \cdot \pi \cdot x^2 \cdot \tan^2 \gamma \cdot \psi_1^2(x)\, dx, \quad (24)$$

$$R_{11} = \int_0^h 2\pi \cdot b \cdot x \cdot \tan \gamma \cdot \psi_1^2(x)\, dx. \quad (25)$$

We select further the necessary basic functions that are included in expression (15). As noted [30] in many cases the proper forms of vibrations of a homogeneous rod of constant cross-section with the same fixed conditions as in the given problem are taken as basic functions. Such basic functions make it possible to find forms that satisfy not only the geometric boundary conditions, as required by the Ritz method, but also the dynamic conditions of the problem. Therefore, we can use in this problem as basic functions the proper forms of longitudinal vibrations of a uniform bar of constant cross-section with one fixed end. According to [30] such forms are as follows:

$$\psi_i(x) = \sin \frac{(2i-1) \cdot \pi \cdot x}{2h}, \ (i = 1, 2, 3, ...). \quad (26)$$

Then:

$$\psi_i'(x) = \frac{(2i-1) \cdot \pi}{2h} \cdot \cos \frac{(2i-1) \cdot \pi \cdot x}{2h}, \ (i = 1, 2, 3, ...). \quad (27)$$

Thus, taking into account expression (15), for an arbitrary n the proper form of longitudinal vibrations will have the following form:

$$\varphi(x) = \sum_{i=1}^{n} \alpha_i \cdot \sin \frac{(2i-1) \cdot \pi \cdot x}{2h}. \quad (28)$$

In particular, to find the first frequency $(n = 1)$, we have the form of longitudinal vibrations of the root crop of this type:

$$\varphi(x) = \alpha_1 \cdot \psi_1(x), \quad (29)$$

or

$$\varphi(x) = \alpha_1 \cdot \sin \frac{\pi \cdot x}{2h}, \quad (30)$$

that is

$$\psi_1(x) = \sin \frac{\pi \cdot x}{2h} \quad (31)$$

Then

$$\psi_i'(x) = \frac{\pi}{2h} \cdot \cos \frac{\pi \cdot x}{2h}. \quad (32)$$

After substitution of (32) into expression (22), of expression (31) into expressions (23), (24), (25), after integrating the obtained expressions, we will have:

$$U_{11} = E \cdot \pi \cdot \tan^2 \gamma \cdot \frac{\left(\pi^2 - 6\right) \cdot h}{24}, \quad (33)$$

$$C_{11} = \frac{c \cdot h^2 \cdot \tan \gamma}{2 \cdot \pi} \cdot \left(\pi^2 + 4\right), \quad (34)$$

$$T_{11} = \rho \cdot \pi \cdot \tan^2 \gamma \cdot \frac{\left(\pi^2 + 6\right) \cdot h^3}{6 \cdot \pi^2}, \quad (35)$$

$$R_{11} = \frac{b \cdot h^2 \cdot \tan \gamma}{2 \cdot \pi} \cdot \left(\pi^2 + 4\right). \quad (36)$$

Substituting expressions (33) – (36) into expression (21), we obtain an analytical expression for finding the first (main) frequency of proper vibrations of a root crop as an elastic body, located in an elastic damping medium:

$$p_1 = \sqrt{\frac{0.51 \cdot E \cdot \tan \gamma + 2.21 \cdot c \cdot h}{h\left(0.84 \cdot \rho \cdot \tan \gamma \cdot h - 2.21 \cdot b\right)}}. \quad (37)$$

3. Numerical Results and Discussion

Based on expression (37), according to numerical calculations, is constructed graphs of the dependence of the first proper frequency p_1 of vibrations of the root crop as a solid elastic body upon the coefficient of elastic deformation of the soil the coefficient b of its damping. In addition, according to [17], the following ranges of values of these coefficients were adopted: $c = 0...20 \cdot 10^5$ N•m^{-3}, $b = 0...10$ N•s^2•m^{-3}. In addition, according to [31], the following averaged statistical values of the physical mechanical characteristics of the root crop, included in expression (37), have been taken. Namely: h=0.25 m, $\gamma = 14^{\circ}$, $E = 18.4 \cdot 10^6$ N•m^{-2}, $\rho = 750$ kg•m^{-3}.

The results are presented in Figure 3 and Figure 4.

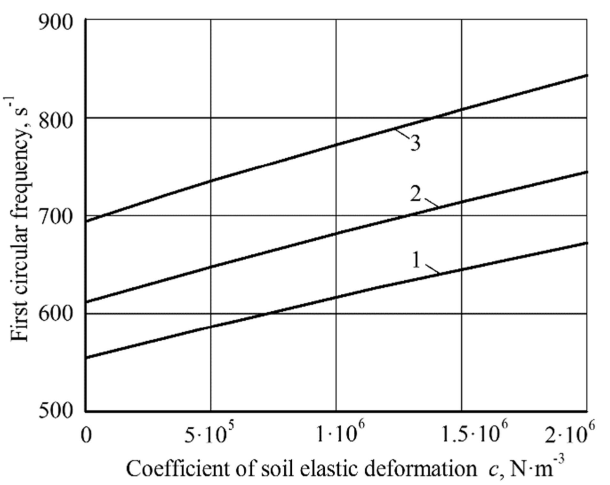


Figure 3. *Dependence of the first proper frequency of longitudinal vibrations body of the root crop upon the coefficient of elastic deformation of the soil (c) at the soil damping coefficient (b): 1 – b=4.0 (N·s²)·m⁻³; 2 – b=6.5 (N·s²)·m⁻³; 3 – b=9.0 (N·s²)·m⁻³.*

There are shown the the dependence of the first (main) proper vibration frequency p_1 of the root crop as a solid elastic body upon coefficient of elastic deformation of the soil (c) at different values of coefficient of its damping (b) (Figure 3).

As it follows from the analysis of the obtained graphs, with an increase in coefficient c, the frequency p_1 of vibrations of the root crop increases practically according to a linear law; besides it also increases with an increase in coefficient b, namely:

1. at b=4.0 (N•s²)•m⁻³ $p_1 = 550...670$ s⁻¹ or $p_1 = 88...107$ Hz;
2. at b=6.5 (N•s²)•m⁻³: $p_1 = 610...740$ s⁻¹ or $p_1 = 97...118$ Hz;
3. at b=9.0 (N•s²)•m⁻³ $p_1 = 690...840$ s⁻¹ or $p_1 = 110...134$ Hz.

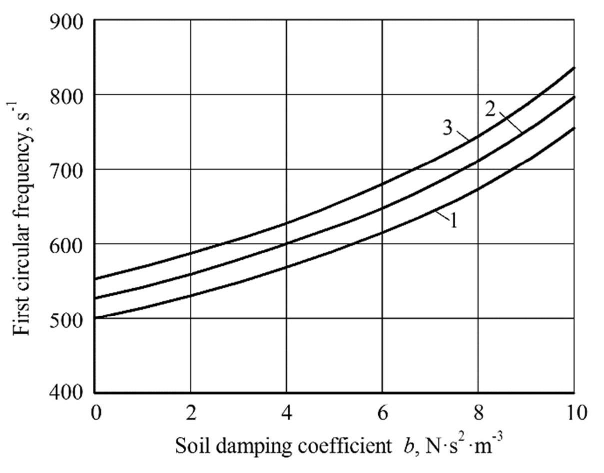


Figure 4. *Dependence of the first proper frequency of longitudinal vibrations of the body of the root crop upon the damping coefficient b of the soil, at the coefficient of elastic deformation of the soil: 1 – $c = 2.0\cdot10^5$ N•m⁻³; 2 – $c = 7.0\cdot10^5$ N•m⁻³; 3 – $c = 12.0\cdot10^5$ N•m⁻³.*

Similarly, Figure 4 shows the graphs of the dependence of the first vibration frequency p_1 of the root crop as a solid elastic body upon the damping coefficient of the soil (b) at different values of the coefficient of its elastic deformation (c).

As these graphs show, with an increase in coefficient b, the frequency p_1 of vibrations of the root crop increases according to a curvilinear law, similar to a parabolic one, while it also increases with an increase in coefficient c, which is consistent with the graphs presented in Figure 3, namely:

1. at $c = 2.0\cdot10^5$ N•m⁻³: $p_1 = 500...750$ s⁻¹ or $p_1 = 80...119$ Hz;
2. at $c = 7.0\cdot10^5$ N•m⁻³: $p_1 = 525...795$ s⁻¹ or $p_1 = 84...127$ Hz;
3. at $c = 12.0\cdot10^5$ N•m⁻³: $p_1 = 550...830$ s⁻¹ or $p_1 = 88...132$ Hz.

The dependencies, resulting from Figure 3, Figure 4, their nature obtained numerical values can be explained by the following circumstances. With an increase in coefficient c of elastic deformation of the soil, the force of elastic resistance of the soil increases, which is transmitted to the body of the root crop during the vibratory process due to sufficiently strong bonds of the root crop with the soil. An increase in the elastic force of the soil means an increase in the amplitude of free vibrations of the soil surrounding the root crop. At the same time, as it is known, with an increase in the elastic properties of the soil, the frequency of free vibrations of the soil increases. Therefore, due to the connections of the root crop with the soil, during the longitudinal deformation of the body of the root crop, the elastic force of the soil of a higher amplitude frequency is transmitted to it from the side of the deformable soil, which leads to an increase in the frequency of proper vibrations of the body of the root crop.

On the other hand, with an increase in the damping coefficient b of the soil, the dissipation of the energy of the soil surrounding the root crop increases. As a result, a smaller part of the energy is transferred to the root crop from the side of the soil, which does not contribute to a decrease in the proper frequency of free vibrations of the body of the root crop.

4. Mathematical Model of Forced Vibrations and Its Numerical Results

The forced vibrations of the root crop will proceed in accordance with the following law:

$$y(x,t) = \varphi(x)\cdot\sin(\omega t), \tag{38}$$

where $\varphi(x)$ – the form of forced vibrations.

To determine the form of forced vibrations of the root crop, expression (38) is substituted into function (10) to obtain the following function:

$$S_2 = \frac{1}{2}\int_{t_1}^{t_2}\int_{o}^{h}\Big\{\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\omega^2\cdot\varphi^2(x)\cdot\cos^2(\omega t) - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\left[\varphi'(x)\right]^2\cdot\sin^2(\omega t)+$$
$$+H\cdot\sigma_1(x-x_1)\cdot\varphi(x)\cdot\sin^2(\omega t) - 2\pi\cdot c\cdot x\cdot\tan\gamma\cdot\varphi^2(x)\cdot\sin^2(\omega t) -$$
$$-2\pi\cdot b\cdot x\cdot\tan\gamma\cdot\omega^2\cdot\varphi^2(x)\cdot\cos^2(\omega t)\Big\}dx\,dt. \tag{39}$$

Integrating expression (39) over t within one period $T = \dfrac{2\pi}{\omega}$, is obtained:

$$S_2 = \frac{\pi}{2\omega}\cdot\int_{o}^{h}\Big\{\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\varphi^2(x)\cdot\omega^2 - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\left[\varphi'(x)\right]^2+$$
$$+H\cdot\sigma_1(x-x_1)\cdot\varphi(x) - 2\cdot\pi\cdot c\cdot x\cdot\tan\gamma\cdot\varphi^2(x) - 2\cdot\pi\cdot b\cdot x\cdot\tan\gamma\cdot\varphi^2(x)\cdot\omega^2\Big\}dx. \tag{40}$$

According to the Ritz method, we consider the value of function (40) on a set of linear combinations of the following form:

$$\varphi(x) = \alpha\cdot\psi(x), \tag{41}$$

where α is a parameter to be determined; $\psi(x)$ is a basic function.

Substituting expression (41) into functional (40), we obtain:

$$S_2 = \frac{\pi}{2\omega}\int_{o}^{h}\Big\{\rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\alpha^2\cdot\psi^2(x)\cdot\omega^2 - E\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\alpha^2\cdot\left[\psi'(x)\right]^2+$$
$$+H\cdot\sigma_1(x-x_1)\cdot\alpha\cdot\psi(x) - 2\cdot\pi\cdot c\cdot x\cdot\tan\gamma\cdot\alpha^2\cdot\psi^2(x) -$$
$$-2\cdot\pi\cdot b\cdot x\cdot\tan\gamma\cdot\alpha^2\cdot\psi^2(x)\cdot\omega^2\Big\}dx. \tag{42}$$

Let us introduce the following notation:

$$\int_0^h \rho\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\psi^2(x)\,dx = T, \tag{43}$$

$$\int_0^h E\cdot\pi\cdot x^2\cdot\tan^2\gamma\cdot\left[\psi'(x)\right]^2 dx = U, \tag{44}$$

$$\int_0^h 2\cdot\pi\cdot c\cdot x\cdot\tan\gamma\cdot\psi^2(x)\,dx = M, \tag{45}$$

$$\int_0^h 2\cdot\pi\cdot b\cdot x\cdot\tan\gamma\cdot\psi^2(x)\,dx = N, \tag{46}$$

$$\int_0^h H\cdot\sigma_1(x-x_1)\cdot\psi(x)\,dx = L. \tag{47}$$

Substituting expressions (43) – (47) into (42), we will have:

$$S_2(\alpha) = \frac{\pi}{2\cdot\omega}\left[\omega^2\cdot(T-N)\cdot\alpha^2 - (U+M)\cdot\alpha^2 + L\,\alpha\right]. \tag{48}$$

Thus, the set of functions (41) and (42) turn into a function of the independent variable α, which has the form (48).

A necessary condition for the stationarity of function (48) (i.e., the existence of an extremum) is the equality to zero of its first derivative, namely:

$$\frac{\partial S_2}{\partial\alpha}\cdot\delta\alpha = 0, \tag{49}$$

from where we obtain the following equation:

$$2\omega^2\cdot(T-N)\cdot\alpha - 2(U+M)\cdot\alpha + L = 0, \tag{50}$$

from which we find the required value of parameter α. It will be equal to:

$$\alpha = \frac{L}{2\left[U+M-\omega^2\cdot(T-N)\right]}. \tag{51}$$

Let us take as the basic function $\psi(t)$ the form of forced longitudinal vibrations of a bar of constant cross-section with one rigidly fixed end which arise under the action of a longitudinal harmonic force of frequency ω applied at point $x=x_1$. According to [30], the form of forced vibrations of the mentioned rod has the following form:

$$\psi(x) = D_1 \cdot \sin(a \cdot x) \quad \text{at} \quad x \le x_1, \tag{52}$$

$$\psi(x) = D_2 \cdot \cos\left[a \cdot (h - x)\right] \quad \text{at} \quad x > x_1, \tag{53}$$

Where

$$D_1 = -\frac{1}{a \cdot E \cdot F} \cdot \frac{\cos\left[a \cdot (h - x_1)\right]}{\cos(a \cdot h)}, \tag{54}$$

$$D_2 = -\frac{1}{a \cdot E \cdot F} \cdot \frac{\sin(a \cdot x_1)}{\cos(a \cdot h)}, \tag{55}$$

$$a = \omega\sqrt{\frac{\mu}{E \cdot F}}, \tag{56}$$

μ – the linear mass (per unit of length) of the elastic conical body of the root crop; F – the cross-sectional area of the elastic conical body of the root crop; E – Young's modulus for the material of the elastic conical body of the root crop; h – the length of the elastic conical body of the root crop; ω – the frequency of forced vibrations of the elastic cone-shaped body of the root crop.

It is easy to verify that the boundary conditions for the functions (52), (53) are satisfied, therefore the adopted functions satisfy the requirements of the Ritz method. To determine parameter α, we will calculate the parameters T, N, U, M, L.

We obtain:

$$\begin{aligned} T = \rho \cdot \pi \cdot \tan^2\gamma \cdot \Bigg\{ & D_1^2 \left[\frac{x_1^3}{6} - \frac{x_1^2 \cdot \sin(2a \cdot x_1)}{4 \cdot a} - \frac{x_1 \cdot \cos(2a \cdot x_1)}{4 \cdot a^2} + \frac{\sin(2a \cdot x_1)}{8 \cdot a^3} \right] + \\ & + D_2^2 \left[\frac{h^3}{6} - \frac{x_1^3}{6} + \frac{x_1^2 \cdot \sin(2a \cdot h - 2a \cdot x_1)}{4 \cdot a} + \frac{h}{4 \cdot a^2} - \frac{x_1 \cdot \cos(2a \cdot h - 2a \cdot x_1)}{4 \cdot a^2} - \frac{\sin(2a \cdot h - 2a \cdot x_1)}{8 \cdot a^3} \right] \Bigg\}, \end{aligned} \tag{57}$$

$$\begin{aligned} N = {} & 2\pi \cdot b \cdot D_1^2 \cdot \tan\gamma \left[\frac{x_1^2}{4} - \frac{x_1 \cdot \sin(2a \cdot x_1)}{4 \cdot a} + \frac{1 - \cos(2a \cdot x_1)}{8 \cdot a^2} \right] + \\ & + 2\pi \cdot b \cdot D_2^2 \cdot \tan\gamma \left[\frac{1 - \cos\left[2a(h - x_1)\right]}{8 \cdot a^2} - \frac{(h - x_1)^2}{4} + \frac{h(h - x_1)}{2} + \frac{x_1 \cdot \sin\left[2a(h - x_1)\right]}{4 \cdot a} \right], \end{aligned} \tag{58}$$

$$\begin{aligned} U = E \cdot \pi \cdot \tan^2\gamma \Bigg\{ & \frac{D_1^2 \cdot a^2 \cdot x_1^3}{6} + \frac{D_2^2 \cdot a^2 \cdot (h^3 - x_1^3)}{6} + D_1^2 \left[\frac{x_1^2 \cdot a \cdot \sin(2a \cdot x_1)}{4} + \frac{x_1 \cdot \cos(2a \cdot x_1)}{4} - \frac{\sin(2a \cdot x_1)}{8 \cdot a} \right] - \\ & - D_2^2 \left[\frac{x_1^2 \cdot a \cdot \sin(2a \cdot h - 2a \cdot x_1)}{4} + \frac{h}{4} - \frac{x_1 \cdot \cos(2a \cdot h - 2a \cdot x_1)}{4} - \frac{\sin(2a \cdot h - 2a \cdot x_1)}{8 \cdot a} \right] \Bigg\}, \end{aligned} \tag{59}$$

$$\begin{aligned} M = {} & 2\pi \cdot c \cdot D_1^2 \cdot \tan\gamma \cdot \left[\frac{x_1^2}{4} - \frac{x_1 \sin(2a \cdot x_1)}{4 \cdot a} + \frac{1 - \cos(2a \cdot x_1)}{8 \cdot a^2} \right] + \\ & + 2\pi \cdot c \cdot D_2^2 \cdot \tan\gamma \left[\frac{1 - \cos\left[2a(h - x_1)\right]}{8 \cdot a^2} - \frac{(h - x_1)^2}{4} + \frac{h(h - x_1)}{2} + \frac{x_1 \sin\left[2a(h - x_1)\right]}{4 \cdot a} \right], \end{aligned} \tag{60}$$

$$L = H \cdot D_1 \cdot \sin(a \cdot x_1) \tag{61}$$

Substituting expressions (57) – (61) into expression (51), we obtain the required value of parameter α, at which function (40) has a stationary value.

Taking into account expressions (41), (52), (53), we obtain expressions for the form of forced vibrations of the solid elastic body of the root crop, fixed in the soil. They look like this:

$$\begin{aligned} & \varphi(x) = \alpha \cdot D_1 \cdot \sin(a\,x), \text{ at } x \le x_1, \\ & \varphi(x) = \alpha \cdot D_2 \cdot \cos\left[a \cdot (h - x)\right], \text{ at } x > x_1, \end{aligned} \tag{62}$$

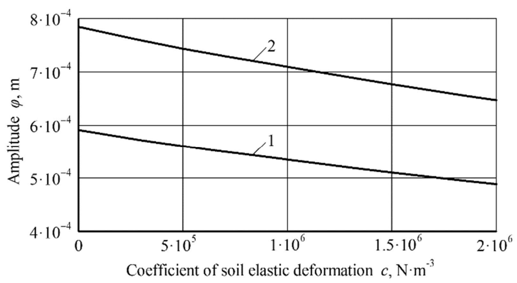
where α is defined according to (51).

Substituting expressions (62) into (38), we finally obtain the law of forced vibrations of a solid elastic body of a root crop, fixed in the soil:

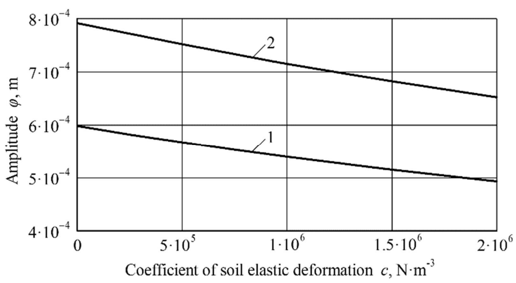
$$y(x,t) = D_1 \cdot \alpha \cdot \sin(ax) \cdot \sin(\omega t)\text{, at } x \le x_1\text{ ,}$$

$$y(x,t) = D_2 \cdot \alpha \cdot \cos\left[a \cdot (h-x)\right] \cdot \sin(\omega t)\text{, at } x > x_1 \quad (63)$$

According to the results of studies of forced vibrations of a root crop as a solid elastic body fixed in the soil, i.e. in an elastic damping medium was drawn up the amplitude of these vibrations. In particular, the value of the amplitude of forced longitudinal vibrations of the root crop were calculated depending on the coefficient of elastic deformation of the soil at the frequency of the disturbing force ν=10 Hz, 15 Hz 20 Hz, the amplitude of the indicated force *H*=500 N. The result of this calculation is shown in Figure 5. In this case the coordinate of the point of capture of the body of the root crop by the vibrating digging working body is equal to $x_1 = 0.15$m. The values of soil elastics deformation coefficient c and its dumping coefficient b, at which the calculations were made, are shown in Figure 5.



(a)



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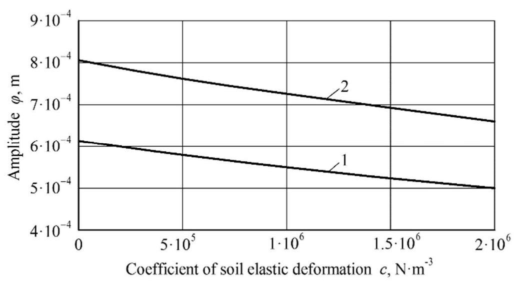


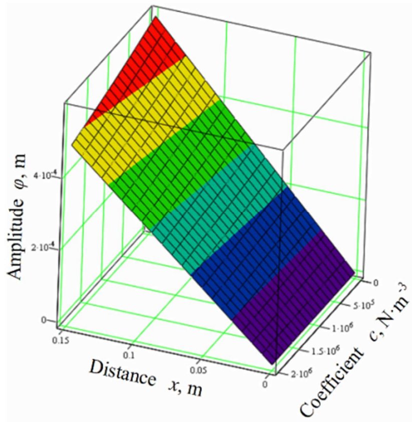
(c)

Figure 5. *Dependence of the amplitude of forced longitudinal vibrations of the root crop body upon coefficient c of elastic deformation of the soil (c) for different values of the disturbing force for the cross-section of the root crop (ν): a) ν =10 Hz, b) ν =15 Hz, c) ν =20 Hz (amplitude of the disturbing force H=500 N, the soil damping coefficient -* $b = 6.5$ *(N·s²)·m⁻³,* $c = 0...20 \cdot 10^5$ *N·m⁻³: 1 – at* $x < 0.15$ m *; 2 – at* $x > 0.15$ m .

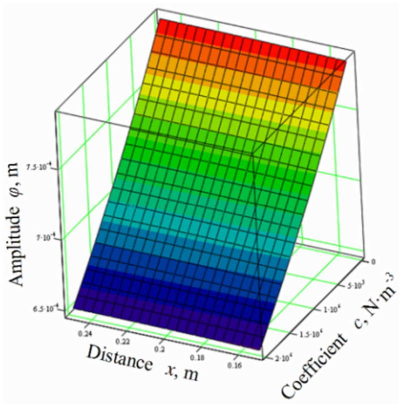
From Figure 5, it can be seen that with an increase in coefficient of the elastic deformation of the soil the amplitude of the forced vibrations of the root crop body decreases, with a change in c from 0 to $20 \cdot 10^5$ N·m⁻³, the indicated amplitude changes:

1. at ν =10 Hz: 0.58…0.48 mm at $x \le 0.15$ m, 0.77…0.65 mm at $x > 0.15$ m;
2. at ν =15 Hz: 0.60…0.49 mm at $x \le 0.15$ m, 0.78…0.66 mm at $x > 0.15$ m;
3. at ν =20 Hz: 0.62…0.50 mm at $x \le 0.15$ m, 0.82…0.67 mm at $x > 0.15$ m.

However, as one can see from these graphs the given calculation results, the amplitude of the forced longitudinal vibrations of the root crop practically does not depend on the vibration frequency of the vibrating digging working body, more precisely, it changes very slightly.



(a)

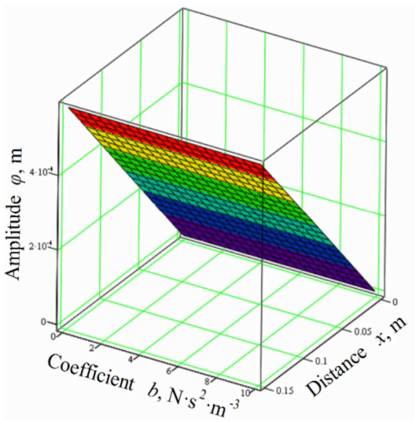


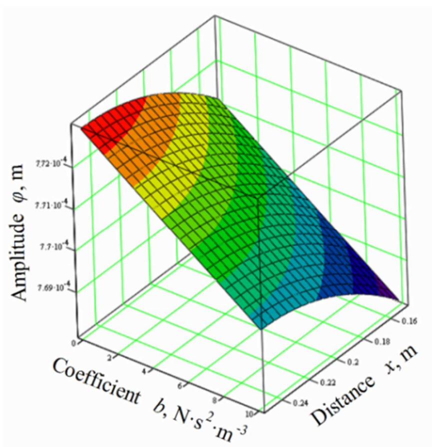
(b)

Figure 6. *Dependence of the amplitude of forced longitudinal vibrations of the body of the root crop upon the coefficient of elastic deformation of the soil c the distance x of the cross-section of the root crop from the conditional point O of its fixation in the soil: a –* $x = 0...0.15$ *m; b –* $x = 0.15...0.25$ *m; (the amplitude of the disturbing force H=500 N, frequency of the disturbing force ν =10 Hz, the soil damping coefficient –b=6.5 (N·s²)·m⁻³, the coordinate of the point of capture of the root crop by the vibrating digging working body* $x_1 = 0.15$ *m).*

Figure 6 shows the graphs of the dependence of the amplitude of the forced longitudinal vibrations of the root crop body upon coefficient of the elastic deformation of the soil and the distance of the cross-section from a conditional point of fixation of the root crop in the soil at the values of the parameters indicated.

As can be seen from Figure 6, the dependence of the amplitude of the forced vibrations of the root crop body upon coefficient *c* of the elastic deformation of the soil is the same as shown in Figure 5. However, the point of capture of the root crop body by the vibrating digging working body ($x \le 0.15$ m), with an increase in the distance x of the root crop cross-section from the conditional point O of its fixation in the soil, the amplitude increases from its almost zero value to 0.48 mm; above the capture point ($x \le 0.15$ m) the amplitude of the distance x of the root crop cross-section from the conditional point O of fixation of the root crop in the soil is practically independent.





(a)

(b)

Figure 7. *Dependence of the amplitude of forced longitudinal vibrations of the body of the root crop upon the damping coefficient b of and the soil the distance x of the cross-section of the root crop from the conditional point O of its fixation in the soil: a –* $x = 0...0.15$ *m; b –* $x = 0.15...0.25$ *m; (the amplitude of the disturbing force H=500 N, frequency of the disturbing force* ν *=10 Hz, coefficient of elastic deformation of the soil* $c = 2 \cdot 10^5$ *N·m*$^{-3}$*, the coordinate of the capture point of the root crop by the vibrating digging working body* $x_1 = 0.15$ *m).*

In Figure 7 there are shown the dependence of the amplitude of forced longitudinal vibrations of the root crop upon the damping coefficient *b* of the soil the distance *x* of its cross-section from the conditional point *O* of fixation in the soil. In this case the soil damping coefficient b varies within the range b=0 to 10 (N•s^2)•m^{-3}.

Figure 7 shows that below the point of capture of the root crop body by the vibrating digging working body ($x \le 0.15$ m), the amplitude of forced vibrations of the root crop body practically does not depend on the change in the soil damping coefficient; however, with an increase in the distance *x* of the root crop section from the conditional point *O* of its fixation in the soil, it increases linearly changes within the range of 0.01 to 0.58 mm. The amplitude of forced vibrations of the root crop body above the capture point ($x > 0.15$ m) decreases insignificantly with an increase in the soil damping coefficient changes within 0.773 to 0.770 mm, also with an increase in the distance x of the root crop body cross-section from the conditional point *O* of its fixation in the soil; its increase according to the parabolic law is rather insignificant varies within 0.768 to 0.773 mm.

Consequently, a conclusion can be made that the amplitude of the forced vibrations of the root crop as an elastic body does not practically depend upon the damping coefficient of the soil, at least within the range of its variation considered.

Dependence of the amplitude of the forced longitudinal vibrations of the root crop body, fixed in the soil, upon the amplitude of the disturbing force, which varies within *H*=100 to 600 N is shown in Figure 8.

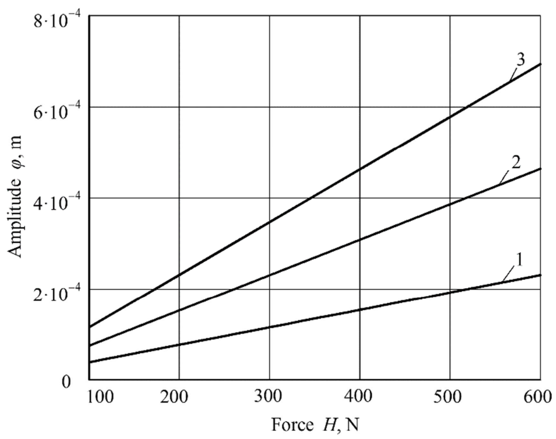


Figure 8. *Dependence of the amplitude of forced longitudinal vibrations of the elastic body of the root crop upon the amplitude of the disturbing force (* $x < x_1$ *,* $\nu = 10$ *Hz,* $c = 2 \cdot 10^5$ *N·m*$^{-3}$*,* $b = 6.5$ *(N·s*^{2}*)·m*$^{-3}$*): 1 –* $x = 0.05$ m *; 2 –* $x = 0.10$ m *; 3 –* $x = x_1 = 0.15$ m .

As one can see from Figure 8, with an increase in the amplitude of the disturbing force H the amplitude of the forced longitudinal vibrations of the root crop body increases according to a law, close to the linear one. Besides, below the capture point ($x < 0.15$ m), increasing the distance of the cross-section of the root crop from the conditional fixing point *O*, the amplitude also increases. So, at $x = 0.05$ m, the amplitude is within 0.03 to 0.23 mm; at $x = 0.1$ m – within 0.07 to 0.46 mm; at $x = 0.15$ m (capture point) – within 0.12 to 0.69 mm. Since the first proper frequency of the root crop as an elastic cone-shaped body has a value not lower than 75 Hz, the frequency of the disturbing force H for technological technical reasons cannot be more than 20 Hz, the resonance case is actually unlikely. In addition, the calculated value of the amplitude of forced longitudinal vibrations of the root crop body, which, with an amplitude of the disturbing force equal to 100...600 N, is within 0.03...0.69 mm, shows that the rupture of the specified root crop at its longitudinal deformation is virtually impossible.

5. Conclusions

1) An equations for calculating the proper frequencies of any order of longitudinal vibrations of the root crop body has been obtained, on the basis of which the the dependence of the first (main) frequency of the longitudinal vibrations of the root crop body upon the elastic deformation coefficients c and the soil damping coefficient b were constructed. When changing the coefficient of elastic deformation of the soil within $c = 0...20 \cdot 10^5$ N•m^{-3}, the first proper frequency of the longitudinal vibrations of the body of the root crop increases within 610 to 740 s^{-1} (97 to 118 Hz) at a value of the soil damping coefficient equal to $b = 6.5$ (N•s^2)•m^{-3}.
2) When the soil damping coefficient varies within 0 to 10 (N•s^2)•m^{-3}, the first proper frequency of longitudinal oscillations of the root crop body changes within 500 to 750 s^{-1} (80 to 119 Hz) at the value of coefficient of elastic deformation of the soil, equal to $c = 2 \cdot 10^5$ N•m^{-3}.
3) There is obtained dependence of the amplitude of these vibration upon the coefficient of elastic deformation c of and the soil the frequency ν of the disturbing force. It has been established that, when coefficient c of elastic deformation of the soil, which is within $c = 0...20 \cdot 10^5$ N•m^{-3}, changes, the amplitude of the disturbing force being equal to $H = 500$ N, its frequency being $\nu = 10$ Hz, the damping coefficient b of the soil being equal to $b = 6.5$ (N•s^2)•m^{-3}, the amplitude of the forced vibrations of the root crop body decreases, it varies within a range of 0.58 to 0.48 mm below the point of capture of the root crop by the vibrating digging working body, within a range of 0.77 to 0.65 mm above the point of capture of the root crop by the vibrating digging working body. The considered amplitude practically does not depend upon the change in the frequency of the disturbing force, or rather changes, but very insignificantly.
4) Below the point of capture of the root crop by the vibrating digging working body, when the distance of the cross-section of the root crop from the conditional point of its fixation in the soil increases, the amplitude of the forced vibrations of the root crop increases changes within 0.01 to 0.58 mm. Above the capture point, the amplitude practically does not depend on the cross-sectional distance to the conditional anchorage point in the soil.
5) The amplitude of the forced longitudinal vibrations of the root crop body below the capture point practically does not depend on the change of the soil damping coefficient; above the capture point, when increasing the soil damping coefficient within the limits of $b = 0...10$ (N•s^2)•m^{-3}, it slightly decreases changes within 0.773 to 0.770 mm.
6) When the amplitude of the disturbing force from the side of the working body changes within the boundaries $H = 100...600$ N , the amplitude of the forced longitudinal vibrations of the root crop body changes within the boundaries $0.30...0.68$ mm.
7) Since the first proper frequency of longitudinal vibrations of the root crop as a solid elastic body is within the boundaries 80 to 119 Hz, the allowed frequency of the disturbing force is not more than 20 Hz, the resonant case in the considered vibratory process is practically excluded.
8) Since the calculated value of the amplitude of the forced longitudinal vibrations of the root crop as a solid elastic body is within the boundaries of $0.30...0.68$ mm, it is obvious that the rupture of the root crop body in the investigated vibratory process is practically impossible.
9) There are created a good theory for sugar beet root crops. However, there are many other root crops in the world - fodder beets, table beets, carrots, horseradish, radishes, etc. Readers can study this theory and them build their own mathematical models for other root crop.

Conflict of Interest Statement

All the authors do not have any possible conflicts of interest.

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