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Well behaved charge analogues of Wyman-Adler exact solution for a *self-bound* star

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Abstract: The exact analytical Wyman-Adler's relativistic solution describing the interior of a charged spherical strange star candidate is found under the assumption and existence of two parameters K and m. The interior *self-bound* star matter, pressure, energy density and the adiabatic sound speed are represented in terms of simple algebraic function. The analytic solution depicts a unique static charged configuration of quark matter with radius R~9 km and total mass M~2.5M$_\odot$. And try to investigate the velocity of sound approximately $1/\sqrt{3}$ which is similar to the attitude of SQM (Strange Quark matter). Based on analytic model in the recent work, the applicable values of physical quantities have been calculated by accepting the estimated masses and radii of some well-known strange star candidates like PSR J1903+327, Her X-1, Cen X-3, EXO 1785-248. The equation of state of the charge matter distribution may play a major role in the study of the interior structure of highly compact charge stellar object in astrophysical study.

Keywords: Exact Solution, Einstein-Maxwell, Reissner–Nordström, Relativistic Astrophysics, Compact Star, Equation of State

1. Introduction

It is well known that, at the pressure free interface, the Reissner-Nordström solution is interesting to observe (present charge) the gravitational collapse of a spherical symmetric distribution of the matter to a point singularity may be avoided [1] if the matter distribution acquires large amount of electric charge. Any compact star is not composed of charged perfect fluid and may be used to make a suitable model of compact object with charge matter for the numerical study of the stellar structure [2, 3, 4]. Pant, Metha and Tewari showed that radiative gravitational radiation (GR) collapse may be contributed to formation of the compact non-singular massive hot object [5-7]. The black hole is never formed due to the apparent horizon formation condition [8]. This could be understood as the formation of a naked singularity. But the main reason is that the star radiates all its mass before it reaches the singularity at r = 0 and t = 0. Nuclear matter is meta-stable and it is well known that after releasing a lot of energy converts into strange quark matter (SQM) to achieve stability. And this QM is more stable matter. The collapse of a neutron star may lead to a strange quark star (SQS) or a hybrid star [9]. Here we consider only the structure properties of SQS. The surface density, $\rho_s = 2\times10^{14}$ g cm^{-3}, is not the most realistic for modeling SQM star [10]. And the actual values of SQM density at zero pressure lie between 4×10^{14} to 10×10^{14} g cm^{-3} [11-13]. The mass-radius relation for an SQS is as $M \propto R^3$ which is different from that of a neutron star. This star does not have the minimum mass. For an SQS with 1M$_\odot$ $\leqslant$M $\leqslant$2M$_\odot$, the radius is about 10 km [14, 15]. Here, we don't want to write the physical condition for a regular and charged fluid sphere for interior solution of the gravitational field equations of a SQM star because [16] was elaborately discussed.

A self-bound strange quark star belongs to a different class compact object than a conventional normal matter neutron star. The surfaces of *bare* strange stars and normal matter neutron stars have significant difference. The main properties of the quark surface which are strong bounding of particles abrupt density change from $4\times10^{14}\ g\,cm^{-3}$ to ~ 0

in ~ 1 fm. Another striking distinction between a strange star and a normal neutron star is their surface electric field [17]. A *bare* strange stars have own ultra-strong electric fields on their surface which is around 10^{18} V/cm [18] and for color superconducting strange matter is 10^{20} V/cm [19-21]. Ray et al. [22] and Malheiro et al. [23] first inquired the influence of energy density of ultra-high electric fields on the bulk properties of compact stars. Weber et al.[24-26] and [27] also have proposed that electric fields of this magnitude increase stellar mass by up to 30% contingent on the strength of the electric field which generated by charge distributions situated neighbor surfaces of strange quark stars. Whereas the strange star's surface electric field, in the case of neutron star, is absent. Actually, these characteristics may allow the observationally distinguish quark stars from neutron stars.

Nevertheless, here we study the SQM star which is nonlinear electrically charged self-bound stars, SQM star's radius R< 10-12 km [12]. A self-bound strange quark star belongs to a different class compact object than a conventional normal matter neutron star [12]. For ordinary strange matter, the electric field is ~ 10^{18} V/cm to up to 10^{19} V/cm if SQS forms a color superconductor [9]. The electric fields are as high as 10^{19-20} V/cm [27] and its determined the electrostatic effects and the surface tension of the interface between vacuum and quark matter [28]. And interesting things is that our model is exactly matching that range [27] of electric field.

Recently, many authors have proposed authentic analytical models of electrically charged compact self-bound stars considering a framework of linear and nonlinear equation based on MIT bag model [29, 30] and metric potential [5-7] successively. *Self-bound* SQM star's modeling requires the use different stellar surface density than modeling a neutron star. An SQM star could have either bare quark-matter surface with vanishing pressure but a large, supernuclear density, or a thin layer of normal matter supported by Coulomb forces above the quark surface. Neutron stars like a part of "normal" stars, with hadronic matter exteriors where the surface pressure and baryon density vanish. Strange star, if it exists, has an extremely abrupt edge; it probably has the hardest smooth surface of any object in the universe. The edge of a neutron star is not nearly as abrupt as that of a strange star. Due to make from material that has been processed in stellar evolution, ordinary iron forms their surface.

By solving Einstein-Maxwell field equations with help of metric coefficient $e^{\nu} = B_N(1+x)^N$ and suitable forms of electric charge distribution functions [31, 32] to represent numerous stellar models of *self-bound* type. The main objective of this work is to present some more relativistic stellar models electrically charged compact stars with nonzero high surface density by satisfying applicable boundary conditions.

The usual physical boundary conditions taken are

1) $P(r=R)=0$

2) $Q(r=0)=0$

3) $e^{\nu(R)} = e^{-\lambda(R)} = \left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right)$; where $q(R) = Q$

The physical quantities such as total mass (M), radius (R), total charge (Q), may be specified by setting one of the following: (i) surface density, or (ii) central density as a parameter. The analytical equations of state for the model charged matter distribution is also obtained in parametric form and analyzed numerically.

A physically acceptable interior solution of the gravitational field equations must comply with the certain (not necessary) physical conditions [33-36] such as (i) the solution must be free from both physical and geometric singularities. (ii) Pressure and density are always positive i.e. P, ρ ≥ 0. (iii) Density and pressure should be maximum at the centre and decreasing monotonically towards the pressure free interface (iv) Pressure P should be zero at boundary r = R i.e. P(r = R) = 0. (v) In order to equilibrium configuration the matter must be stable against the collapse of local region. *Le Chatelier's* principle state that P must be a monotonically non-decreasing function of ρ, $\frac{dP}{d\rho} \geq 0$. (vi) For causality condition, $\sqrt{\frac{dP}{d\rho}}$, the hydrodynamic phase velocity of sound waves in the neutron star matter, would not exceeds the velocity of light. (vii) the energy momentum tensor must be nonnegative, i.e. $\rho - 3P \geq 0$, $0 \leq r < R$. This is known as trace condition [37, 38, 33, 40, 42]. (viii) Redshift z should be positive, finite and monotonically decreasing toward the boundary of the sphere. (ix) Allowable mass-to-radius ratio (M/R) for isotropic fluid spheres of the form $\frac{2M}{R} \leq \frac{8}{9}$ (c = G = 1) and [39] proved for the mass-radius ratio in a lower bound for a compact object with charge

$$\frac{3}{2}\frac{Q^2}{R^2}\frac{(1+\frac{Q^2}{18R^2})}{(1+\frac{Q^2}{12R^2})} \leq \frac{2M}{R}$$

For the metric function $e^{\nu} = B_N(1+x)^N$ the pressure and energy density for the charged matter distribution become from the Einstein-Maxwell gravitational field equations become,

$$\frac{\kappa}{C}P = \left[\frac{1+(2N+1)x}{x(1+x)}\right]Z - \frac{1}{x} + \frac{Cq^2}{x^2} \tag{1.1}$$

$$\frac{\kappa}{C}\rho = -2\frac{dZ}{dx} - \frac{Z}{x} - \frac{1}{x}\left(\frac{Cq^2}{x} - 1\right) \tag{1.2}$$

The equation of "pressure isotropy" yields the following solution,

$$e^{-\lambda} = \frac{x}{(1+x)^{N-2}[1+(1+N)x]^{\frac{2}{(1+N)}}} \int \frac{(1+x)^{N-1}[1+(1+N)x]^{\frac{(1-N)}{(1+N)}}}{x^2}\left(\frac{2Cq^2}{x}-1\right)dx + A_N \frac{x}{(1+x)^{N-2}[1+(1+N)x]^{\frac{2}{(1+N)}}} \quad (1.3)$$

where A_N is a constant of integration.

2. Einstein–Maxwell Field Equations for Interior Solutions of Perfect Fluid Sphere

2.1. Field Equations

We consider a static, spherically symmetric star whose interior metric is given in Schwarzschild coordinates Tolman [43] $x^{\mu} = (t, r, \theta, \phi)$ [1]

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (2.1)$$

For the metric (2.1), the Einstein-Maxwell field equations become,

$$\frac{\nu'}{r}e^{-\lambda} - \frac{(1-e^{-\lambda})}{r^2} = \kappa P - \frac{q^2}{r^4} \quad (2.2)$$

$$\left(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} + \frac{\nu'-\lambda'}{2r}\right)e^{-\lambda} = \kappa P + \frac{q^2}{r^4} \quad (2.3)$$

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa\rho + \frac{q^2}{r^4} \quad (2.4)$$

where prime (′) denotes the r -derivative and where $q(r)$ represents the total charge contained within the sphere of radius r defined by,

$$q(r) = 4\pi\int_0^r e^{\frac{\lambda}{2}}\rho_{ch}\, u^2 du \quad (2.5)$$

Eq. (2.5) can be interpreted as the relativistic version of Gauss's law.

To transform the system (2.2)—(2.4) into relatively simpler form we assume the following ansatz [40, 44, 45],

$$e^{\nu} = B_N(1+Cr^2)^N$$

N is a positive integer and B_N, $C > 0$ are two constants to be determined by the appropriate physical boundary conditions. Eliminating the pressure P from (2.2) and (2.3) one obtain the equation of "pressure isotropy"

$$\left(\frac{\nu''}{2} - \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} - \frac{\nu'+\lambda'}{2r}\right)e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \frac{2q^2}{r^4} \quad (2.6)$$

Equation (2.6) is a second order differential equation in ν and first order in λ . At this moment it is convenient to introduce the following transformations $e^{-\lambda} = Z$, $x = Cr^2$ equations (2.2) and (2.4) become,

$$\frac{\kappa}{C}P = \left[\frac{1+(2N+1)x}{x(1+x)}\right]Z - \frac{1}{x} + \frac{Cq^2}{x^2} \quad (2.7)$$

$$\frac{\kappa}{C}\rho = -2\frac{dZ}{dx} - \frac{Z}{x} - \frac{1}{x}\left(\frac{Cq^2}{x} - 1\right) \quad (2.8)$$

And the equation of "pressure isotropy" (2.6) can be written in terms of auxiliary variable x as,

$$\frac{dZ}{dx} + \left[\frac{(N^2-2N-1)x^2 - 2x - 1}{x(1+x)(1+(1+N)x)}\right]Z = \frac{(1+x)}{x(1+(1+N)x)}\left(\frac{2Cq^2}{x} - 1\right) \quad (2.09)$$

which yields the following solution [10],

$$Z = \frac{x}{(1+x)^{N-2}[1+(1+N)x]^{\frac{2}{(1+N)}}}\int \frac{(1+x)^{N-1}[1+(1+N)x]^{\frac{(1-N)}{(1+N)}}}{x^2}\left(\frac{2Cq^2}{x}-1\right)dx$$
$$.......+ A_N\frac{x}{(1+x)^{N-2}[1+(1+N)x]^{\frac{2}{(1+N)}}} \quad (2.10)$$

where A_N is a constant of integration.

In order to obtain closed form solution one can imagine several plausible distributions to integrate the equation of pressure isotropy (2.6). Various authors presented variety of solutions previously for different suitable choices of charge distributions. Some of the solutions will be found elsewhere (See Table 1, [31]). In this work we consider the following model distributions:

Model I:

$$\frac{2Cq^2}{x^2} = K\,x[1+(1+N)x]^{\frac{(N-1)}{(N+1)}}[1+(N+m)x]^m(1+x)^{(1-N)} \quad (2.11a)$$

Model II:

$$\frac{2Cq^2}{x^2} = K\,x[1+(1+N)x]^{\frac{(N-1)}{(N+1)}}\left[1+\frac{1}{2}(N+m)x\right]^m(1+x)^{(1-N)} \quad (2.11b)$$

Where $K \geq 0$ and m is a nonzero real number.

These distributions are chosen, in term of x, in such a way that electric field intensity vanishes at the center and remains continuous and bounded in the interior of the star for a wide range of values of the parameters m and K. Thus these choices are physically reasonable and useful in the study of the gravitational behavior of charged stellar objects. It has been shown in Maurya and Gupta [47, 48] for the uncharged and charged cases respectively that the

[1] Throughout the work we will use $c = G = 1$ except in the tables and figures.

ansatz for the metric function $e^{\nu} = B_N(1+x)^N$ where N is a positive integer produces an infinite family of analytic solutions of the self-bound type. Some of these were previously known (N = 1, 2, 3, 4, and 5). The most relevant case is for N = 2, for which the velocity of sound $\approx 1/\sqrt{3}$ throughout most of the star, somewhat similar to the behavior of strange quark matter [49]. In this work we keep our interest particularly on to obtain the charged analogue of the case N = 2 and derive corresponding equations of state.

3. New Charged Stellar Models

For the case N = 2, Einstein–Maxwell system yields some new charged analogues of well-known Wyman-Adler-Kuchowicz exact solution in general relativity,

Model I:

$$e^{\upsilon} = B_2(1+x)^2 \tag{3.1a}$$

$$e^{-\lambda} = \frac{K}{(m+1)(m+2)}\frac{x}{(1+3x)^{2/3}}\{1+(m+2)x\}^{m+1} + 1 + A_2\frac{x}{(1+3x)^{2/3}} \tag{3.1b}$$

$$\frac{2Cq^2}{x^2} = K\,x[1+(1+N)x]^{\frac{(N-1)}{(N+1)}}\left[1+(N+m)x\right]^m(1+x)^{(1-N)} \tag{3.1c}$$

$$\frac{\kappa}{C}P = \frac{K\{1+(m+2)x\}^m}{2(m+1)(m+2)}\frac{(3m^2+19m+26)x^2+(m^2+5m+16)x+2}{(1+x)(1+3x)^{2/3}} + \frac{4}{(1+x)} + A_2\frac{1+5x}{(1+x)(1+3x)^{2/3}} \tag{3.1d}$$

$$\frac{\kappa}{C}\rho = -\frac{K}{2(m+1)(m+2)}\frac{\{1+(m+2)x\}^m\{(21m^2+61m+38)x^3+(22m^2+66m+42)x^2+(5m^2+17m+14)x+2\}}{(1+x)(1+3x)^{5/3}} - A_2\frac{3+5x}{(1+3x)^{5/3}} \tag{3.1e}$$

Model II:

$$e^{\upsilon} = B_2(1+x)^2 \tag{3.2a}$$

$$e^{-\lambda} = \frac{2^n K}{(m+1)(m+2)}\frac{x}{(1+3x)^{2/3}}\{\frac{1}{2}+(m+2)x\}^{m+1} + 1 + A_2\frac{x}{(1+3x)^{2/3}} \tag{3.2b}$$

$$\frac{2Cq^2}{x^2} = K\,x[1+(1+N)x]^{\frac{(N-1)}{(N+1)}}\left[1+\frac{1}{2}(N+m)x\right]^m(1+x)^{(1-N)} \tag{3.2c}$$

$$\frac{\kappa}{C}P = \frac{K\{1+(2m+4)x\}^m}{2(m+1)(m+2)}\frac{(3m^2+19m+26)x^2+(m^2+5m+11)x+2}{(1+x)(1+3x)^{2/3}} + \frac{4}{(1+x)} + A_2\frac{1+5x}{(1+x)(1+3x)^{2/3}} \tag{3.2d}$$

$$\frac{\kappa}{C}\rho = -\frac{K}{2\,(m+1)(m+2)}\frac{\{1+(2m+4)x\}^m\{(21m^2+61m+38)x^3+(22m^2+66m+43)x^2+(5m^2+17m+14)x+1\}}{(1+x)(1+3x)^{5/3}} - A_2\frac{3+5x}{(1+3x)^{5/3}} \tag{3.2e}$$

Equation (3.1d), (3.1e) and (3.2d), (3.2e) constitute the equation of state.

4. Determination of Constants and Physical Quantities Using Boundary Conditions

4.1. Determination of the Arbitrary Constant A_2

To specify A_2 the boundary condition $P(r=R)=0$ can be utilized,

Model I:

$$A_2 = -\frac{K\{1+(m+2)X\}^m}{2(m+1)(m+2)}\frac{\left[(3m^2+19m+26)X^2+(m^2+5m+16)X+2\right]}{1+5X} - \frac{4(1+3X)^{2/3}}{(1+5X)}$$

Model II:

$$A_2 = -\frac{K\{1+(2m+4)X\}^m}{2(m+1)(m+2)}\frac{\left[(3m^2+19m+26)X^2+(m^2+5m+11)X+1\right]}{1+5X} - \frac{4(1+3X)^{2/3}}{(1+5X)}$$

where, $X = CR^2$.

4.2. The Constant B_2

The constant B_2 can be specified by the boundary condition $e^{\upsilon(R)} = e^{-\lambda(R)}$, which yields,

Model I:

$$B_2 = \frac{K}{(m+1)(m+2)}\frac{x}{(1+x)^2(1+3x)^{2/3}}\{1+(m+2)x\}^{m+1} + \frac{1}{(1+x)^2} + A_2\frac{x}{(1+x)^2(1+3x)^{2/3}}$$

Model II:

$$B_2 = \frac{K}{(m+1)(m+2)}\frac{x}{(1+x)^2(1+3x)^{2/3}}\{1+(2m+4)x\}^{m+1} + \frac{1}{(1+x)^2} + A_2\frac{x}{(1+x)^2(1+3x)^{2/3}}$$

4.3. Determination of the Total Charge to Radius Ratio

From Eq. (1.4), using $X = CR^2$, we obtain the charge to radius ratio

Model I:

$$\frac{Q^2}{R^2} = \frac{K}{2}\frac{X^2\{1+(m+2)X\}^m(1+3X)^{1/3}}{1+X} \tag{4.3a}$$

Model II:

$$\frac{Q^2}{R^2} = \frac{K}{2}\frac{X^2\{1+(2m+4)X\}^m(1+3X)^{1/3}}{1+X} \tag{4.3b}$$

4.4. Total Mass to Radius Ratio

Using the boundary condition iii) of Sect. 1 and with the help of Eq. (2.3) we can establish the equation of mass to radius ratio (compactness parameter).

Model I:

$$\frac{2M}{R} = -\frac{K\{1+(m+2)X\}^{m+1}}{(m+1)(m+2)}\frac{X}{(1+3X)^{2/3}} - A_2\frac{X}{(1+3X)^{2/3}} + \frac{Q^2}{R^2} \tag{4.4a}$$

Model II:

$$\frac{2M}{R} = -\frac{K\{1+(2m+4)X\}^{m+1}}{(m+1)(m+2)}\frac{X}{(1+3X)^{2/3}} - A_2\frac{X}{(1+3X)^{2/3}} + \frac{Q^2}{R^2} \tag{4.4b}$$

4.5. The Central and Surface Redshift z_c, z_s

The central and surface redshifts of the charged fluid sphere are given by,

$$z_c = \sqrt{e^{-\upsilon(0)}} - 1 = \frac{1}{\sqrt{B_N}} - 1$$

$$z_s = \sqrt{e^{-\upsilon(R)}} - 1 = \frac{(1+X)^{-\frac{N}{2}}}{\sqrt{B_N}} - 1$$

5. Physical Analysis

5.1. Pressure and Density Gradients

To analyze the analytical equation of state, obtained parametrically in equations. (3.1d), (3.1e) and (3.2d), (3.2e), we differentiate the pressure and density equations with respect to the auxiliary variable x. Due to the complicacy of those equations we prefer to choose the following particular case for which the pressure and density gradients become,

Solution:

Model I:

$$\frac{\kappa}{C}\frac{dP}{dx} = \frac{K\{1+(m+2)x\}^{m-1}}{2(m+1)(m+2)}\frac{P_n(x)}{(1+x)^2(1+3x)^{5/3}} + A_2\frac{2-10x^2}{(1+x)^2(1+3x)^{5/3}} - \frac{4}{(1+x)^2}$$

$$\frac{\kappa}{C}\frac{d\rho}{dx} = -\frac{K\{1+(m+2)x\}^{m-1}}{2(m+1)(m+2)}\frac{Q_n(x)}{(1+x)^2(1+3x)^{8/3}} + 10A_2\frac{1+x}{(1+3x)^{8/3}}$$

where,

$$\begin{aligned} P_n(x) = {} & (9m^4+78m^3+217m^2+220m+52)x^4 + (15m^4+134m^3+415m^2+532m+33m^2+59m+222)x^3 \\ & + (7m^4+60m^3+185m^2+244m+59m^2+177m+214)x^2 \\ & + (m^4+8m^3+23m^2+28m+25m^2+83m+78)x + 3m^2+9m+10 \end{aligned}$$

$$\begin{aligned} Q_n(x) = {} & (63m^4+330m^3+583m^2+388m+76)x^5 + (150m^4+824m^3+1581m^2+1229m+326)x^4 \\ & + (124m^4+706m^3+1476m^2+1318m+400)x^3 + (42m^4+252m^3+620m^2+674m+208)x^2 \\ & + (5m^4+32m^3+117m^2+178m+44)x + 7m^2+21m+2 \end{aligned}$$

Model II:

$$\frac{\kappa}{C}\frac{dP}{dx} = \frac{K\{1+(2m+4)x\}^{m-1}}{2(m+1)(m+2)}\frac{P_n(x)}{(1+x)^2(1+3x)^{5/3}} + A_2\frac{2-10x^2}{(1+x)^2(1+3x)^{5/3}} - \frac{4}{(1+x)^2}$$

5.2. Calculating Physical Quantities Using Surface/Central Density as Parameter

For a given surface density

Solution I:

Radius

$$R^2 = -\frac{1}{\kappa\rho_S}\left[\frac{K\,X}{2(m+1)(m+2)}\frac{\{1+(m+2)X\}^m\{(21m^2+61m+38)X^3+(22m^2+66m+42)X^2+(5m^2+17m+14)X+2\}}{(1+X)(1+3X)^{5/3}}+A_2\frac{X(3+5X)}{(1+3X)^{5/3}}\right]$$

Total Mass

$$M = \frac{R}{2}\left[-\frac{K\{1+(m+2)X\}^{m+1}}{(m+1)(m+2)}\frac{X}{(1+3X)^{2/3}}+\frac{K}{2}\frac{X^2\{1+(m+2)X\}^m(1+3X)^{1/3}}{1+X}-A_2\frac{X}{(1+3X)^{2/3}}\right]$$

Total Charge

$$Q = R\sqrt{\left(\frac{K}{2}\frac{X^2\{1+(m+2)X\}^m(1+3X)^{1/3}}{1+X}\right)}$$

Central density

$$\rho_C = -\frac{3X}{\kappa R^2}\left[\frac{K}{(m+1)(m+2)}+A_2\right]$$

Solution II:
Radius

$$R^2 = -\frac{1}{\kappa\rho_S}\left[\frac{K\,X}{2(m+1)(m+2)}\frac{\{1+(2m+4)X\}^m\{(21m^2+61m+38)X^3+(22m^2+66m+43)X^2+(5m^2+17m+14)X+1\}}{(1+X)(1+3X)^{5/3}}+A_2\frac{X(3+5X)}{(1+3X)^{5/3}}\right]$$

Mass

$$M = \frac{R}{2}\left[-\frac{K\{1+(2m+4)X\}^{m+1}}{2(m+1)(m+2)}\frac{X}{(1+3X)^{2/3}}+\frac{K}{2}\frac{X^2\{1+(2m+4)X\}^m(1+3X)^{1/3}}{1+X}-A_2\frac{X}{(1+3X)^{2/3}}\right]$$

Charge

$$Q = R\sqrt{\left[\frac{K}{2}\frac{X^2\{1+(2m+4)X\}^m(1+3X)^{1/3}}{1+X}\right]}$$

Central density

$$\rho_C = -\frac{3X}{\kappa R^2}\left[\frac{K}{2(m+1)(m+2)}+A_2\right]$$

5.3. Physical Analysis of the Models

Table 1a. *Some values of parameters* (m, K, X) *for which well-behaved charge fluid sphere can be generated using solution I*

m	$(K, X_{\max})$	A_2	B_2	$\frac{1}{c^2}\left(\frac{P}{\rho}\right)_c$	$\sqrt{\frac{1}{c^2}\left(\frac{dP}{d\rho}\right)_c}$	$\frac{2M(km)}{R(km)}$	$\frac{Q(km)}{R(km)}$	$\frac{Q(km)}{M(km)}$	z_c	z_s
0.025	(1.12, 0.48)	-3.4108	0.1750	0.11646	0.52641	0.865366	0.34551	0.79852	1.390	0.6152
0.05	(1.11, 0.47)	-3.3877	0.1796	0.11692	0.52482	0.858017	0.34008	0.792711	1.3957	0.6053
0.1	(1.1, 0.45)	-3.3566	0.1890	0.11669	0.521439	0.845084	0.33037	0.781863	1.2999	0.5862
0.5	(0.85, 0.351)	-3.0807	0.2505	0.12711	0.520182	0.735008	0.25979	0.706922	0.9978	0.4788
1	(0.62, 0.39)	-2.9523	0.2206	0.13149	0.531199	0.818074	0.308706	0.754713	1.1289	0.5316
5	(0.05, 0.154)	-3.1301	0.4785	0.092773	0.56844	0.509465	0.150312	0.590077	0.4456	0.2527
10	(0.003, .075)	-3.4184	0.6730	0.056708	0.562818	0.29086	0.071756	0.493407	0.2189	0.1338
20	(0.001,0.021)	-3.7968	0.8859	0.01784	0.552522	0.097314	0.020945	0.430468	0.0624	0.0406

Table 1b. *Maximum mass and the various physical variables of charged fluid spheres for given central density*

m	$(K, X_{\max})$	$\rho_s(\times10^{14}\,g\,cm^{-3})$							
		8.4				**9.5**			
		M(M⊙)		**R(Km)**		$\rho_{c,15}$		Q_{20}	
0.025	(1.12, 0.48)	2.87	2.703	9.873	9.284	2.56	2.89	3.657	3.72
0.05	(1.11, 0.47)	2.83	2.66	9.805	9.221	2.53	2.86	3.867	3.637
0.1	(1.1, 0.45)	2.75	2.589	9.686	9.108	2.46	2.79	3.711	3.49
0.5	(0.85, 0.351)	2.22	2.089	8.981	8.445	2.1	2.38	2.7.6	2.544
1	(0.62, 0.39)	2.40	2.26	8.731	8.209	2.4	2.71	3.126	2.939
5	(0.05, 0.154)	1.173	1.197	7.423	6.980	1.41	1.59	1.294	1.217
10	(0.003, 0.075)	0.6014	0.566	6.1456	5.779	1.09	1.23	0.5114	0.481
20	(0.001, 0.021)	0.123	0.1157	3.757	3.532	0.91	1.03	0.9125	0.858

Where central density $\rho_c = \rho_{c,15} \times 10^{15}\ g\,cm^{-3}$ and surface density $\rho_s = \rho_{s,14} \times 10^{14}\ g\,cm^{-3}$ and charge $Q = Q_{20} \times 10^{20}\ C$

Table 2a. *Some values of parameters* (n, K, X) *for which well-behaved charge fluid sphere can be generated using solution*

m	$(K, X_{\max})$	A_2	B_2	$\frac{1}{c^2}\left(\frac{P}{\rho}\right)_c$	$\sqrt{\frac{1}{c^2}\left(\frac{dP}{d\rho}\right)_c}$	$\frac{2M(km)}{R(km)}$	$\frac{Q(km)}{R(km)}$	$\frac{Q(km)}{M(km)}$	z_c	z_s
0.025	(1.12, 0.48)	-3.1465	0.1701	0.12251	0.53352	0.86839	0.34724	0.79973	1.3921	0.6163
0.05	(1.11, 0.47)	-3.1406	0.1791	0.1219	0.53039	0.8640	0.34349	0.79512	1.3628	0.6074
0.1	(1.1, 0.45)	-3.1396	0.1881	0.11965	0.524293	0.856848	0.336976	0.786548	1.3057	0.5901
0.5	(0.85, 0.351)	-3.0481	0.2467	0.11796	0.51208	0.78265	0.28593	0.73069	1.0133	0.4902
1	(0.62,0.35)	-3.0945	0.2395	0.1037	0.51927	0.87997	0.33283	0.75645	1.04327	0.5135
5	(0.05, 0.084)	-3.4257	0.6441	0.05594	0.558605	0.345015	0.092502	0.53622	0.2460	0.1494
10	(0.003, .039)	-3.6603	0.8041	0.03094	0.555909	0.175222	0.041052	0.468576	0.1152	0.0734
20	(0.001,0.0105)	-3.8933	0.9401	0.00914	0.550154	0.050459	0.010474	0.415161	0.0313	0.0206

Table 2b. *Maximum mass and the various physical variables of charged fluid spheres for given surface density*

m	$(K, X_{\max})$	$\rho_s(\times 10^{14}\ g\,cm^{-3})$							
		8.4				**9.5**			
		M(M⊙)		**R(Km)**		$\rho_{c,15}$		Q_{20}	
0.025	(1.12, 0.49)	2.78	2.614	9.4098	8.848	2.72	3.07	3.864	3.633
0.05	(1.11, 0.48)	2.75	2.589	9.3633	8.805	2.69	3.04	3.806	3.578
0.1	(1.1, 0.45)	2.671	2.115	9.264	8.711	2.58	2.92	3.62	3.404
0.5	(0.85, 0.357)	2.424	2.279	8.788	8.264	2.35	2.66	3.137	2.95
1	(0.524, 0.333)	2.246	2.112	8.469	7.964	2.2	2.49	2.818	2.65
5	(0.03, 0.098)	0.852	0.801	6.652	6.256	1.19	1.35	0.799	0.752
10	(0.005, 0.035)	0.2496	0.235	4.658	4.380	0.956	1.08	0.199	0.187
15	(0.001, 0.019)	0.105	0.098	3.595	3.381	0.90	1.02	0.074	0.069

Table 3. *Physical values of energy density and pressure for different strange stars Model I*

Strange star candidate	**(m, K, X)**	**M(M⊙)**	**R(Km)**	$P_{c,35}$	$\rho_{c,15}$	$\rho_{s,14}$
PSR J1614-2230	(0.1, 0.63,0.31)	1.97	9.69	2.04	1.49	10.03
Vela X-1	(0.1, 0.75, 0.25)	1.78	9.56	1.51	1.561	9.76
Her X-1	(0.1, 0.2, 0.12)	0.85	8.10	0.788	0.92	0.767
Cen X-3	(0.1, 0.41, 0.23)	1.49	9.178	1.61	1.24	9.22
EXO 1785-248	(0.1, 0.245, 0.21)	1.30	8.849	1.58	1.21	9.02

For the particular set of values of (m, K, X) for which the fluid distribution satisfies the following inequalities $P(r) \geq 0,\ \rho(r) > 0$, $dP/dr < 0,\ d\rho/dr < 0$ and $dP/d\rho \geq 0$ and the speed of sound satisfies $0 \leq \sqrt{dP/d\rho} \leq 1$ and monotonically decreasing with increasing radius are reported in Table 1a and 2a. A fluid sphere satisfying these inequalities will be termed as *well-behaved*. Though there is no explicit relation in among m , K and X , these inputs various charged fluid spheres can be generated. The mass and corresponding radius of compact charged fluid spheres, obtained by specifying one of the following: i) central density or, ii) surface density, is reported in Tables 1b and 2b. For a particular choice of stellar surface density $\rho_s = 8.4 \times 10^{14}\ g\,cm^{-3}$, the total mass and other physical quantities are calculated and numerical results have been reported in the Table 1a.[2]

Model I:

For $m = 5$ the range of values $K \geq 0.05$, $X \leq 0.154$ are obtained over which the fluid distribution satisfies the above inequalities. Numerical investigation shows that X decreases as K increases. The maximum values of compactness parameter is obtain $(2M/R)_{\max} = 0.509465$, using Eq. (4.4a) at $K = 0.05$, $X_{\max} = 0.154$. Corresponding to the values of *K* and *X*, the total charge to radius ratio, and total charge to total mass ratio are $Q/R = 0.150312$ and $Q/M = 0.590077$ using Eq. (4.3a). We find out the total mass and other physical quantities are calculated as M = 1.173 $M_\odot$, R = 7.423 Km , $\rho_C = 1.41 \times 10^{15}\ g\,cm^{-3}$ and $Q = 1.294 \times 10^{20}\ C$ for choosing the stellar surface density $\rho_s = 8.4 \times 10^{14}\ g\,cm^{-3}$ as parameter.

Model II:

[2] The following physical constants, in their conventional values, have been used for the numerical calculation:

$C = 1 = 2.997 \times 10^{8}\ ms^{-1},\ \ G = 1 = 6.674 \times 10^{-11}\ Nm^{2}kg^{-2}\ \ M_\Theta = 1.486\ Km = 2 \times 10^{30}\ Kg$

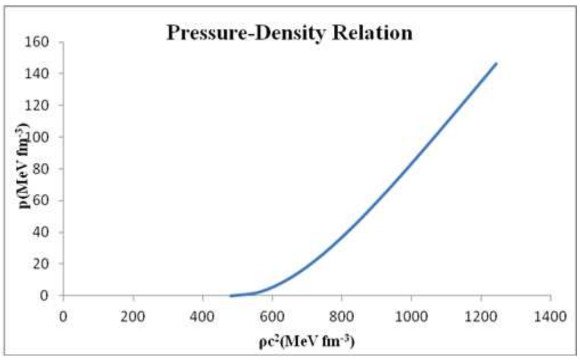
For arbitrary value of m = 0.5, $K = 0.85$, $X_{\max} = 0.375$, corresponding to these values of K and X the compactness parameter, the speed of sound is 0.512005 , total charge to radius ratio and total charge to total mass ratio are found to be $(2M/R)_{\max} = 0.735008$, $Q/R = 0.25979$ and $Q/M = 0.706922$. In order to choosing stellar surface density $\rho_s = 9.5\times10^{14}\ g\,cm^{-3}$ as parameter the mass and other physical values comes out to be M = 2.089$M_\odot$, R = 8.264 Km, $\rho_C = 2.66\times10^{15}\ g\,cm^{-3}$.

The maximum mass of charge star depends on the set of lowest values of K and corresponding set of highest values of X. the values of K and X have been plugged in simultaneously as to satisfy $dP/dx < 0$, $d\rho/dx < 0$ and the speed of sound satisfy $0 \le \sqrt{dP/d\rho} \le 1$ and monotonically decreasing with increasing radius.

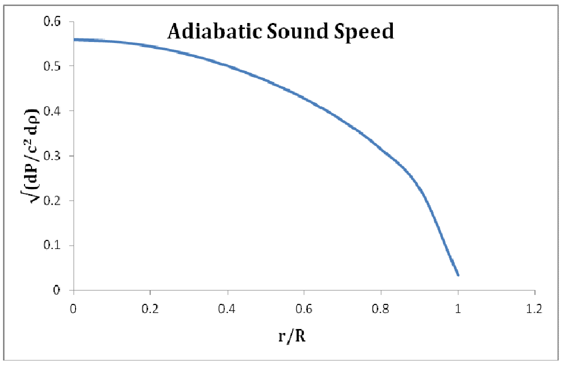
The behaviors of various physical variables in the interior of the star have been investigated and found regular and well behaved throughout the fluid sphere. It has been observed that the speed of sound always remain less than the speed of light and the condition of causality is satisfied. For large ρ (at center), $\Gamma > 4/3$ and there is a minimum value of $\rho = \rho_s$ the surface value below which Γ becomes infinitely large.

6. Model for Some Well Known Strange Star Candidates

From last few decades astrophysicist has been analysis not only theoretical but also observational relativistic stellar objects for estimating mass and radius of their known compact objects such as PSR J1903+327, X-ray pulser Her X-1, X-ray burster 4U 1820-30, RX J185635-3754 are not compatible with the standard neutron star models [50,51]. More recent review is found in Weber [53]. Base on the analytic model development so far, to get an estimate of the range of various physical parameters of some potential strange star candidates we have calculated the values of the relevant physical quantities, such as central pressure, and central/surface density, by using the refined mass and predicted radius of 5 pulsars recently reported in Gangopadhyay et al. [52]. The values are reported in Table 3



(a)



(b)

Fig 1. *(a) Pressure-density profile and (b) Behavior of speed of sound for a charged fluid sphere for potential strange star candidate PSR J1614−2230 with mass 0.673M⊙ and radius 6.146 km generated by the input (m, K, X) = (10,0.003, 0.075)*

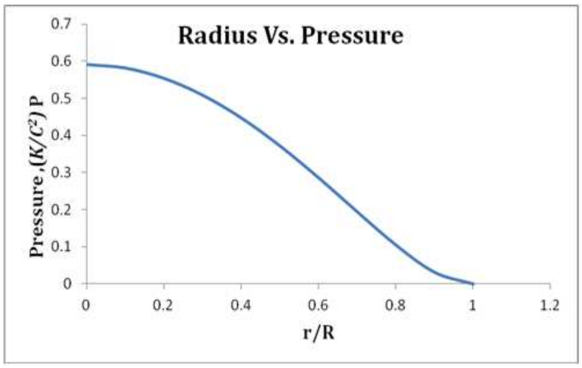


Fig 2. *Behavior of the isotropic pressure* P

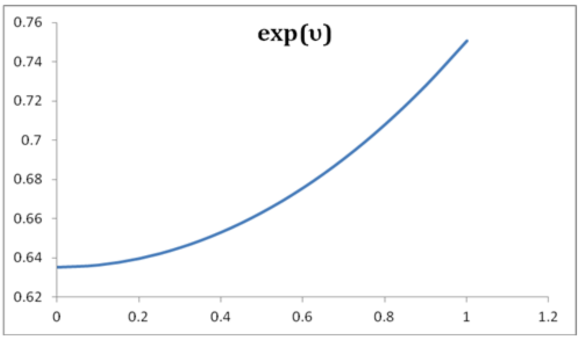


Fig 3. *Metric function* e^{υ}

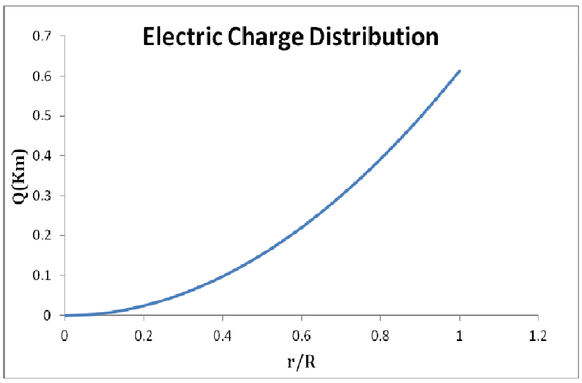


Fig 4. *Electric charge distribution within the same fluid sphere*

7. Conclusions

In this work we have demonstrated some new interior solutions of Einstein-Maxwell field equations for a static spherically symmetric distribution of perfect fluid with particular forms of charge distribution to construct electrically charge stellar models. Our analytical analysis shows that, in the presence of charge, the solutions satisfy all the physical requirements to construct physically acceptable electrically charge stellar models. The charged analogues of Tolman IV and VII models, obtained by Gupta and Maurya [30], and Kiess [54], as the neutral ones, exhibit the physical features required for the construction of physically realizable relativistic compact stellar structure. However, various authors usually have chosen 2×10^{14} g cm^{-3} as stellar surface density to calculate the mass and radius of the charged fluid sphere which may have given rise to the stellar configuration as massive as 4-6 $M_\odot$ with much lower central density. Such massive configuration may not serve as a realistic model for a strange quark star.

A wide range of values, constant parameters, are allowed to specify the maximum mass of charged fluid spheres. Numerical studies show that the solutions obtained in this work can generate charged fluid sphere with maximum mass 2.87 $M_\odot$, radius 9.409 km, central and surface densities on the order 3.07×10^{15} g cm^{-3} and 9.5×10^{14} g cm^{-3} respectively, with electric charge on the order 10^{20} C. Moreover, the speed of sound is obtained $\sim 1/\sqrt{3}$ at the center and remains almost the same throughout most of the fluid sphere. This behavior is like MIT bag model.

An analytical stellar model with such physical features is most likely to present an approximated realistic model of strange quark star. And hence the analytical EOS given by our models, besides the usual linear EOS based on phenomenological MIT bag model, could play a significant role in the description of internal structure of electrically charged *bare* strange quark stars.

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