



Gravitational and Electromagnetic Field of an Isolated Positively Charged Particle

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Abstract: A particle which is positively charged with spherically symmetry and non-rotating in empty space is taken to find out a metric or line element. The particle is under the influence of both gravitational and electro-magnetic field and the time component of this metric is depend on the combine effect of these two fields. Therefore in this work especial attention is given in Einstein gravitational and Maxwell's electro-magnetic field equations. Einstein field equations are individually considered for gravitational and electro-magnetic fields in empty space for an isolated charged particle and combined them like two classical waves. To solve this new metric initially Schwarzschild like solution is used. There after a simple elegant and systematic method is used to determine the value of space coefficient and time coefficient of the metric. Finally to solve the metric the e-m field tensor is used from Maxwell's electro-magnetic field equations. Thus in the metric the values of space and time coefficient is found a new one. The space and time coefficient in the new metric is not same in the metric as devised by Reissner and Nordstrom, The new space and time coefficient gives such an information about the massive body that at particular mass of a body can stop electro-magnetic interaction. Thus the new metric able to gives us some new information and conclusions.

Keywords: Metric, Gravitational Field, e-m Field, e-m Field Tensor

1. Introduction

It was the year 1915, Albert Einstein [1] proposed a new field theory known as 'The general theory of relativity'. The new field theory was one of the greatest advances in modern physics. Einstein's field equations are very simple and elegant, but it is difficult to find the exact analytical solution, because these are based on the fact that the equation is a set of nonlinear differential equations. The exact solution has got physical meanings only in case of some simplified assumptions; among the solutions the most famous of which are Schwarzschild solution given in 1916 [2]; Kerr's rotating black hole solution [3] and FRW models [4] for cosmology.

After the general relativity T Kaluza [5] in 1921 and later Oskar Klein [6, 7] in 1926 try to unify the relativity as a geometrical theory of gravitation and electro-magnetic (e-m) field. The metric for gravitational field due to an electron was given by Reissner and Gunner Nordstrom [8-12] in 1921. Schwarzschild metric is understood in the year 1958 to describe a black hole [13] and in the year 1963 Kerr [3, 14]

generalised the solution for rotating black hole. The metric for charged, rotating body was described by Newman [15, 16] on the basis of Reissner-Nordstrom and Kerr metric.

The model of universe was first put forwarded by Einstein on the development of his general theory of relativity. Later on the same theory was developed with de-Sitter [17] and finally another model forwarded by Freidman [18] which is non-static, isotropic and homogeneous in the year 1922 as well as by Robertson [19] and Walker [20] in the year 1935 known as FRW model. Today different methods of solution of Einstein field equations are proposed by different authors [21-28].

Most of the cosmic celestial bodies are rotating and rarely a few at rest; therefore in this reference, Kerr solution for rotating body is important in astrophysics. The metric for non-rotating charged cosmic celestial bodies are first devised by Reissner and Gunner Nordstrom. Considering the Newtonian potential in the above metric and on differentiation we get an equation of force. In this equation of force if the mass of the body considered as zero then the

force is varying as the inverse cube of the distance, which is impossible. This means that there may have some discrepancy in mathematical derivation process of the metric. This compelled me to think the matter seriously. The motivation of this derivation simply came from the use of Schwarzschild solution in empty space for both fields and combined them like two classical waves [29]. A more enlightening way to find this solution is attempted. The metric in this paper for gravitational and e-m field in empty space of a charged particle the time coefficient g_{44} is found more elegant using e-m field tensor. Further the new metric gives us some of physical characteristics of the charged cosmic celestial non-rotating bodies.

2. Schwarzschild and Nordstroem Solution

The exact solution of the field equation given by Einstein is usually expressed in metric or line element. The original field equation of Einstein's in empty space is given by

$$R_{\mu\nu} = 0 \quad (1)$$

The solution of the above Einstein's field equations in empty space of an isolated particle continually at rest at the origin was first given by Schwarzschild and the metric is

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{2m}{r}\right) dt^2 \quad (2)$$

Or in ordinary units,

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 \quad (3)$$

This metric is spherically symmetric and may be regarded as the gravitational field of a non-rotating point mass M at rest at origin.

The metric for a non-rotating charged cosmic celestial body was given by Reissner and Gunner Nordstrom which is,

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{4\pi\epsilon^2}{r^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{2m}{r} + \frac{4\pi\epsilon^2}{r^2}\right) dt^2 \quad (4)$$

Equation (4) gives the Newtonian potential,

$$\phi = \frac{-m}{r} + \frac{2\pi\epsilon^2}{r^2}$$

Furthermore the potential force is,

$$\frac{\partial \phi}{\partial r} = \frac{m}{r^2} - \frac{4\pi\epsilon^2}{r^3} \quad (5)$$

If we consider $m = 0$ in the above equation then the force is inversely proportional to the cube of the distance, which is impossible. Considering this problem we proceed for the solution of the gravitational and e-m fields in metric form for

charged particle in a new simple and elegant way.

3. Mathematical Formulation of the New Metric

Consider a positively charged particle, say proton, is isolated at origin at rest in empty space. The proton has both mass and charge, therefore it has both gravitational and e-m field. The fields due to this proton are assumed to be spherically symmetric. The interaction range of the gravitational field is from infinity to 10^{-33} cm and e-m field is from infinity to 10^{-8} cm. Hence both gravitational and e-m field have together the interaction range from infinity to 10^{-8} cm. The nature of both gravitational and e-m forces is inversely proportional to the square of the distance between two protons but gravitational force is proportional to the product of two masses and in case of e-m force it is proportional to the product of two charges. Since the proton is under the influence of both gravitational and e-m fields in empty space therefore Einstein field equation (1) is applicable.

The fundamental metric is given by,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (6)$$

The most general form of the metric for gravitational field of an isolated particle at origin at rest in empty space satisfying the condition of spherically symmetric is given by,

$$ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^\nu dt^2 \quad (7)$$

Here λ and ν are the functions of r only.

Considering in a similar fashion the most general form of the metric for e-m field of an isolated positively charged particle (proton) at origin at rest in empty space satisfying the condition of spherically symmetric is given by,

$$ds^2 = -e^a dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^b dt^2 \quad (8)$$

Also here a and b are the functions of r only.

The equations (7) and (8) are individual metric for each field of the isolated particle at origin at rest. As mentioned earlier actually the isolated particle is under the influence of both gravitational and e-m fields together. Therefore the most general form of the metric [29] for positively charged particle under the influence of both gravitational and e-m field at rest at origin in empty space satisfying the condition of spherically symmetric is the combination of (7) and (8), i.e.

$$ds^2 = -\frac{1}{2}(e^\lambda + e^a)dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \frac{1}{2}(e^\nu + e^b)dt^2 \quad (9)$$

Further instead of working with (9) due to its troublesome solution we used individual line element (7) and (8) to find out the value of e^λ , e^a , e^ν & e^b .

The solution of (7) becomes,

$$ds^2 = -\left(1 + \frac{A}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 + \left(1 + \frac{A}{r}\right) dt^2 \quad (10)$$

Here A is an integrating constant to be determined and it is related with the gravitational field of the proton.

Again the solution of (8) becomes,

$$ds^2 = -\left(1 + \frac{B}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 + \left(1 + \frac{B}{r}\right) dt^2 \quad (11)$$

Here B is an integrating constant to be determined and it is related with the charge of the proton.

In the metric (10) we put the constant $A = -2m$. This is done in order to facilitate the physical interpretation of m as the mass of the gravitating particle. But in the metric (11) for e-m field the constant B is still unknown to us.

$$ds^2 = -\left\{ \frac{\left(1 - \frac{m}{r} + \frac{B}{2r}\right)}{\left(1 - \frac{2m}{r}\right)\left(1 + \frac{B}{r}\right)} \right\} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 + \left(1 - \frac{m}{r} + \frac{B}{2r}\right) dt^2 \quad (14)$$

The relation between m and the proton mass m_p is

$$m = \frac{Gm_p}{c^2} \quad (15)$$

But still the value of B is unknown. To find out B let we continue the following steps.

3.1. Equation of Motion of a Particle

Now to find out the value of B , let we consider a particle is in motion with very low velocity in static field. Since the velocity of the particle is non-relativistic the geodesic equation for a particle is

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma_{\mu\nu}^\alpha \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0 \quad (16)$$

The metric in Riemannian space is given by the (6). Let us now assume that in (6), $g_{\mu\nu}$ are not constant, but differ from the values in flat space by infinitesimal amount. Therefore,

$$g_{\mu\nu} = \xi_{\mu\nu} + \eta_{\mu\nu} \quad (17)$$

Here $\eta_{\mu\nu}$ are very small quantities and functions of x, y, z ; but independent of time t . This means that,

$$\frac{\partial \eta_{\mu\nu}}{\partial x^4} = \frac{\partial g_{\mu\nu}}{\partial x^4} = 0 \quad (18)$$

Furthermore,

$$\Gamma_{\mu\nu}^\alpha = g^{\rho\alpha} \Gamma_{\rho, \mu\nu}$$

This gives,

In the (9) the value of g_{44} is,

$$\frac{1}{2}(e^\nu + e^b) = \left(1 - \frac{m}{r} + \frac{B}{2r}\right) \quad (12)$$

Furthermore,

$$\frac{1}{2}(e^\lambda + e^a) = \left\{ \frac{\left(1 - \frac{m}{r} + \frac{B}{2r}\right)}{\left(1 - \frac{2m}{r}\right)\left(1 + \frac{B}{r}\right)} \right\} \quad (13)$$

Hence the (9) becomes,

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2}(\xi^{\rho\alpha} + \eta^{\rho\alpha}) \left(\frac{\partial \eta_{\rho\mu}}{\partial x^\nu} + \frac{\partial \eta_{\rho\nu}}{\partial x^\mu} - \frac{\partial \eta_{\mu\nu}}{\partial x^\rho} \right) \quad (19)$$

Neglecting $\eta_{\mu\nu}$, because these are very small quantities and putting $\rho = \alpha$ we obtain,

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2} \left(\frac{\partial \eta_{\alpha\mu}}{\partial x^\nu} + \frac{\partial \eta_{\alpha\nu}}{\partial x^\mu} - \frac{\partial \eta_{\mu\nu}}{\partial x^\alpha} \right) \quad (20)$$

Taking as $x^1 = x$, $x^2 = y$, $x^3 = z$ and $x^4 = ct$

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$$

gives

$$ds^2 = -v^2 dt^2 + c^2 dt^2 = c^2 dt^2 \left(1 - \frac{v^2}{c^2}\right),$$

Since the velocity v is non-relativistic and hence $v \ll c$ then it gives,

$$ds = c dt = dx^4 \quad (21)$$

So we can write,

$$\frac{dx^1}{ds} = \frac{dx^2}{ds} = \frac{dx^3}{ds} = 0 \quad \text{and} \quad \frac{dx^4}{ds} = 1 \quad (22)$$

Therefore by virtue of (16)

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma_{44}^\alpha = 0 \quad (23)$$

Using (20) the above equation yields

$$\frac{d^2 x^\alpha}{ds^2} = -\Gamma_{44}^\alpha = -\frac{1}{2} \left(\frac{\partial \eta_{44}}{\partial x^\alpha} \right) \text{ for } \alpha = 1, 2, 3 \quad (24)$$

Again using the (21),

$$\frac{d^2 x^\alpha}{dt^2} = -\frac{\partial}{\partial x^\alpha} \left(\frac{1}{2} c^2 g_{44} \right) \quad (25)$$

This is the equation for motion of a particle.
Now Newton's equations of motion are

$$\frac{d^2 x^\alpha}{dt^2} = -\frac{\partial \phi}{\partial x^\alpha} \quad (26)$$

From (25) and (26)

$$-\frac{\partial}{\partial x^\alpha} \left(\frac{1}{2} c^2 g_{44} \right) = -\frac{\partial \phi}{\partial x^\alpha}$$

Here $\alpha = 1, 2, 3$
Or

$$-\frac{\partial}{\partial r} \left(\frac{1}{2} c^2 g_{44} \right) = -\frac{\partial \phi}{\partial r} \quad (27)$$

3.2. The e-m Field Tensor

We have found the equation of motion for a particle in static field. Now we are going to find out the e-m field tensor for a charged particle which is also required to find out the value of constant B . Since the field in (8) is symmetrical and our assumption implies that the field is purely electrostatic. So the magnetic field intensities are,

$$H_x, H_y, H_z = 0 \quad (28)$$

The general potential K^μ is defined in terms of e-m potential and scalar potential ψ as,

$$k^\mu = (A_x, A_y, A_z, \psi) \quad (29)$$

The associate covariant vector K_μ of K^μ is defined as,

$$K^\mu = g_{\mu\nu} K^\nu = g_{\mu\mu} K^\mu; \text{ Since } g_{\mu\nu} = 0 \text{ for } \mu \neq \nu$$

For this reason we can write

$$k^\mu = (-A_x, -A_y, -A_z, \psi) \quad (30)$$

Now the e-m field tensor $F_{\mu\nu}$ can be written as,

$$F^{\mu\nu} = K_{\mu,\nu} - K_{\nu,\mu} = \frac{\partial K_\mu}{\partial x^\nu} - \frac{\partial K_\nu}{\partial x^\mu} \quad (31)$$

And $\vec{H} = \nabla \times \vec{A}$

In the view of this, (28) gives

$$A_x, A_y, A_z = 0$$

This means that ψ is a function of r only.
Therefore using the (31) we can write

$$F_{12}, F_{23}, F_{31}, F_{24}, F_{34} = 0 \text{ and } F_{14} = -\frac{\partial \psi}{\partial r} \quad (32)$$

This implies that the only non-vanishing components of $F_{\mu\nu}$ is F_{14} and $F_{14} = -F_{41}$.

The current density J^μ can be written as,

$$J^\mu = F_\nu^{\mu\nu} = \frac{\partial F^{\mu\nu}}{\partial x^\nu} + F^{\alpha\nu} \Gamma_{\alpha\nu}^\mu + F^{\mu\alpha} \Gamma_{\alpha\nu}^\nu \quad (33)$$

The value of $F^{\alpha\nu} \Gamma_{\alpha\nu}^\mu = 0$ and this yield,

$$\sqrt{-g} J^\mu = \frac{\partial(\sqrt{-g} F^{\mu\nu})}{\partial x^\nu}$$

This gives us

$$\sqrt{-g} q = \frac{\partial(\sqrt{-g} F^{41})}{\partial r}$$

In the above equation symbol q is considered as charge. But there is no charge and no current in the space surrounding the charged particle which is at origin. Therefore,

$$\frac{\partial(\sqrt{-g} F^{41})}{\partial r} = 0 \quad (34)$$

Furthermore,

$$F^{41} = g^{44} g^{11} F_{41} = -F_{41}$$

and

$$F^{14} = g^{11} g^{44} F_{14} = -F_{14}$$

These gives,

$$F^{41} = -F_{41} = -F^{14} = F_{14} = -\frac{\partial \psi}{\partial r}$$

From the above equations the (34) can be written as,

$$\frac{\partial}{\partial r} \left[e^{(a+b)/2} r^2 \sin \theta (-e^{-(a+b)}) F_{41} \right] = 0$$

This yield,

$$\frac{\partial}{\partial r} \left[r^2 e^{-(a+b)/2} F_{14} \right] = 0$$

Integrating the above equation we get

$$r^2 e^{-(a+b)/2} F_{14} = \text{constant} = \varepsilon \text{ (say)}$$

$$F_{14} = \frac{\varepsilon}{r^2} e^{(a+b)/2} \quad (35)$$

The symbol ε is considered as an absolute constant. The constant ε is related with the charge of the particle which is situated at origin. This means $4\pi\varepsilon$ ($=q$) is charge.

After rigorous calculation the (8) gives us $b = -a$, and this gives the field tensor in (35) as

$$F_{14} = \frac{\varepsilon}{r^2}$$

Also from (32) the field tensor is

$$F_{14} = -\frac{\partial\psi}{\partial r}$$

Therefore from the above two equations we get,

$$-\frac{\partial\psi}{\partial r} = \frac{\varepsilon}{r^2} \quad (36)$$

3.3. The Value of Constant B

Suppose

$$-\frac{\partial\phi}{\partial r} = -\frac{\partial\psi}{\partial r}$$

Then (36) comparing with (27) we get,

$$-\frac{\partial}{\partial r} \left(\frac{1}{2} c^2 g_{44} \right) = \frac{\varepsilon}{r^2} \quad (37)$$

Integrating above equation,

$$ds^2 = - \left\{ \frac{\left(1 - \frac{Gm_p}{rc^2} + \frac{Kq}{rc^2} \right)}{\left(1 - \frac{2Gm_p}{rc^2} \right) \left(1 + \frac{2Kq}{rc^2} \right)} \right\} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{Gm_p}{rc^2} + \frac{Kq}{rc^2} \right) dt^2 \quad (42)$$

This is the equation for proton continually at rest at origin. If we consider for a body having mass M and the total charge Q on the body then (42) becomes,

$$ds^2 = - \left\{ \frac{\left(1 - \frac{GM}{rc^2} + \frac{KQ}{rc^2} \right)}{\left(1 - \frac{2GM}{rc^2} \right) \left(1 + \frac{2KQ}{rc^2} \right)} \right\} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{GM}{rc^2} + \frac{KQ}{rc^2} \right) dt^2 \quad (43)$$

Or

$$ds^2 = - \left\{ \frac{\left(1 - \frac{1}{rc^2} (GM - KQ) \right)}{\left(1 - \frac{2GM}{rc^2} \right) \left(1 + \frac{2KQ}{rc^2} \right)} \right\} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \left(1 - \frac{1}{rc^2} (GM - KQ) \right) dt^2 \quad (44)$$

And,

$$g_{44} = \frac{2\varepsilon}{rc^2} + k \quad (38)$$

In the above equation k is considered as an integrating constant.

At $r \rightarrow \infty$, $g_{44} = 1$ gives $k = 1$, hence

$$g_{44} = \left(1 + \frac{2\varepsilon}{rc^2} \right) \quad (39)$$

The above equation compare with the time coefficient of the (11) i.e.

$$g_{44} = e^b = \left(1 + \frac{B}{r} \right)$$

Gives,

$$B = \frac{2\varepsilon}{c^2} \quad (40)$$

3.4. The new metric

Now from (15) and (40) the (12) becomes,

$$\frac{1}{2} (e^\nu + e^b) = \left(1 - \frac{Gm_p}{rc^2} + \frac{\varepsilon}{rc^2} \right)$$

Considering $4\pi\varepsilon = q$, the charge of the particle and $\frac{1}{4\pi} = K$, is a constant then the above equation becomes,

$$\frac{1}{2} (e^\nu + e^b) = \left(1 - \frac{Gm_p}{rc^2} + \frac{Kq}{rc^2} \right) \quad (41)$$

The line elements (14) becomes

$$g_{\mu\nu} = \begin{bmatrix} -\left\{ \frac{\left(1 - \frac{GM}{rc^2} + \frac{KQ}{rc^2}\right)}{\left(1 - \frac{2GM}{rc^2}\right)\left(1 + \frac{2KQ}{rc^2}\right)} \right\} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2 \sin^2 \theta & 0 \\ 0 & 0 & 0 & \left(1 - \frac{GM}{rc^2} + \frac{KQ}{rc^2}\right) \end{bmatrix} \quad (45)$$

4. Results and Discussion

Considering (42) the Newtonian-like potential is

$$\left(1 - \frac{Gm_p}{rc^2} + \frac{Kq}{rc^2}\right) = 1 + 2\phi \quad (46)$$

$$\phi = \frac{-Gm_p}{2rc^2} + \frac{Kq}{2rc^2}$$

This gives,

$$-\frac{\partial\phi}{\partial r} = -\frac{Gm_p}{2r^2c^2} + \frac{Kq}{2r^2c^2} \quad (47)$$

If $m_p = 0$, then the last term of r. h. s. gives,

Force = $\frac{Kq}{2r^2c^2}$, This means that the force $\propto \frac{1}{r^2}$. The potential force in (47) satisfied the inverse square law whereas in Nordstrom metric [10, 11] the inverse square law is not satisfied. In Nordstrom metric the force is inversely proportional to the cube of the distance, which is impossible.

Equation (42) becomes singular at $r=0$; but this singularity also occurs in Newton's theory. The first term of r. h. s. of (42) the numerator becomes zero when,

$$r = \frac{Gm_p}{c^2} - \frac{Kq}{c^2} = (m - k) \quad (48)$$

Here $\frac{Kq}{c^2} = k$ (say)

Again in the same term of (42) the denominator becomes zero when,

$$r = \frac{2Gm_p}{c^2} = 2m \quad (49)$$

Hence from (48) and (49)

$$2m > (m - k)$$

This means that there is a singularity like Schwarzschild solution when $r = 2m$. For points $0 \leq r \leq (m - k)$, $ds^2 < 0$ i.e. the interval is purely space-like. Hence there is a finite singular region for $0 \leq r \leq (m - k)$. Thus $r = (m - k)$ represents the boundary of the isolated particle and the solution holds in empty space outside the isolated particle whose radius must greater than $(m - k)$.

5. Conclusion

In equation (42) or (43) when $m_p = 0$ or $M = 0$ then without massive body charge cannot exist alone. Since charge is zero and hence represents flat space-time. Let we consider in the (44),

$$GM - KQ = 0$$

Means,

$$GM = KQ$$

This gives that the metric (44) also represents flat space-time. This means that the time components is depend not only gravitational field but also the combine effect of both gravitational and e-m field. In the other hand the (44) gives conclusion that if gravitational field strength increases in such a level that gravitational field can stop e-m field. This means that if the body gain mass such a particular level then the gravitational field can stop e-m interaction.

It is known to us that in the universe many heavenly bodies are not in rest but they are rotating with an angular velocity. So we can improve the above (43) for a rotating charged body.

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